Non-uniformly correlated partially coherent pulses

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Abstract: We consider partially coherent plane-wave pulses with non-uniform correlation distributions and study their propagation in linear second-order dispersive media. Particular models for coherence functions are introduced both in time and frequency domains. It is shown that the maximum peak of the pulse energy can be accelerating or decelerating and also self-focusing effects are possible due to coherence-induced propagation effects.

OCIS codes: (030.1640) Coherence; (320.5550) Pulses; (030.6600) Statistical optics.

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1. Introduction

Stochastic fluctuations affect inevitably the behavior of many types of light fields. While the optical coherence theory has traditionally concentrated on studying such effects occurring in stationary fields [1], similar phenomena concern also optical pulses that play a crucial role, for example, in optical telecommunications [2]. The influence of the coherence properties on the evolution of pulses upon propagation has been studied in various media, including linear [3, 4] and nonlinear [5] dispersive media, coherent linear absorbers [6] and scattering random media [7]. A typical conclusion found in linear dispersive media is that decreased coherence degrades the performance of optical systems by leading to faster temporal spreading of pulses.

In most of the studies concerning coherence properties of pulses, the basic Gaussian Schell-model (GSM) pulses [8] have been chosen to represent the field. However, it is possible to have ensembles of pulses with correlation distributions that do not follow the simple Schell model where the degree of coherence between two instants of time is assumed to depend only on their temporal separation. For instance, it has been shown to be possible to introduce partially coherent pulses that propagate as dispersion-free localized waves [9] or maintain their shape upon propagation in resonant linear absorbers [10]. The coherence distributions may also differ from the basic Gaussian Schell-model due to the noise properties of the pulsed source [11], or their shape may be distorted upon propagation in nonlinear media [5]. Besides, a scheme for experimental generation of pulses with predetermined coherence distributions has been suggested [12].

Recently we introduced a model for optical scalar beams with new type of non-uniform spatial correlations [13]. The modified coherence distribution was shown to lead to self-focusing and lateral shifts of the beam maxima in free-space propagation. The propagation of such beams has been studied also in random media [14] and the model has been extended for electromagnetic beams [15]. Due to the space-time analogy between paraxial propagation of partially coherent beams and the evolution of partially coherent plane-wave pulses in linear second-order dispersive media [16], it can be expected that corresponding coherence-induced effects could be seen also in the temporal evolution of pulses.

In this paper we apply a model similar to that of Ref. [13] to the correlation functions of partially coherent pulses. The basic theory for defining non-uniform correlation functions is discussed in Sect. 2. In Sects. 3 and 4 the model is introduced in time and frequency domains, respectively, and the propagation characteristics of the resulting pulses in linear dispersive media are studied. Finally, the conclusions are presented in Sect. 5.

2. Theory

Let us consider partially coherent plane-wave pulses propagating in the direction of the positive z-axis. In the space-time domain the coherence properties of the pulses can be defined by their mutual coherence function $\Gamma(t_1,t_2) = \langle U^*(t_1)U(t_2)\rangle$, where $U(t)$ represents the complex analytic signal of pulse realizations at time t, and the angle brackets denote ensemble average. The average intensity of the partially coherent pulses is then obtained as $I(t) = \Gamma(t,t)$. The degree of coherence of the pulses is defined as the normalized form of the mutual coherence function, $\gamma(t_1,t_2) = \Gamma(t_1,t_2)/\sqrt{I(t_1)I(t_2)}$. Its absolute value varies between 0 and 1, the extremes corresponding to fully incoherent or coherent fields, correspondingly.

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In general, since the exact forms of all the pulse realizations are not known, the correlation properties of the pulse ensemble must be described by choosing a suitable mathematical form for the mutual coherence function. In order to be a proper correlation function, $\Gamma(t_1,t_2)$ must correspond to a non-negative definite kernel [1]. As has been shown for correlation functions in the spatial domain [17], a necessary and sufficient condition for the non-negative definiteness is that the mutual coherence function can be represented in the form

$$\Gamma(t_1,t_2) = \int p(v)V^*(t_1,v)V(t_2,v)\mathrm{d}v, \tag{1}$$

where V is an arbitrary kernel and p is an arbitrary non-negative function. Starting from this condition, a wide variety of different temporal correlation distributions can be defined for partially coherent pulses.

The coherence properties of pulses can be studied also in the space-frequency domain through the cross-spectral density $W(\omega_1,\omega_2)$ that measures the correlations between two angular frequencies ω_1 and ω_2. Here, we present all the frequencies with respect to the central frequency of the pulses ω_0, i.e. the real frequencies are obtained by $\omega' = \omega + \omega_0$. W is connected to Γ through the generalized Wiener-Khintchine theorem [1],

$$W(\omega_1,\omega_2) = \frac{1}{(2\pi)^2}\iint \Gamma(t_1,t_2)\exp[-i(\omega_1 t_1 - \omega_2 t_2)]\mathrm{d}t_1\mathrm{d}t_2, \tag{2}$$

and to the average spectrum of the pulses as $S(\omega) = W(\omega,\omega)$. Further, the degree of spectral coherence is given analogously to the corresponding time-domain function by $\mu(\omega_1,\omega_2) = W(\omega_1,\omega_2)/\sqrt{S(\omega_1)S(\omega_2)}$. Inserting Eq. (1) into (2) and assuming that the kernel $V(t,v)$ has inverse Fourier transform with respect to the variable t, $\tilde{V}(\omega,v)$, we readily get

$$W(\omega_1,\omega_2) = \frac{1}{(2\pi)^2}\int p(v)\tilde{V}^*(\omega_1,v)\tilde{V}(\omega_2,v)\mathrm{d}v. \tag{3}$$

This proves that for partially coherent fields defined by Eq. (1) the non-negative definiteness condition is naturally fulfilled also in the frequency domain. On the other hand, it is equally possible to first define the correlation function in the space-frequency domain by choosing kernels $\tilde{V}$ and weight function p in Eq. (3). It should be noted that even though Eqs. (1) and (3) are formally analogous, they do not generally result in a pair of corresponding coherence functions that have similar forms in the time and frequency domain.

Equations (1) and (3) resemble the elementary-pulse representations introduced for partially coherent fields in temporal and spectral domains [18, 19]. However, in the elementary-pulse representations the coherence functions are defined as an integrated continuum of shifted identical pulses, i.e., the kernel functions in the above equations are assumed to be of the form $V(t,v) = V(t-v)$ or $\tilde{V}(\omega,v) = \tilde{V}(\omega - v)$. Allowing the elementary fields to change as a function of v we can define more versatile coherence functions that do not necessarily follow the basic Schell-model.

Based on the theory presented above, various different coherence functions with non-uniform correlation distributions can be defined. In the following sections, we concentrate on a basic case where both the weight and kernel functions are Gaussian. Starting from such model first in the time domain and then in the frequency domain, we study the effects of varying coherence distributions on pulse propagation in second-order dispersive media.

3. Non-uniformly correlated temporally Gaussian pulses

First, we consider partially coherent pulses with non-uniform correlation function obtained assuming Gaussian weight and kernel functions in temporal domain. Inserting

$$p(v) = (\sqrt{\pi}a)^{-1}\exp\left(-v^2/a^2\right) \tag{4}$$

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and

$$V_0(t,v) = \exp\left(-\frac{t^2}{2T_0^2}\right)\exp\left[-i\kappa v(t-t_c)^2\right], \tag{5}$$

where a and κ are positive real constants and t_c is a real constant, into Eq. (1) we get mutual coherence function

$$\Gamma_0(t_1,t_2) = \exp\left(-\frac{t_1^2+t_2^2}{2T_0^2}\right)\exp\left\{-\frac{[(t_2-t_c)^2-(t_1-t_c)^2]^2}{T_c^4}\right\}, \tag{6}$$

where $T_c = \sqrt{2/\kappa a}$. If the latter exponential term in Eq. (5) was replaced by $\exp(-ivt)$, we would get conventional GSM pulses. Based on the definition of the kernel function, Eq. (5), the correlation function is obtained as a weighted superposition of linearly chirped Gaussian modes for which the amount and sign of chirp changes as a function of the variable v. In addition, the chirping is decentered compared to maximum intensity of the modes by the time constant t_c. This leads to a situation where the correlations are highest around t_c. An example of the shape of the mutual coherence function and the corresponding degree of coherence are illustrated in Fig. 1. The intensity of the defined partially coherent pulses has Gaussian shape, as can easily be seen by definition from Eq. (6).

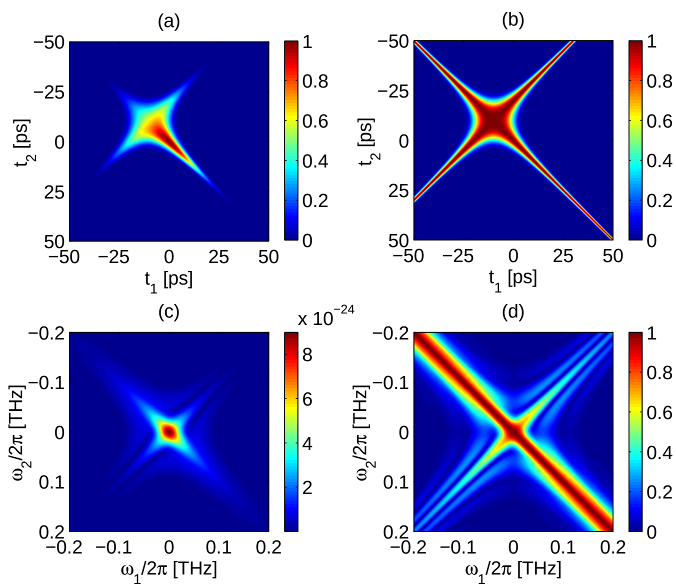

Fig. 1. (a) The mutual coherence function $\Gamma(t_1,t_2)$ and (b) the degree of coherence $\gamma(t_1,t_2)$ given by Eq. (6) with $T_0 = 15$ ps, $T_c = 10$ ps and $t_c = -10$ ps. (c) The corresponding cross-spectral density $W(\omega_1,\omega_2)$ and (d) degree of spectral coherence $\mu(\omega_1,\omega_2)$ calculated numerically by Eq.(2).

In the frequency domain we can solve the corresponding modes of Eq. (3) by taking the inverse Fourier transform of Eq. (5), and using a well-known integration formula (Eq. 3.323-2 in page 337 of Ref. [20]):

$$\tilde{V}_0(\omega,v) = [\pi A(v)]^{-1/2}\exp\left[-(\omega+2\kappa v t_c)^2/A(v) - i\kappa v t_c^2\right], \tag{7}$$

where $A(v) = 2/T_0^2 + 4i\kappa v$. Even though these modes still are chirped and have Gaussian shapes, each of them is spectrally shifted depending both on the variable v and the parameter t_c. Further, because of the more complicated dependence on v, we cannot get an analytical

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closed form solution for the integral of Eq. (3). However, $W(\omega_1, \omega_2)$ can be easily calculated numerically using Eq. (2). The resulting cross-spectral density and the degree of spectral coherence of the partially coherent pulses are also illustrated in Fig. 1, which shows a clear difference to the corresponding coherence functions in the time domain. Unlike the intensity, the spectrum is not Gaussian but it has a slightly sharper line shape.

For studying the coherence-induced propagation effects, we assume now a medium characterized by the second-order dispersion coefficient β_2 [2], and study the propagation of pulses whose coherence function at the starting plane $z = 0$ is the same as in Fig. 1. The basic methods for modeling the propagation of the coherence functions in dispersive media are described in several articles [3, 4, 16]. Unfortunately, the propagated intensity distribution of non-uniformly correlated pulses cannot be expressed analytically in closed form. Our numerical simulation results for normally dispersive media are shown in Fig. 2. The intensity distribution of the pulses follows a skewed trajectory in the (I, z)-plane, which means that the maximum peak is accelerating and also focused and increased for a specific range of propagation distances. Analogously to the similar beams in the spatial domain [13], the maximum temporal shift of the peak equals approximately the decentering parameter t_c of the coherence function.

If we assume anomalously dispersive media with negative β_2, the trajectory of the intensity distribution is reversed and the pulse maximum is thus decelerating. Similar change is observed if we choose a positive value for the parameter t_c in normally dispersive media. Likewise, a positive t_c in anomalously dispersive media leads to similar accelerating as shown in Fig. 2. Thus, by choosing properly the parameter t_c, we can obtain accelerating or decelerating behavior in both types of dispersive media.

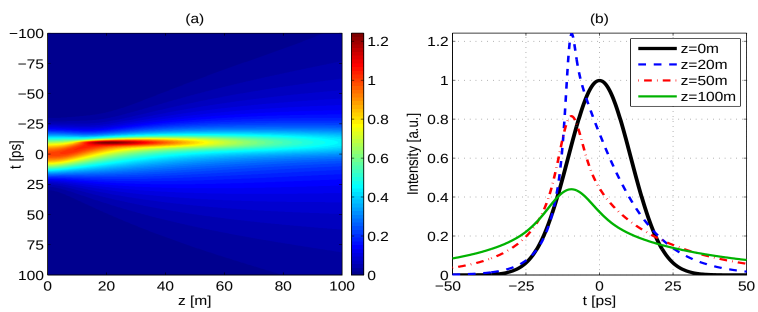

Fig. 2. (a) Evolution of the intensity distribution $I(t)$ of pulses given at $z = 0$ by Eq. (6) with $T_0 = 15$ ps, $T_c = 10$ ps and $t_c = -10$ ps in a medium with $\beta_2 = 50$ ps^2/km. (b) Pulse shapes at selected propagation distances. The time coordinate is measured in the reference frame moving at the group velocity of the pulse.

4. Non-uniformly correlated spectrally Gaussian pulses

When the non-uniform correlation function is specified as in Eq. (6) in the time domain, the propagation characteristics of the pulses in second-order dispersive media are analogous to the beams discussed in Ref. [13] as shown in the previous section. On the other hand, it is possible to define a similar form for the cross-spectral density of the partially coherent pulses in the frequency domain. In that case some differences in the pulse evolution in the same media are observed as shown below.

Let us now assume that the cross-spectral density is given by Eq. (3) with the weight function

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of Eq. (4) and modes

$$\tilde{V}_0(\omega,v) = \exp\left(-\frac{\omega^2}{2\Omega_0^2}\right)\exp\left[-ibv(\omega-\omega_c)^2\right], \tag{8}$$

that correspond to a spectral representation linearly chirped Gaussian pulses with the chirping centered around angular frequency ω_c. Identically to the previous section, we get

$$W_0(\omega_1,\omega_2) = \exp\left(-\frac{\omega_1^2+\omega_2^2}{2\Omega_0^2}\right)\exp\left\{-\frac{[(\omega_2-\omega_c)^2-(\omega_1-\omega_c)^2]^2}{\Omega_c^4}\right\}, \tag{9}$$

where $\Omega_c = \sqrt{2/ba}$. In the temporal domain the modes given by the Fourier transform of Eq. (8) are now analogous to Eq. (7) and the shapes of the correlation functions are qualitatively similar to those shown in Fig. 1 but switched between the spectral and temporal domains. Thus, now the spectrum has Gaussian shape and the intensity distribution differs from it.

The propagation of the pulses given by Eq. (9) in the same dispersive medium as in the previous section is illustrated in Fig. 3. In contrast to the previous example, now the pulse maximum is decelerating in the normally dispersive media and there is no increase in the pulse peak intensity upon propagation. Furthermore, the temporal shift is not restricted to a specific value, while the parameter ω_c affects to the angle at which the peak intensity is directed. However, when the pulses spread upon further propagation, their maximum finally returns to the center. Similar considerations as above about changing the sign of parameter ω_c or the type of dispersion for controlling which way the maximum is shifted are valid also in this case.

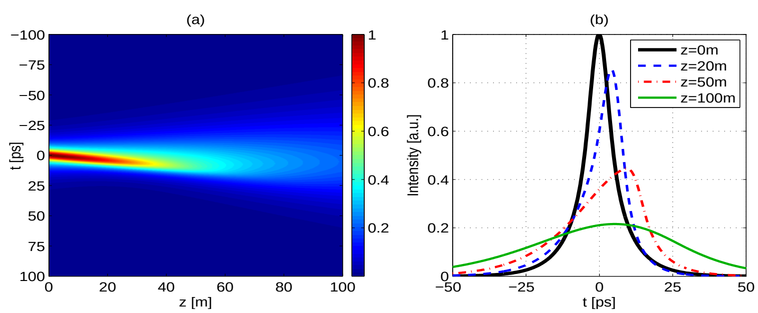

Fig. 3. (a) Evolution of the intensity distribution $I(t)$ of pulses given at $z = 0$ by Eq. (9) with $\Omega_0 = 0.3$ THz*rad, $\Omega_c = 0.3$ THz*rad and $\omega_c = -0.2$ THz*rad in a medium with $\beta_2 = 50$ ps^2/km. (b) Pulse shapes at selected propagation distances. The time coordinate is measured in the reference frame moving at the group velocity of the pulse.

5. Conclusions

We have studied the propagation of pulses with non-uniform correlation distributions in second-order dispersive media. The modified coherence properties lead to various changes upon propagation, including possible self-focusing and acceleration or deceleration of the intensity maximum of the pulses. Similar effects can be obtained both in normally and anomalously dispersive media. Our results imply that tailoring the coherence distributions offers a novel way to manipulate the propagation characteristics of pulsed fields.

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