

# Self-mixing interferometer: analysis of the output signals

Enrico M. Randone and Silvano Donati

Università di Pavia, Dipartimento di Elettronica, Via Ferrata 1, 27100, Pavia, Italy  
[enrico.randone@unipv.it](mailto:enrico.randone@unipv.it) [silvano.donati@unipv.it](mailto:silvano.donati@unipv.it)

**Abstract:** This paper presents the calculation of amplitude and relative phase of signals at the three outputs available in a self-mixing interferometer, i.e. the front output on the target side, the back output on the rear of the chip, and, for diode laser, the junction voltage output. Front and rear outputs are observed to be in phase for the He-Ne laser while they are in phase opposition in the diode laser. This discrepancy can now be explained theoretically. It will also be shown how the junction voltage output is always in phase opposition with respect to the rear output. Experimental measurements were carried out on two sources: a laser diode and a He-Ne laser, validating the calculations.

©2006 Optical Society of America

OCIS codes: (120.3180) Interferometry; (230.0250) Optoelectronics

---

## References and links

- 1 S. Donati: "Laser interferometry by induced modulation of the cavity field," J. Appl. Phys. **49**, 495-497 (1978).
- 2 S. Donati: Electrooptical instrumentation, (Prentice Hall, 2004), Chap. 2.2.
- 3 S. Donati, G. Giuliani, and S. Merlo: "Laser diode feedback interferometer for measurement of displacement without ambiguity," IEEE J. Quantum Electron. **31**, 113-119 (1995).
- 4 S. Shinoara, H. Naito, H. Yoshida, H. Ikeda, and M. Sumi: "Compact and versatile self-mixing type semiconductor laser doppler velocimeter with direction discrimination circuit," IEEE Trans. Instrum. Meas. **38**, 574-577 (1989).
- 5 W. Wang, W. J. O. Boyle, K. T. W. Grattan, and A. W. Palmer: "Self-mixing Interference in a diode laser for optical sensing applications," IEEE J. Lightwave Technol. **12**, 1577-1587 (1992).
- 6 S. Donati, and S. Merlo, "A PC-interfaced, compact laser-diode feedback interferometer for displacement measurements," IEEE Trans. Instrum. Meas. **45**, 942-947 (1996).
- 7 M. Norgia, S. Donati, and D. d'Alessandro, "Interferometric measurements of displacement on a diffusing target by a speckle-tracking technique," IEEE J. Quantum Electron. **37**, 800-806 (2001).
- 8 T. Bosch, N. Servagent, and S. Donati, "Optical feedback interferometry for sensing applications," Opt. Eng. **40**, 20-27 (2001).
- 9 S. Donati, and C. Mirasso, eds., "Optical Chaotic Cryptography," IEEE J. Quantum Electron. **38**, 1338 (2002).
- 10 R. Lang, and K. Kobayashi: "External optical feedback effects on semiconductor laser properties," IEEE J. Quantum Electron. **16**, 347-355 (1980).
- 11 R. Juskaitis, N. P. Rea, and T. Wilson: "Semiconductor laser confocal microscopy," Appl. Opt. **33**, 578-584 (1994).
- 12 R. H. Webb, and F. J. Rogomentich: "Microlaser microscope using self-mixing detection for confocality," Opt. Lett. **20**, 533-535 (1995).
- 13 see Ref.2, Sect. A1.1, A1.2.
- 14 M. Kumar, J. T. Boyd, H. E. Jackson, and B. L. Weiss: "Birefringent properties of GaAlAs multiple quantum well planar optical waveguides," IEEE J. Quantum Electron. **28**, 1678-1689 (1992).

---

## 1. Introduction

The principle of self mixing interferometer was demonstrated in a He-Ne laser source [1] about 30 years ago by one of the Authors, but it was only in the nineties [2-5] that the method underwent a sharp technical development with the advent of semiconductor lasers suitable to self-mixing. Since then, the method has increasingly gained acceptance and, with the new successful applications reported in the last years, the potentialities of the approach and the

substantial advantages of the self-mixing configuration, with respect to conventional interferometers, have been confirmed [2-7].

In addition the theory of operation of self-mixing can be regarded as well established, both at weak and moderate levels of injection [2-8] as well as at the strong level at which optical chaos is generated [9].

However, some features of operation still deserve a theoretical explanation. The feature considered in this paper is about the outputs of the self-mixing signals carrying the interferometric information, in form of a cosine function,  $\cos 2ks$  [or of a distorted cosine,  $F(2ks)$  at middle-high injection], where  $s$  is the path length external to the laser and  $k = 2\pi/\lambda$  is the wavenumber.

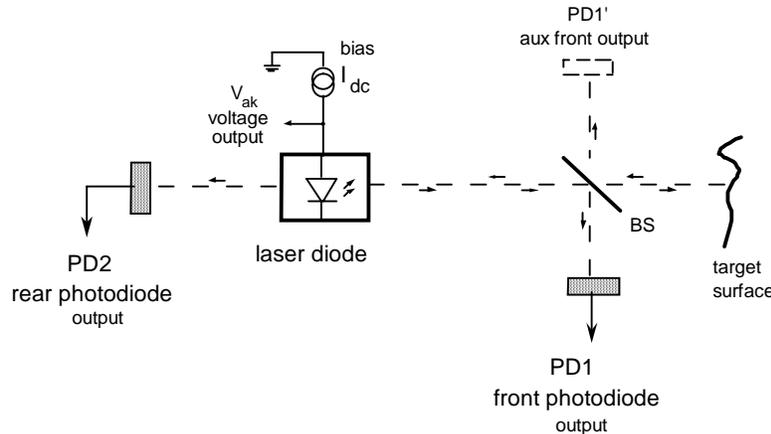


Fig. 1. Different pickups of the output signal from a self-mixing interferometer: from rear and front mirror, and from the junction voltage of the laser diode.

As it is well known, we can take the optical output of the interferometer from two sides (Fig. 1): i) from a photodiode PD2 placed behind the laser chip, opposite to the target side, usually the monitor photodiode provided by the manufacturer for monitoring the emitted power (this is the most commonly used and most suitable output in applications), and ii) from a photodiode PD1 collecting, by a beamsplitter, a fraction of the front output aimed to the target (PD1 can be placed everywhere along the propagating beam). Additionally, we may use, iii) a third output, that of the junction voltage  $V_{ak}$  across the laser diode, which is intended to be biased by a current source  $I_{dc}$  (in a He-Ne laser, this output is a pump-related quantity).

As it was noted in a seminal paper by Grattan et al. [5], the front and rear outputs in semiconductor laser diodes are seen experimentally to be in phase opposition (or, phase shifted by  $\pi$ ) in normal biasing conditions.

Contrary to this, in the He-Ne laser interferometer, the Author [1, 3] and other researchers, had always noticed the phase concordance of the front and rear outputs (or, zero phase shift).

In Ref. [5], the following conjecture was developed to explain the phase opposition. As the wavevectors  $\underline{k}_1$  and  $\underline{k}_2$  of fields at the two mirror outputs are oppositely directed, that is  $\underline{k}_1 = -\underline{k}_2$ , their scalar products with the target displacement  $\underline{s}$  has opposite sign. Now, the feedback in the Lang and Kobayashi equation [10] giving the self-mixing modulation is introduced by a term proportional to  $\underline{k}\underline{s}$ , and thus, the self-mixing signals from the two mirror outputs shall be in phase opposition.

The conclusion is a fallacy: the two supposedly opposite terms  $\underline{k}_1\underline{s}$  and  $\underline{k}_2\underline{s}$  interact with the *same* (and single) cavity field, and they cannot generate two different signals modulated onto the same carrier. Both outputs derive from the same internal field, so only some extra field added to one output can bring the two outputs to become opposite in phase. This is just

what is found in this paper: it is the reflection at the output mirror of returning field to give a contribution opposite to that coming out from the laser. The reflected field adds vectorially to the transmitted field exiting from the mirror. Then, depending on the relative amplitudes, the total field may or may not change sign with respect to the internal field. As it will be calculated in the next section, the reflected contribution increases respect to the transmitted contribution as the gain increases above the laser threshold. Only in semiconductor lasers the increase is large enough to produce the sign reversal, whereas in a He-Ne laser a gain large enough is never attained. This explains the different behavior of He-Ne lasers versus semiconductor lasers.

## 2. Analysis

Let us consider  $E_0$ , the electric field just before the exit of the output mirror and directed outward the laser, and the roundtrip in the cavity gain as well as the external back-and-forth path to the target (Fig. 2).

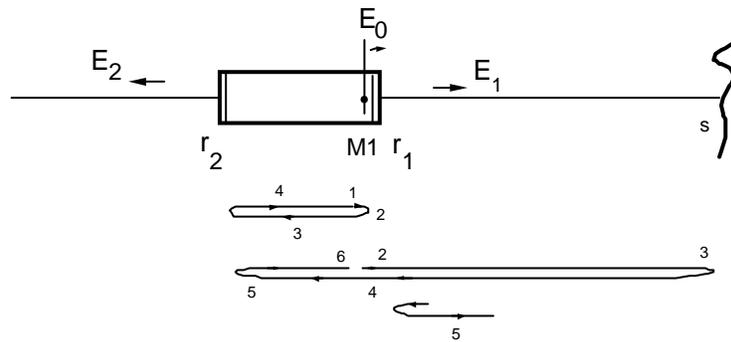


Fig. 2. Electric field  $E_0$  has unity gain along path 1-4, and the self-mix contribution comes from propagation on the 1-6 (bottom) path. Another contribution to  $E_1$  comes from the reflection at mirror M1, point 4 of the bottom feedback path.

The perturbation  $\Delta E$  collected by the back-reflection from the external target, following the round trip on path 1-6 in Fig. 2, is easily written as:

$$\Delta E = E_0 i t_1 e^{2iks} A i t_1 e^{\gamma L + 2ikL} r_2 = E_0 (-t_1^2) e^{2iks} A/r_1 \quad (1)$$

where:

$t_1, t_2$  (and  $r_1, r_2$ ) are the field transmissions (and reflections) coefficients at mirrors 1 and 2. Later, we will use also the power coefficients,  $T_1 = t_1^2, R_1 = r_1^2$ , etc...;

$i$  is the  $90^\circ$  phase shift added at each mirror transmission (respect to  $0^\circ$  of the reflection) and at the target back-diffusion (we should account for a phase difference  $\psi$  between reflected and incoming fields, and of  $90^\circ - \psi$  between transmitted and incoming fields, but it is just their sum,  $90^\circ$ , that actually appears in the final result);

$e^{2iks}$  is the propagation term for the path to target and back;

$A$  is the (complex) field attenuation introduced by the target, including diffusion at the target surface;

$e^{\gamma L}$  is the active medium gain (on the round trip  $2L$ , for the field amplitude), equal to  $1/r_1 r_2$  because in the unperturbed regime of oscillation it shall be  $|r_1 r_2 e^{\gamma L + 2ikL}| = 1$ ;

$\gamma$  is the power gain per unit length of the active medium;

$e^{2ikL}$  is the phase of the roundtrip loop internal to the laser, with  $|e^{2ikL}| = 1$ , in the unperturbed regime of oscillation;

Looking at Fig. 2, we can write the outputs  $E_1$  and  $E_2$  from the front and back mirrors as:

$$E_1 = E_0 i t_1$$

$$E_2 = E_0 r_1 e^{\gamma L/2 + iks} \quad ; \quad t_2 = E_0 \sqrt{(r_1/r_2)} \quad ; \quad t_1 \quad (2)$$

where the last equation follows from  $e^{\gamma L/2 + iks} = 1/\sqrt{(r_1 r_2)}$  in the unperturbed regime of oscillation;

The extra field  $\Delta E$  added by the external back-reflection is the perturbation term in the Lamb [1] (or Lang-Kobayashi [4]) equation, written as:

$$(d/dt) E = [\alpha - \beta E^2 - \Gamma] E + (c/2L) \text{Re}\{\Delta E\} \quad (3)$$

where:  $\alpha$  is the linear field gain per unit time (equal to  $\gamma c/2$  in term of the power gain per unit length  $\gamma$  and of the light velocity  $c$ ),  $\beta$  is the gain saturation coefficient,  $\alpha - \beta E^2$  is the effective gain and is the usually accepted approximation of the more rigorous  $\alpha/(1+\beta E^2)$ ,  $\Gamma$  is the field loss per unit time and is equal to  $-(\ln r_1 r_2) (c/2L)$  when mirrors dominate,  $L$  is the laser cavity optical length ( $n_{\text{eff}} L$  when the effective index is considered) and,  $\text{Re}\{\Delta E\}$  (equal to  $-A [E_0 t_1^2 / r_1] \cos 2ks$ , as from Eq. (1)) is the self-mixing or induced modulation term.

Starting with  $A = 0$  for the solitary laser in Eq. (3), we look for the stationary solution of the field letting  $(d/dt)E = 0$ , and solving for the quiescent point  $E_0$  as:  $E_0 = \sqrt{(\alpha - \Gamma)/\beta}$ . Then, we allow for the returning field  $\Delta E$  to be different from zero (but small) and solve Eq. (3) with  $E = E_0 + \Delta E_{\text{sm}}$ , obtaining the perturbed field solution  $\Delta E_{\text{sm}}$ , in the self-mixing condition as:

$$\Delta E_{\text{sm}} = \Delta E \cos 2ks (c/2L) / 2(\alpha - \Gamma). \quad (4)$$

Recalling that  $\alpha = \gamma c/2$  and  $\Gamma = -(\ln r_1 r_2) (c/2L)$  and rearranging terms, we can rewrite this equation as:

$$\Delta E_{\text{sm}} = \Delta E \cos 2ks (2\gamma L + \ln R_1 R_2)^{-1} \quad (5)$$

where  $\Delta E$  is the contribution returning from the target, given by Eq. (1), and the relation  $R_1 = r_1^2$ ,  $R_2 = r_2^2$  have been used.

Now, turning to outputs  $E_1$  and  $E_2$  in the self-mixing condition, their expressions are still given by Eq. (2) with  $E_0$  replaced by  $E_0 + \Delta E_{\text{sm}}$ :

$$E_{01} + \Delta E_{\text{sm}1} = i t_1 E_0 [1 - (t_1^2 / r_1) A \cos 2ks (2\gamma L + \ln R_1 R_2)^{-1}] \quad (6)$$

$$E_{02} + \Delta E_{\text{sm}2} = \sqrt{(r_1/r_2)} \quad ; \quad i t_2 E_0 [1 - (t_1^2 / r_1) A \cos 2ks (2\gamma L + \ln R_1 R_2)^{-1}]. \quad (7)$$

As it can be seen, the outgoing fields  $E_{01}$  and  $E_{02}$ , as well their impressed self-mix modulations  $\Delta E_{\text{sm}1}$  and  $\Delta E_{\text{sm}2}$  are in phase, and differ in amplitude just by a factor  $(t_2/t_1)\sqrt{(r_1/r_2)}$ . Thus, for these contributions, the associated modulation indexes,  $\Delta E_{\text{sm}1}/E_{01}$  and  $\Delta E_{\text{sm}2}/E_{02}$  are just the same.

Now, let us take account also of the reflection  $\Delta E_{\text{ref}}$  that the returning field experiences (point 5, bottom path in Fig. 2) when impinging on mirror M1. This field  $\Delta E_{\text{ref}}$  is easily written as:

$$\Delta E_{\text{ref}} = E_0 \quad ; \quad i t_1 e^{2iks} \quad ; \quad A r_1 = \Delta E r_1^2 / i t_1 \quad (8)$$

where Eq. (1) has been used to write the last term. Adding  $\Delta E_{\text{ref}}$  to the output  $E_1$  gives for the total field exiting from the front mirror:

$$E_{01} + \Delta E_{\text{tot}} = i t_1 E_0 \{1 - (t_1^2 / r_1) A \cos 2ks [(2\gamma L + \ln R_1 R_2)^{-1} - r_1^2 / t_1^2]\}. \quad (9)$$

Finally, what is actually observed as the self-mix signal is the photodetector output, where the photodetected current is a quantity proportional to the modulation indexes, that is  $m = \Delta E_{\text{tot}}/E_0$ . Using Eq. (6), Eq. (7), Eq. (9),  $r_1^2 = R_1$  and,  $t_1^2 = T_1$  these quantities are written as:

$$m_1 = \Delta E_{\text{tot}1}/E_{01} = (-t_1^2 / r_1) A \cos 2ks [(2\gamma L + \ln R_1 R_2)^{-1} - R_1/T_1] \quad (10)$$

$$m_2 = \Delta E_{\text{tot}2}/E_{02} = (-t_1^2 / r_1) A \cos 2ks [(2\gamma L + \ln R_1 R_2)^{-1}]. \quad (11)$$

Thus, the ratio of modulation indexes between front and back output is:

$$m_1/m_2 = 1 - (R_1/T_1) (2\gamma L + \ln R_1 R_2). \quad (12)$$

Let us now comment Eq. (12), for He-Ne and diode lasers. In a low-gain active media like the He-Ne gas,  $2\gamma L$  is relatively small (typically a few percent per pass) and thus unlikely to dominate over the loss  $\ln R_1 R_2$ . So, the term subtracted to unity at the right-hand side of Eq. (12) keeps small and the modulation index ratio  $m_1/m_2$  is positive (front and back outputs are in phase). In a semiconductor laser diode, the gain per pass is high (typically  $2\gamma L = 3-5$ ) and overwhelms the loss due to mirror reflectivity (typically  $R_{1,2} = 0.3$  for the cleaved facets). Then, the quantity subtracted to unity at the right-hand side of Eq. (12) is easily larger than one above threshold and front and rear outputs are in phase opposition. For a high-gain medium we may also expect the following behavior at increasing pump current. At threshold, where gain and losses are equal, i.e.  $2\gamma L \approx -\ln R_1 R_2$ , Eq. (12) shows that outputs signals are equal ( $m_1/m_2 \approx 1$ ) and in phase. If we increase the gain we reach a value  $2\gamma L \approx -\ln R_1 R_2 + R_1/T_1$ , at which  $m_1 = 0$ , i.e. the front signal disappears. Increasing the gain further above threshold, i.e.  $2\gamma L \gg -\ln R_1 R_2$ , the front output recovers in amplitude but becomes in phase opposition respect to the rear output signal ( $m_1/m_2 < 0$ ), see Eq. (12).

Let us examine the other output placed on the front mirror beam (Fig. 1), that is the auxiliary photodetector PD1' looking through the beamsplitter in the target direction. It may appear that the reflection contribution is missing there. Actually, once the contribution has been collected by the self-mixing process, it appears everywhere in the beam, thus the signal available at the PD1' output coincides with that of PD1.

Now, let us turn to the voltage output signal (Fig. 1). The voltage  $V_{ak}$  across the junction is related to the carrier density through the well known expression  $N = N_0 \exp(eV_{ak}/2kT)$ , hence a signal  $\Delta V_{ak} = (2kT/e) \Delta N/N_0$  is developed when the carrier density is varied by  $\Delta N$ . Writing the Lang and Kobayashi equation for  $N$  as:

$$(d/dt) N = J/ed - N/\tau_r - G(N - N_0)E^2 \quad (13)$$

where, as usual,  $J$  is the current density,  $\tau_r$  is the recombination time,  $d$  is the active region thickness,  $G$  is the modal (power) gain (with  $G(N - N_0)\gamma = c$ ), and  $N_0$  is the carrier density at transparency. As the electric field  $E = E_0$  is perturbed by the self-mixing phenomenon to the new value  $E = E_0 + \Delta E$ , where  $\Delta E$  is given by Eq. (4) or Eq. (5), the small signal solution for  $\Delta N$  is found from Eq. (13) as:

$$\Delta N = - (1/\tau_r + GE_0)^{-1} G(N - N_0)\Delta E \quad (14)$$

This expression shows how the signal across the diode, written from the above expressions as:

$$\Delta V_{ak} = - (1/\tau_r + GE_0)^{-1} (2kT/e) G(N - N_0)/N_0 \Delta E \quad (15)$$

is in phase opposition respect to that of the back mirror output, see Eq. (7).

It is important to note how, if no photodiode is allocable in the rear or front path, the junction voltage is the only option possible. This is, for example, the usual output chosen in confocal microscope self-mixing readout [11-12], and in VCSEL lasers with no rear photodiode. On the other hand, if we can use a photodiode to pick up the signal, it is definitely better to use it respect to the junction output, because of the better S/N ratio. Indeed, while signals are comparable (or the photodiode provides the larger one), noise of the junction is larger than shot noise of the photodetected current  $I_{ph} = \sigma E_0^2$ . Indeed, the differential resistance  $r_{diff} = dV_{ak}/dI = 2kT/eI_{dc}$  found across the laser diode carries Johnson noise, which is given, in rms current, by  $i_n = [4kTB/r_{diff}]^{1/2}$ , or by  $i_n = [2eI_{dc}B]^{1/2}$  after substituting the value of  $r_{diff}$ . Thus, the current noise across the junction is the shot noise of the bias current, a quantity much larger than current  $I_{ph}$  detected by the photodiode and accompanied by the shot noise  $i_n = [2eI_{ph}B]^{1/2}$ .

### 3. Experiment

We have used the basic self-mixing setup, as depicted in Fig. 1, for both the He-Ne laser and the GaAlAs diode laser. With the He-Ne, the output beam (front output) is sent directly onto a loudspeaker covered with a white paper spot, or with a super-diffusing (of the Scotchlite tape type) spot. With the diode laser, we had to use a collimating objective lens, projecting the spot on the target, to attain a high ( $C > 1$ , [3]) injection level on the diffuser. The loudspeaker was driven at 20-100Hz to create an ac-selfmix signal easily to measure. For both sources, the photodetectors used were 1-mm diameter Si photodiodes, backed by a trans-impedance op-amp stage (with typical  $R = 10$ -100k $\Omega$  feedback load) and then sent directly to the oscilloscope. The feedback level was adjusted through a neutral-density filter inserted on the path to the target.

Two laser sources have been used:

- 1: a He-Ne laser from NEC (GTL1791), built around a 17cm tube with internal mirrors and emitting a single-longitudinal mode of about 0.5mW at  $\lambda = 632.8$ nm;
- 2: a GaAlAs triple-QW semiconductor laser from Hitachi (HL8325G), emitting up to 20mW at  $\lambda \approx 832$ nm on a single longitudinal mode with side modes suppression at least of 30dB.

The diode laser was fed by a constant current supply, and TEC-controlled to avoid mode hopping, whereas the He-Ne was fed at fixed constant current and thermally shielded from ambient to avoid strong thermal drifts during the measurements.

No polarizer was inserted in the optical path. For the He-Ne, this means that we made no polarization selection of the modes. Indeed, as it is well known [13], internal mirrors He-Ne's oscillate on two orthogonally-polarized adjacent longitudinal modes, when two of them fit under the atomic line. We found that the two-mode regime was affected by strong mode coupling, making the measurement erratic. So, we worked in single mode only, by letting the mode pattern to slowly drift as temperature varies, until the main mode is centered under the Lamb's peak of the atomic line. Looking at the mode amplitude in the Lamb's dip condition [13] allowed us to ensure single longitudinal mode operation for a stable self-mixing measurement.

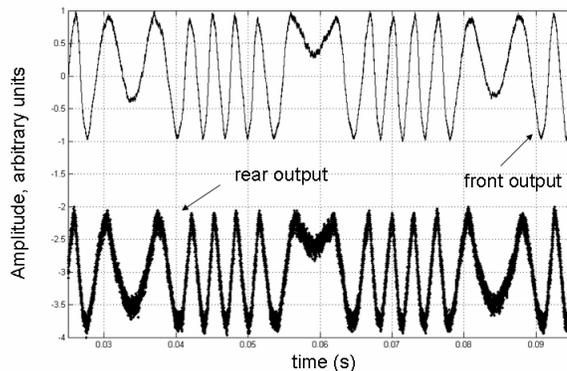


Fig. 3. Self-mixing waveforms detected at the front PD1 (top trace) and rear PD2 (bottom trace) photodiodes in a 17cm length, 0.5mW He-Ne laser subjected to self-mixing feedback from a remote ( $s = 40$ cm) loudspeaker driven with a sinusoidal signal at 40Hz, showing the in-phase condition. Pre-amplifier output voltages had dc components of 192mV and 11mV, respectively, and the modulation indexes were  $m_1 = 0.239$  (front output) and  $m_2 = 0.358$  (rear output). The trace of PD2 is noisier than that of PD1 but we didn't use balanced noise performance for the two preamplifiers of the photodiodes.

The waveforms of PD1 and PD2 on the He-Ne source are shown in Fig. 3. The in phase character of the signals was independent from the level of feedback, from less than 0.02% to about 10% in power, or in terms of  $C$  [3], from  $10^{-3}$  to  $\approx 0.2$ . Because of the negative-resistance V-I characteristics, it was not possible to vary the supply current (and hence  $\gamma L$ ) appreciably.

Experimentally, the reflectivity of the front mirror was measured to be  $R_1 = 0.970 \pm 0.005$ , whereas the rear mirror reflectivity was calculated from the ratio  $P_1/P_2 = (1 - R_1)/(1 - R_2)$  of front-to-rear dc powers. Using the measured value  $P_1/P_2 = 18 (\pm 0.5)$ , we found  $R_2 = 0.9983 \pm 0.0001$ . This provides us with the value of loss  $\ln R_1 R_2 = -0.032$ . The measured value of modulation index ratio was  $m_1/m_2 = 0.67 \pm 0.01$  (the data of Fig. 3). If we go on the diagram of Fig. 5, we can see that we need  $2\gamma L = 0.05 \pm 0.001$  to fit theoretical data. A roundtrip gain (in power) of  $\approx 5\%$  per pass is just what typically expected [13] from a He-Ne laser in normal working conditions, so we may conclude that an adequate fitting is achieved for the He-Ne.

The semiconductor laser (Hitachi HL8325G) is a GaAlAs/GaAs structure with a typical value  $n_{\text{eff}} = 3.32$  for the effective refractive index [14], so that  $R_1 = R_2 = 0.29$  for the two cleaved untreated facets and the total loss is  $\ln R_1 R_2 = -2.48$ .

On this device, we started looking at the output waveforms found at the nominal drive current of  $I_{\text{dc}} \approx 70\text{mA}$  (the threshold being at 40mA). At a target distance of  $s = 40\text{cm}$ , on a loudspeaker driven at 30Hz for a swing of  $\approx 18\mu\text{m}$  peak-to-peak, the value of modulation index ratio was  $m_1/m_2 = -0.57$ . We found that front and rear outputs were in phase opposition independent from the feedback level (for  $C = 10^{-3}$  to  $>1$ ), and distance (from 20 to 150cm). A sample of the several measurements is shown in Fig. 4, where the front PD1 and rear PD2 output signals are compared together with the signal taken at the junction voltage  $V_{\text{ak}}$ . Also for  $V_{\text{ak}}$  the phase relationship respect to PD2 were observed not to change at different target distance and strength of injection.

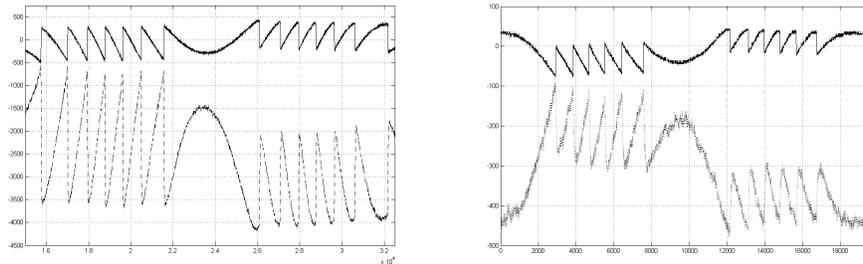


Fig. 4. Typical self-mixing waveforms obtained with a semiconductor laser well above threshold. Left side: front (top) and rear (bottom) photodiode outputs are in phase-opposition. Right hand side shows the front output (top) and junction-voltage (bottom) output. Ratio of the modulation indexes of PD1 and PD2 was  $m_1/m_2 = -0.57$  at  $I_{\text{dc}} = 70\text{mA}$  bias current. Peak-to-peak amplitudes were approximately:  $I_{\text{ph1}} \approx 50\mu\text{A}$ ,  $I_{\text{ph2}} \approx 5\mu\text{A}$ ,  $\Delta V_{\text{ak}} \approx 100\mu\text{V}$ , and distance to target was 40cm.

Thereafter, we looked for the signal null as a function of bias current, and found it at  $I_{\text{dc}} = 61\text{mA}$ . At this current, from Eq. (10) the corresponding gain value is evaluated as  $(2\gamma L + \ln R_1 R_2)^{-1} = R_1/T_1$  whence  $2\gamma L = T_1/R_1 - \ln R_1 R_2 = 4.94$ .

In addition, swinging from the threshold current  $I_{\text{dc}} = 40.5\text{mA}$  to the maximum permissible current  $I_{\text{dc}} = 90\text{mA}$ , the modulation-index ratio  $m_1/m_2$  varied with continuity from +1, the value expected for  $2\gamma L = -\ln R_1 R_2 = 2.48$ , to about -3 (see Fig. 4), in accordance with the qualitative trend of Eq. (10) and Eq. (11), in which the sign of the quantity  $(2\gamma L + \ln R_1 R_2)^{-1} - R_1/T_1$  determines the phase concordance or opposition as pointed out in the discussion following Eq. (12). Now, recalling Eq. (13), we see that the gain per unit length  $\gamma = G(N - N_0)/c$  can be expressed at the equilibrium ( $dN/dt = 0$ ) as  $\gamma = A I_{\text{dc}} + B$ , where  $A$  and  $B$  are two constants. Fitting them to the curve of Fig. 4 we can plot the experimental results for the laser

diode as in Fig. 6 (left). By inspection, the agreement is satisfactory and thus confirms the theoretical explanation presented above.

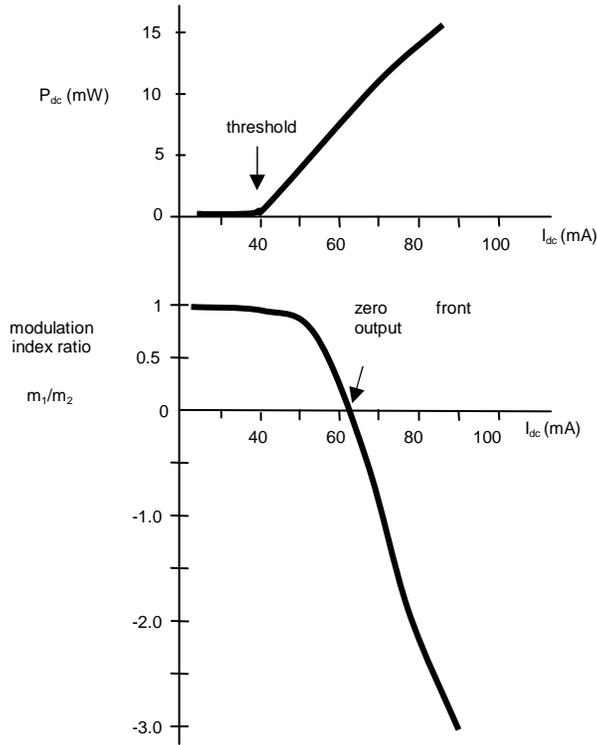


Fig. 5. The I-P characteristic of the laser diode (threshold is 40mA) and the modulation index ratio  $m_1/m_2$  as a function of dc drive current. Inversion of the sign is at  $\approx 60$ mA.

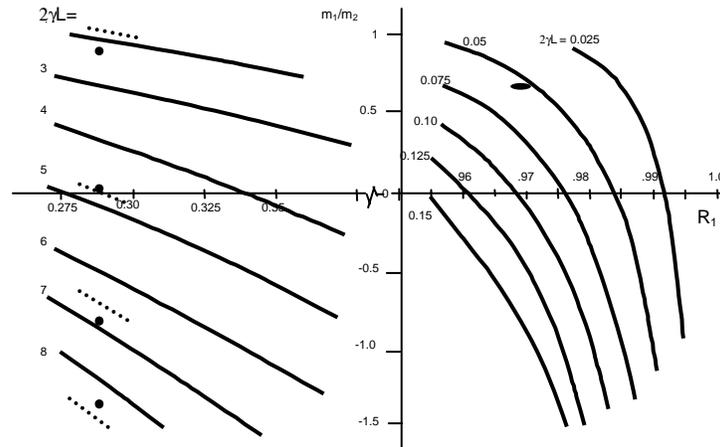


Fig. 6. The ratio  $m_1/m_2$  of modulation indexes for the front and rear mirror outputs, as given by Eq. (12), plotted versus the front mirror reflectivity  $R_1$ , for some values of the (power) round trip gain  $2\gamma L$ : left, for the diode laser, right, for the He-Ne laser. Ellipse is the experimental point for a 633nm He-Ne laser with  $R_1 = 0.97$ , in which the self-mixing outputs have  $m_1/m_2 = 0.67$ , implying a gain per pass  $2\gamma L = 0.06$ . Dots are for the 832nm GaAlAs diode laser with  $R_1 = R_2 = 0.288$ : the measured values of  $m_1/m_2$  plotted here should match theoretical values (from top to bottom) of  $2\gamma L = 2.4, 4.94, 6.5$  and  $9.0$ , indicated by the short dotted lines.

#### **4. Conclusions**

We have derived expressions describing the modulation indexes associated with the several signal outputs available in a self-mixing interferometer, that is the rear mirror (or monitor) output, usually the preferred in applications, the front mirror output, useful for remote detection of the signal, and the voltage across junction, the obvious choice when no photodiode is available.

Theoretical results have been compared to experimental data, exhibiting a very good match of the general trend and a satisfactory agreement to numerical values, for a He-Ne laser and a semiconductor GaAlAs laser. Thus, we have reconciled apparently contradictory results reported by several Authors about the phase and amplitude of the different outputs. Last, we may conclude that our theory, directly derived from Lamb (or Lang and Kobayashi) equations, well explains the details of lasers dynamics in the self-mixing regime.

#### **Acknowledgments**

This work was performed under a FAR grant of the Ministry of University and Scientific and Technological Research (MURST). One Author (S.D.) wishes to thank Professor Chih-Chung Yang of the Graduate School of Electro-Optical Engineering, National Taiwan University, for the laboratory equipment made available during the course of this work.