Self-imaging of three-dimensional images by pulsed wave fields

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Abstract: Recently, the classical Talbot effect (self-imaging of optical wave fields) has attracted a renewed interest, as the concept has been generalized to the domain of pulsed wave fields by several authors. In this paper we discuss the self-imaging of three-dimensional images. We construct pulsed wave fields that can be used as self-imaging "pixels" of a three-dimensional image and show that their superpositions reproduce the spatial separated copies of its initial three-dimensional intensity distribution at specific time intervals. The derived wave fields will be shown to be directly related to the fundamental localized wave solutions of the homogeneous scalar wave equation – focus wave modes. Our discussion is illustrated by some spectacular numerical simulations. We also propose a general idea for the optical generation of the derived wave fields. The results will be compared to the work, published so far on the subject.

OCIS codes: (070.6760) Talbot effect; (320.5540) Pulse shaping; (070.2580) Fourier optics; (320.2250) Femtosecond phenomena.

References and links

1. W. D. Montgomery, "Self-Imaging Objects of Infinite Aperture, "J. Opt. Soc. Am. **57**, 772-778 (1967).
2. J. Turunen and A. T. Friberg, "Self-imaging and propagation-invariance in electromagnetic fields," Pure Appl. Opt. **2**, 51-60 (1993).
3. Z. Bouchal and J. Wagner, "Self-reconstruction effect in free propagation of wavefield," Opt. Commun. **176**, 299-307 (2000).
4. J. Wagner and Z. Bouchal, "Experimental realization of self-reconstruction of the 2D aperiodic objects," Opt.Commun. **176**, 309-311 (2000).
5. R. Piestun, Y. Y. Schechner and J. Shamir, "Self-imaging with finite energy," Opt. Lett. **22**(4), 200-202 (1997).
6. J. Durnin, J. J. Miceli, Jr., and J. H. Eberly, "Diffraction-free beams," Phys. Rev. Lett. **58**, 1499–1501 (1987).
7. R. Piestun and J. Shamir, "Generalized propagation-invariant wave fields," J. Opt. Soc. Am. A **15**, 3039-3044 (1998).
8. Z. Bouchal and M. Bertolotti, "Self-reconstruction of wave packets due to spatio-temporal couplings," J. Mod. Opt. **47**, 1455 - 1467 (2000).
9. Z. Bouchal, "Self-reconstruction ability of wave field" Proc. SPIE, vol. 4356, 217-224, (2001).
10. J. Salo and M. M. Salomaa, "Diffraction-free pulses at arbitrary speeds," J. Opt. A: Pure Appl. Opt. **3**, 366-373 (2001).
11. H. Wang, C. Zhou, L. Jianlang and L. Liu, "Talbot effect of a grating under ultrashort pulsed-laser illumination,"Microwave and Opt. Technol. Lett. **25**, 184-187 (2000)
12. J. Azaña and M. A. Muriel, "Technique for multiplying the repetition rates of periodic trains of pulses by means of a temporal self-imaging effect in chirped fiber grating," Opt. Lett. **24**, 1672-1674 (1999).
13. P. Saari, H. Sõnajalg, "Pulsed Bessel beams," Laser Physics **7**, 32-39 (1997).
14. A. A. Maznev, T. F. Crimmins and K.A. Nelson, "How to make femtosecond pulses overlap," Opt. Lett. **23**, 1378-1380 (1998).
15. Zs. Bor and B. Rácz, "Group velocity dispersion in prisms and its application to pulse compression and traveling-wave excitation," Opt. Commun. **54**, 165-170 (1985).
16. P. Di Trapani, D. Caironi, G. Valiulis, A. Dubietis, R. Danielius and A. Piskarskas, "Observation of Temporal Solitons in Second-Harmonic Generation with Tilted Pulses," Phys. Rev. Lett. **81**, 570-573 (1998).

17. P. Saari, J. Aaviksoo, A. Freiberg, K. Timpmann, "Elimination of excess pulse broadening at high spectral resolution of picosecond duration light emission," Opt. Commun. **39**, 94-98 (1981).
18. O. Svelto, *Principles of Lasers* (3rd ed. Plenum Press 1989).
19. I. Besieris, M. Abdel-Rahman, A. Shaarawi, and A. Chatzipetros, "Two fundamental representations of localized pulse solutions of the scalar wave equation," Progr. In Electromagn. Research **19**, 1 (1998).
20. P. Saari and K. Reivelt, "Evidence of X-shaped propagation-invariant localized light waves," Phys. Rev. Lett. **79**, 4135-4138 (1997).
21. K. Reivelt and P. Saari, "Optical generation of focus wave modes," J. Opt. Soc. Am. A **17**, 1785-1790 (2000).
22. K.Reivelt and P.Saari, "Optical generation of focus wave modes: errata," J. Opt. Soc. Am. A **18**, 2026-2026, (2001).
23. K. Reivelt and P. Saari, "Optically realizable localized wave solutions of homogeneous scalar wave equation," Phys. Rev. E (to be published) [accepted for publication].

1. Introduction

Self-imaging, also known as Talbot effect is, in its original sense, a well-known phenomenon in classical wave optics where certain wave fields reproduce their transversal intensity distribution at periodic spatial intervals in the course of propagation (see Refs. [1-5], and references therein). The effect has been studied extensively by means of Fresnel diffraction theory and the angular spectrum representation of scalar wave fields. As the result, the general, physically transparent conditions have been formulated the transversal intensity distribution of a wave field have to obey to be self-imaging [1-5]. In recent years the effect has been studied mostly in connection with the so-called propagation-invariant wave fields – monochromatic Bessel beams (see Refs. [6-7], and references therein). In fact, the propagation-invariant wave fields constitute a special class of self-imaging wave fields, the mathematical description of the two being much the same.

In recent years the effect has attracted a renewed interest, as the concept has been generalized to the domain of pulsed wave fields by several authors (see Refs. [8-12], and references therein). The phenomenon has been discussed in the context of fiber optics [12] and also as a property of spatial, wideband wave fields [8-11]. However, to our best knowledge the spatio-temporal self-reconstruction of three-dimensional images has not been discussed in literature so far. Also, in our opinion the effect of self-imaging of pulsed wave fields has not been given a sufficiently well comprehensible physical explanation so far.

In this paper we construct pulsed, spatio-temporally localized, self-imaging wave fields that can be used as self-imaging "pixels" of a three-dimensional image and show that their superposition reproduce spatial separated copies of its initial three-dimensional intensity distribution at specific time intervals. Our approach is different from those used in literature so far – we use the concept of tilted pulses to give a intuitive understanding of the temporal evolution of the coupling of the monochromatic components of the pulsed wave fields. This approach also proposes a general idea for their optical generation. The derived wave fields will be shown to be directly related to the fundamental localized wave solutions of the homogeneous scalar wave equation – focus wave modes. The results will be compared to the work, published so far on the subject.

2. Monochromatic self-imaging

The self-imaging of monochromatic wave fields can be introduced as follows. Consider the general monochromatic solution of the homogeneous scalar wave equation

$$\nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0\,, \tag{1}$$

written as a superposition of homogeneous plane waves:

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$$\Psi(x,y,z,t)=\exp[-i\omega t]\int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\phi\, A(k,\theta,\phi) \times \exp[ik(x\cos\phi\sin\theta + y\sin\phi\sin\theta + z\cos\theta)], \quad (2)$$

where $k^2=\omega^2/c^2$ is the wave number and $A(k,\theta,\phi)$ is the angular spectrum of plane waves of the wave field in the spherical coordinates. Defining the self-imaging condition of the wave field (2) as [1-5]

$$\Psi(x,y,z,t)=\Psi(x,y,z+d,t), \quad (3)$$

we get the condition

$$kd\cos\theta = \psi + 2\pi q, \quad (4)$$

where q is an integer and ψ is an arbitrary phase factor. The relation (4) implies, that a monochromatic wave field (2) periodically reconstructs its initial transversal amplitude distribution if only its angular spectrum of plane waves is sampled so that the condition

$$k_z = k\cos\theta = \frac{\psi + 2\pi q}{d}, \quad (5)$$

where k_z is the z component of the wave vector, is satisfied for every plane wave component of the wave field.

In what follows we are mainly concerned with the self-imaging of transversal images. For this purpose the solution (2) is often represented in cylindrical coordinates (see, e.g., Ref. [3,13]):

$$\Psi(\rho,z,t)=\exp[-i\omega t]\sum_n c_n \exp[in\phi]\int_0^{\pi} d\theta \sin\theta\, A(k,\theta) \times J_n(k\rho\sin\theta)\exp[ikz\cos\theta], \quad (6)$$

where J_n is the n-th order Bessel function. Applying the condition (5) and including only the axially symmetric terms in the summation, we get the following expression for the general cylindrically symmetric, monochromatic, self-imaging wave field:

$$\Psi(\rho,z,t)=\exp[-i\omega t]\sum_q a_q\, J_0\left[k\rho\sqrt{1-\left(\frac{2\pi q}{d}\right)^2}\right]\exp\left[i\frac{2\pi q}{d}z\right]. \quad (7)$$

Here we have denoted $a_q = A[k, \arccos(2\pi q/kd)]$ and $\psi = 0$ is assumed. The wave field $J_0[k\rho\sin\theta]\exp[ikz\cos\theta - i\omega t]$ in Eqs. (6) and (7) is a special case of the so-called nondiffracting beams – the zeroth-order Bessel beam [6].

The on-axis amplitude of the superposition (7),

$$\Psi(0,z,t)=\exp[-i\omega t]\sum_q a_q \exp\left[i\frac{2\pi q}{d}z\right], \quad (8)$$

can be recognized as the Fourier series representation of a periodic function along the optical axis. As the Bessel beams have sharply peaked beam profiles, the relation is very useful in qualitative analysis of the spatial shape of the wave field (7).

3. Self-imaging of pulsed wave fields

Consider a set of overlapping, non-spreading optical pulses that (1) are transversally localized, i.e., their transversal intensity distribution have a single narrow intense peak, (2) have equal carrier frequency, (3) propagate at equal group velocities, but (4) have different phase velocities. Obviously the interference of such pulses would behave as a propagating "light bullet" with modulated peak intensity (see movie on Fig. 1). In complete analogy with the monochromatic self-imaging we could suggest that if the component pulses satisfy certain conditions, the wave field could periodically reconstruct its initial (localized) transversal intensity distribution and the resulting temporal evolution could be perceived as a spatial arrow of sequentially visible light spots (see Fig. 2 for a comparable example of the temporal evolution of the proposed pulsed self-imaging wave field and the spatial amplitude of a monochromatic self-imaging wave field). In this sense, such superposition could be considered self-imaging and, due to its spatial localization, it could be used as a pulsed "self-imaging pixel" of a transversal or even spatial image.

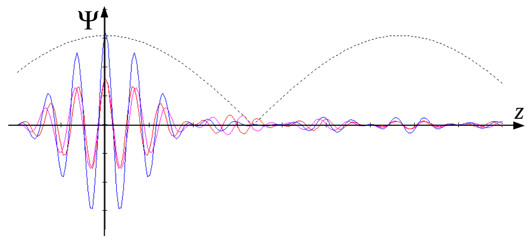

Fig. 1. (0.85 MB) The video clip of the temporal evolution of the superposition of two interfering optical pulses (red and cyanide line) that have equal carrier wavelengths and group velocities but different phase velocities. The blue line and the dotted line denote the amplitude and the envelope of the sum of the two waves respectively.

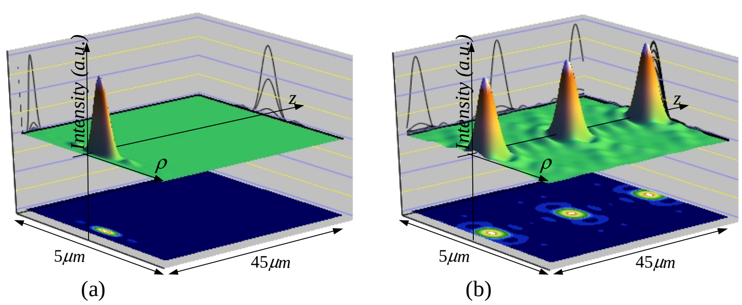

Fig. 2. (a) The temporal evolution of the spatial intensity distributions of the pulsed self-imaging wave field (the 1.2 MB movie); (b) The spatial amplitude of a monochromatic self-imaging wave field (see also text in Sec. 4).

To realize the proposed general idea we have to define a pulsed, non-spreading, transversally localized wave field the, on-axis phase and group velocities of which differ and are also independently variable. Let us start by considering a two-dimensional weighted superposition of the monochromatic plane waves of the form

$$\Psi^{(T)}(x,z,t)=\int_0^\infty dk\,A(k)\exp\left[ik\left(x\sin\theta(k)+z\cos\theta(k)-ct\right)\right]. \tag{9}$$

Here the function $\theta(k)$ is the wavelength-dependent angle between the direction of propagation of the plane waves and the optical axis (z-axis) and $A(k)$ is the frequency spectrum. The wave field (9) can be recognized as a quite general form of a class of solutions of the linear homogeneous wave equation – the angularly dispersed plane wave pulses, also referred to as the tilted pulses (see, e.g., Refs. [14-17], and references therein for various relevant discussions). The tilted pulses are not transversally localized. However, the on-axis phase and group velocities of the tilted pulses are generally different [14–17] and the wave field provides us with a general idea on the origin of the phenomenon – the difference can be explained by the fact, that the pulse fronts of the tilted pulses are not parallel to the phase fronts [see Fig. 3(a)]. Indeed, if we choose a direction of propagation that subtends a non-zero angle θ with the optical axis, we observe an increasing (or decreasing) difference between the pulse front and the phase fronts, thus, the phase fronts move forward (or backward) inside the pulse envelope along the optical axis [see Fig. 3(b)]. In this section we introduce the polychromatic self-imaging in terms of tilted pulses. The spatial localization will be considered in next section.

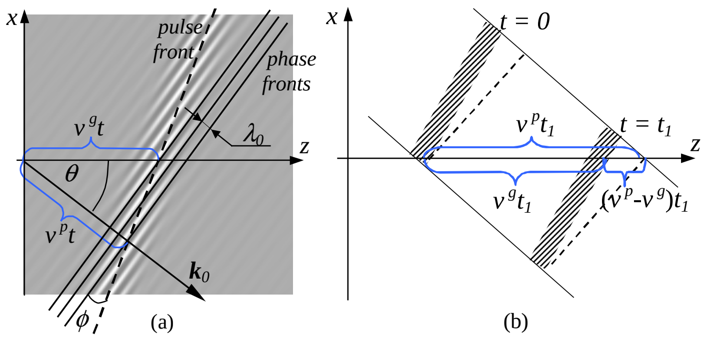

Fig. 3. (a) An excerpt of the spatial amplitude distribution of a tilted pulse. v^g and v^p denote its group and phase velocities respectively, $\boldsymbol{k_0}$ and λ_0 are the wave vector and the wavelength of the plane wave of the carrier wave number, t is an arbitrary time and ϕ is the tilt angle of the pulse; (b) On the phase and group velocities of the tilted pulses (see text).

The function $\theta(k)$ in Eq. (9) can be specified from the condition, that the envelope of the pulse along the optical axis should not change in the course of propagation. Also, this property should hold for the ultrashort optical pulses, the frequency spectrum of which spans the entire visible and near-infrared spectrum. Let us choose the function as

$$\theta^{(T)}(k,\beta) = \arccos\left[\frac{\gamma(k-2\beta)}{k}\right], \tag{10}$$

where γ and β are constants. In this case the group velocity v^g of the pulse along the optical axis,

$$v^g(k) = \left(\frac{dk_z}{d\omega}\right)^{-1}, \tag{11}$$

is constant over the spectral range of the pulse. Indeed, from the Eq. (10) we get

$$k_z = k\cos\theta^{(T)}(k,\beta) = \gamma k - 2\beta\gamma \tag{12}$$

and the substitution in Eq. (11) yields

$$v^g(k) = \frac{c}{\gamma}, \tag{13}$$

i.e., the group velocity of the pulse does not depend on the wave number and is determined by the constant γ. Consequently, the pulse envelope along the optical axis does not spread, irrespective of the exact form of the frequency spectrum $A(k)$ in Eq. (9). For the phase velocity of such pulse along the optical axis we obtain

$$v^p(\beta) = \frac{c}{\cos\theta^{(T)}(k_0,\beta)} = \frac{ck_0}{\gamma(k_0-2\beta)}, \tag{14}$$

where k_0 is the mean (or carrier) frequency of the wave field.

It may be of interest to note, that the function $\theta^{(T)}(k,\beta)$ in Eq. (10) can be interpreted as the definition of the support of angular spectrum of plane waves of the tilted pulse, i.e. the

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definition of a volume in the k-space where the plane wave components of the wave field have non-zero amplitudes. In our case the support of angular spectrum is a line in k_x-k_z plane (see Fig. 4(a) for an example). The parameter 2β in Eq. (10) has an interpretation as being the wave number of the wave vector component directed perpendicularly to the optical axis on this support.

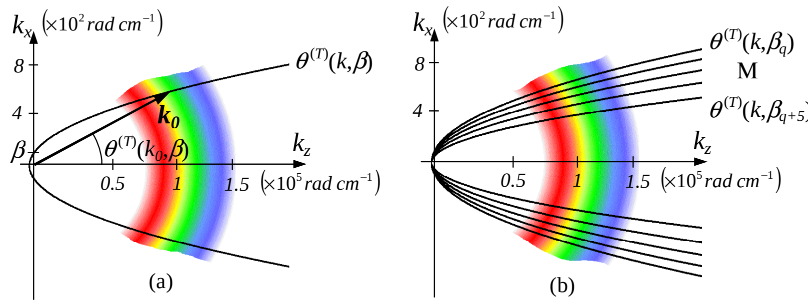

Fig. 4. The examples of the supports of the angular spectrums of plane waves of (a) the specified tilted pulse for $\beta = 40$ rad m^{-1} and $\gamma = 1$ ($\boldsymbol{k_0}$ is the wave vector of the carrier wavelength), (b) the self-imaging superposition of the tilted pulses (see text). The “rainbow” on the pictures denotes the visible spectral region.

Consider a discrete superposition of a set of the tilted pulses

$$\Psi^{(SI)}(x,z,t) = \sum_q a_q \int_0^\infty dk\, A(k) \exp\left[ik\left(x \sin\theta^{(T)}(k,\beta_q) + z\cos\theta^{(T)}(k,\beta_q) - ct\right)\right], \qquad (15)$$

where the function $\theta^{(T)}(k,\beta_q)$ is determined by the Eq. (10). The expression can be given a readily interpretable form if we approximate the x and z components of the wave vector by

$$\begin{aligned} k_x(k,\beta) &= k_x(k_0,\beta) + \left.\frac{dk_x(k,\beta)}{dk}\right|_{k_0}(k-k_0) \\ k_z(k,\beta) &= k_z(k_0,\beta) + \left.\frac{dk_z(k,\beta)}{dk}\right|_{k_0}(k-k_0) \\ &= k_z(k_0,\beta) + \gamma(k-k_0) \end{aligned} \qquad (16)$$

where k_0 is the carrier wave number and we have also used the Eq. (12). Substitution relation (16) into Eq. (15) yields

$$\Psi^{(SI)}(x,z,t) \cong \sum_q a_q C_q(x, z\gamma - ct) \exp\left[ik_0\left(x\sin\theta^{(T)}(k_0,\beta_q) + z\cos\theta^{(T)}(k_0,\beta_q) - ct\right)\right], \quad (17)$$

where $\exp[i\, k_0\, (x \sin\theta^{(T)}(k_0,\beta_q) + y \cos\theta^{(T)}(k_0,\beta_q) - c\, t)]$ is the carrier wavelength plane wave component of the tilted pulse (15) and

$$C_q(x, z\gamma - ct) = \int_{-k_0}^{\infty} dk\, A(k+k_0) \exp\left[ik\left(x \left.\frac{dk_x(k,\beta_q)}{dk}\right|_{k_0} + z\gamma - ct\right)\right] \qquad (18)$$

is an approximation to the non-spreading traveling envelope of the pulse. In near axis volume we can write

$$\Psi^{(SI)}(x,z,t) \cong C(x, z\gamma - ct) \sum_q a_q \exp\left[ik_0\left(x\sin\theta^{(T)}(k_0,\beta_q) + z\cos\theta^{(T)}(k_0,\beta_q) - ct\right)\right]. \quad (19)$$

The Eq. (19) is essentially a product of a propagating pulse $C(x, z\gamma - ct)$ and of a term, that is a mathematical equivalent of the superposition of the monochromatic carrier-wavelength plane waves that propagate at angles $\theta^{(T)}(k_0,\beta_q)$ to the optical axis. According to our general idea, the latter term should be self-imaging in the conventional, monochromatic sense of the term– in this case the product (19) behaves as a pulse that vanishes and reconstructs itself periodically. The condition for such behavior can be written in complete analogy with the Eq. (4) of the Sec. 2:

$$kd\cos\theta^{(T)}\left(k_0,\beta_q\right)=\psi+2\pi q \quad (20)$$

Applying the Eq. (12) yields

$$\frac{2\pi q}{d}=\gamma k_0-2\gamma\beta, \quad (21)$$

and we get a discrete set of constants β for the superposition (15):

$$\beta_q=\frac{k_0}{2}-\frac{\pi q}{\gamma d} \quad (22)$$

(see Fig. 4(b) for an example). The direction of propagation of the carrier wavelength plane wave component of the tilted pulse (15) can be found by combining the Eqs. (10) and (22): we can write

$$\theta^{(T)}\left(k,\beta_q\right)=\arccos\left(\frac{\gamma\left(k-k_0\right)}{k}+\frac{2\pi}{kd}q\right), \quad (23)$$

so that

$$\theta^{(T)}\left(k_0,\beta_q\right)=\arccos\left(\frac{2\pi}{k_0 d}q\right). \quad (24)$$

Note, that for the real values of the angle $\theta^{(T)}(k, \beta_q)$ the condition

$$0<q<\frac{k_0 d}{2\pi} \quad (25)$$

have to be satisfied.

The longitudinal shape of the superposition in Eq. (19) can be easily evaluated for the most practical case of a uniform superposition of $(2n + 1)$ tilted pulses, centered around some carrier spatial frequency $k_z(k_0,\beta_Q)$ (see Fig. 5). The spatial amplitude corresponding to such superposition can be expressed as

$$\Psi(z)=A_0\frac{\sin\left[\tfrac{1}{2}(2n+1)\Delta k_z z\right]}{\sin\left(\tfrac{1}{2}\Delta k_z z\right)}, \quad (26)$$

where Δk_z is the interval between the spatial frequencies (see Ref. [18] for example). For this case the self-imaging distance is $d = 2\pi/\Delta k_z$ and the width of the peaks of the resulting function is $\Delta z \approx 2\pi/(2n+1)\Delta k_z$. The result of the evaluation of Eq.(24) for a superposition of seven tilted pulses is shown on Fig. 5(b).

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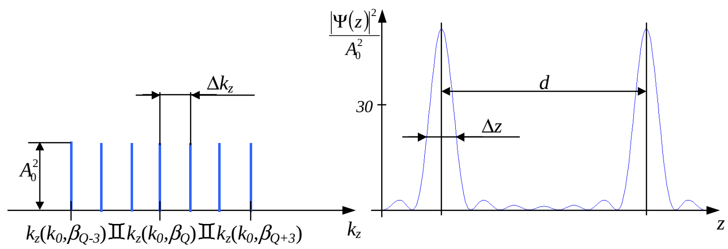

(a) (b)

Fig. 5. (a) The Fourier spectrum and (b) the spatial amplitude of a train of sinusoidal waves.

4. Self-imaging of the transversal and three-dimensional images by pulsed wave fields.

The transversal amplitude distribution of the tilted pulses is not localized by any means and it may not be obvious how the proposed concept could be used to construct pulsed self-imaging wave fields that carry arbitrary non-trivial transversal images – the property the term "self-imaging" actually referred to [3,13]. In monochromatic self-imaging the independent "pixels" of an image are formed by introducing cylindrically symmetric superpositions of plane waves – the Bessel beams [3,4]. This approach can also be used in the polychromatic case.

Consider the polychromatic superposition of the Bessel beams:

$$\Psi(\rho,z,t)=\int_0^\infty dk\,A(k)\,J_0[k\rho\sin\theta(k)]\exp[ik(z\cos\theta(k)-ct)]\,. \tag{27}$$

It can be easily shown that the Eq. (27) is essentially a cylindrically symmetric superposition of the tilted pulses in Eq. (9), the angle $\theta(k)$ now denoting the wavelength dependent cone angle of the Bessel beams. Consequently, the on-axis behavior of (27) is identical to that of the tilted pulses in previous section, provided that the function $\theta(k)$ is the same in both cases. Thus (1) the on-axis group and phase velocities of the wave field (27) differ and overlapping pulses with different phase velocities give rise to the evolving interference, (2) the support of the angular spectrum of plane waves in Eq. (10) yields a pulsed wave field with non-spreading longitudinal amplitude distribution (in this context the support of the angular spectrum on Fig. 4(a) should be considered a plane section of a three-dimensional axially symmetric surface) and (3) the condition (20) determines the set of constants β_q that provide the self-imaging of the superposition of the wave fields. As a result, we get a cylindrically symmetric pulsed self-imaging wave field that is essentially the axially symmetric superposition of the self-imaging tilted pulses in Eq. (15):

$$\begin{aligned}\Psi^{(SI)}(\rho,z,t)=\sum_q a_q\int_0^\infty dk\,A(k)\\ \times J_0\left[k\rho\sin\theta^{(T)}(k,\beta_q)\right]\exp\left[ik\left(z\cos\theta^{(T)}(k,\beta_q)-ct\right)\right]\end{aligned}\,, \tag{28}$$

and the Eq. (10) further simplifies the result to

$$\begin{aligned}\Psi^{(SI)}(\rho,z,t)=\sum_q a_q\exp\left[-i2\gamma\beta_q z\right]\int_0^\infty dk\,A(k)\\ \times J_0\left[k\rho\sqrt{1-\left(\tfrac{\gamma(k-2\beta_q)}{k}\right)^2}\right]\exp\left[ik(z\gamma-ct)\right]\end{aligned}\,. \tag{29}$$

The goal of the representation (29) is obvious – it can be shown, that the spatial amplitude distribution of the wave field can be designed to consist of a micrometer diameter central peak on a sparse, low intensity background (see Fig. 6). In fact, the expression under the summation sign in Eq. (29) is the integral representation of a wave field known as the fundamental localized wave solution of the homogeneous scalar wave equation – the focus wave mode (FWM), the spatial localization and diffractionless propagation being its definitive properties (see Refs. [19-23], and references therein). In this paper we have shown, that a superposition of the FWM's can be constructed so that the micrometer diameter central peak periodically vanishes due to the interference between its components. Thus, the wave field (29) can be used as pulsed "self-imaging pixel".

It is important to note, that our discussion is closely related to those in Refs. [8] and [10]. The three publications consider essentially the same problem, however the analysis is different in each occasion. For example, the Eq. (6) in Ref. [10] is a close relative to the Eq. (29), the two differ only by the definition of the *z*-component of the wave vector: in our approach the Eq. (23) yields

$$k_{z,q} = \gamma(k - k_0) + \frac{2\pi}{d} q \,, \tag{30}$$

in Ref. [10] the corresponding relation reads (see Eq. (3) of the Ref. [10])

$$k_{z,q} = \gamma k + \frac{2\pi}{d} q \,, \tag{31}$$

As the result, the summation integer q in our approach is positive for all practical configurations, the summation integer in Ref. [10] [see Eq (31)] has mostly negative values. However, we hope that our approach gives a more intuitive and physically transparent picture of the phenomenon. A different approach is used in Ref. [11]: in this paper the authors use the Weyl type angular spectrum of plane waves and apply the "complete sampling" of the spatio-temporal spectrum as

$$U'(\nu,\psi,\omega) = U(\nu,\psi,\omega) \sum_{m=-M}^{M} \sum_{n=0}^{N} \delta(\omega - \omega_m)\delta(\nu - \nu_{mn}) \,, \tag{32}$$

where $U(\nu,\psi,\omega)$ is the Fourier spectrum of the spatio-temporal amplitude of the wave field at the input transverse plane $z = 0$, ν and ψ are the radial spatial frequency and polar angle in the Fourier plane respectively, m and n are integers. Such an approach is very useful when the self-imaging of arbitrary transversal (or three-dimensional) images is considered because the inevitable effect of filtering of spatial frequencies of initial image is clearly manifested in the results.

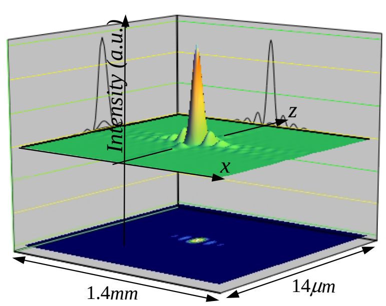

Fig. 6. The instantaneous intensity distribution of a FWM if $\beta = 40$ rad m^{-1}, $\theta^{(T)}(k_0,\beta) = 0.22$ deg, $\gamma = 1$ and the frequency spectrum $A(k)$ has a rectangular shape and extends over the wavelengths 400-800*nm*.

A numerical example of the self-imaging behavior of a superposition of five FWM's has been depicted on Fig. 2(a). In this example the self-imaging distance $d = 2\times10^{-5} m$, $\gamma = 1$, $k_0 = 1\times10^{7}$ rad m^{-1}, $q = 27, 28, \ldots, 31$ with β_q being determined by Eq. (22) ($\beta_q = 7.6\times10^{5}$ rad $m^{-1}\ldots$ 1.3×10^{5} rad m^{-1}) and $\theta^{(T)}(k_0,\beta_q)$ by Eq. (24) [$\theta^{(T)}(k_0,\beta_q) = 13 \ldots$ 32 deg, note, that the angle is considerably larger than that chosen in the example on Fig. 6

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resulting in lower side lobes and different scale along the x axis]. The frequency spectrum $A(k)$ spans the entire visible spectrum (400 to 800nm). The wave field on Fig. 2(b) is the monochromatic, self-imaging superposition of the Bessel beams with cone angles $\theta^{(T)}(k_0,\beta_q)$ – the cylindrically symmetric generalization of the second term in the product in Eq. (19).

As the second example we demonstrate the self-imaging transmission of a non-trivial spatial image, depicted on Fig. 7(a). The image consists of eight "pixels"– the self-imaging superpositions of FWM's – specified in previous example.

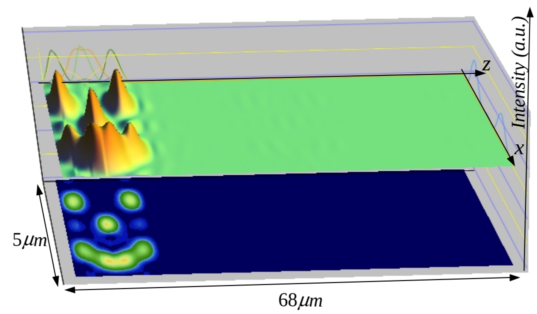

Fig. 7. (1.4 MB movie) A numerical example of the evolution of a self-imaging spatial image (smiling human face) consisting of eight self-imaging pixels, each consisting of five FWM's of different constant β (see text).

The numerical examples clearly show that the concept, in principle, is applicable for constructing wave fields that self-image three-dimensional image. Still, the experimental realization of such wave fields is not trivial – though the optical generation of the localized wave solutions has been a subject of intense research over the past decade, the task has been accomplished only for the case of Bessel–X pulses (see Refs. [19-22], and references therein). However, in our recent publication we proposed a realizable, physically transparent solution to the problem of optical generation of FWM's (see Ref. [21]). Also, we have proposed an elucidation to the well-known issue of infinite energy content of FWM's and derived a feasible, finite energy FWM's that preserves all the peculiar properties of the infinite energy FWM's [23]. Thus, such situation is likely to change in near future. Also, we believe that the approach in Ref. [21] is also applicable for the optical generation of the superpositions of FWM's.

5. Conclusion

In this paper we have discussed the spatio-temporal self-imaging of three-dimensional images by the novel class of solutions of the homogeneous scalar wave equation – the pulsed self-imaging wave fields. We gave a physically transparent interpretation to the behavior of the wave fields and connected the phenomenon with two widely known concepts in wave optics – the tilted pulses and localized wave solutions of the homogeneous scalar wave equation. The numerical examples clearly show that the concept, in principle, is applicable for constructing wave fields that self-image three-dimensional image.

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