

Spectral coherence properties of temporally modulated stationary light sources

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Abstract: It is shown that partially spectrally coherent pulses of light with controlled spectral coherence properties can be generated by temporal modulation of beams emitted by stationary light sources. A method for generation of spectrally Gaussian Schell-model-type pulses is presented.

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References and links

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1. Introduction

Usually ideal pulsed light is thought to be fully coherent, but recently we speculated on implications of partial spectral coherence of optical pulses [1]. In this paper we present a practical example of spectrally partially coherent pulses, which is consistent with the so-called spectral Gaussian Schell model assumed in Ref. [1]. This is done by considering temporal modulation of stationary fields. While the method is applicable to the production of pulses with a wide range of spectral coherence function, we concentrate here on just one special case because it permits us to obtain simple and illustrative analytical results.

The Gaussian Schell-model [2] is a popular model for the description of spatially partially coherent light fields: here both the intensity distribution and the complex degree of spatial coherence are assumed to be Gaussian. The model allows, e.g., a smooth transition between incoherent and coherent sources with Gaussian intensity distributions [3–8].

A corresponding model was applied to describe the spectral and temporal coherence properties of pulses [1]. Here it provides a smooth transition between spectrally fully coherent and incoherent fields, the former representing conventional coherent pulses and the latter representing stationary fields, all having the same spectrum.

Here we describe a method to generate pulses that may be described temporally and spectrally by the Gaussian Schell model. The method involves Gaussian temporal modulation of a quasi-planar wave emitted by a stationary source with a Gaussian spectrum. In practice the source can be, e.g., a light-emitting diode and the beam can be chopped, e.g., by electro-optic or acousto-optic modulators.

The basic mathematical description of temporally modulated light emitted by stationary sources is provided in Section 2. The theory is applied to Gaussian emission spectra and modulation profiles in Section 3, where we show that the result is a Gaussian Schell-model pulse. The results are interpreted in Section 4 and discussed further in Section 5. Some comments on the analogy with spectrally and spatially Gaussian Schell-model sources are provided, among other things. Finally, some conclusions are drawn in Section 6.

2. Fundamentals

The geometry under investigation is simple. A collimated beam of light emitted by a statistically stationary source is passed through a modulator. After the temporal modulation we have a plane-wave-type pulse, and we proceed to investigate its temporal as well as spectral coherence properties.

Let us denote by $S_s(\omega)$ the spectrum of the stationary light source at $z = 0$. According to the Wiener-Khinchine theorem [9], the mutual coherence function of the source is given by

$$\Gamma_s(\tau) = \int_{-\infty}^{\infty} S_s(\omega) \exp(-i\omega\tau) d\omega, \quad (1)$$

where $\tau = t_2 - t_1$, t_1 and t_2 represent two arbitrary instants of time, and we have used the subscript s to indicate that the field is stationary. The complex degree of coherence of the light field is then given by

$$\gamma_s(\tau) = \Gamma_s(\tau)/\Gamma_s(0), \quad (2)$$

where $\Gamma_s(0)$ is the (constant) intensity of the light beam.

Assume now that the field emitted by the stationary source is modulated temporally by a deterministic but possibly complex-valued function $M(t)$. Since $M(t)$ is deterministic, the modulator does not affect the complex degree of coherence. The transmission of each temporal realization of the field through the modulator is obtained by multiplication with $M(t)$ and, according to the definition of the mutual coherence function, the transmitted field at $z_1 = z_2 = 0$ is described by

$$\Gamma(0, 0; t_1, t_2) = \Gamma_s(0)M^*(t_1)M(t_2)\gamma_s(t_2 - t_1) \quad (3)$$

After propagation in free space at the speed of light c the mutual coherence function takes the form

$$\Gamma(z_1, z_2; t_1, t_2) = \Gamma_s(0)M^*(t_1 - z_1/c)M(t_2 - z_2/c)\gamma_s(t_2 - t_1 - (z_2 - z_1)/c). \quad (4)$$

Clearly the intensity and the complex degree of temporal coherence of the modulated field are given by

$$I(z, t) = \Gamma(z, z; t, t) = \Gamma_s(0)|M(t - z/c)|^2 \quad (5)$$

and

$$\begin{aligned}\gamma(z_1, z_2; t_1, t_2) &= \frac{\Gamma(z_1, z_2; t_1, t_2)}{[I(z_1, t_1)I(z_2, t_2)]^{1/2}} \\ &= \gamma_s(t_2 - t_1 - (z_2 - z_1)/c) \exp[i\Delta\Phi(z_1, z_2; t_1, t_2)],\end{aligned}\quad (6)$$

respectively, where $\Delta\Phi(z_1, z_2; t_1, t_2) = \arg M(t_1 - z_1/c) - \arg M(t_2 - z_2/c)$. Obviously we can control the pulse intensity by means of the absolute value of $M(t)$ and the complex degree of temporal coherence by the functional form of $S_s(\omega)$ and the argument of $M(t)$. For real-valued M , we always obtain a Schell-model temporal pulse, i.e., the degree of temporal coherence is a function of the time difference only, not depending on the choice of the origin of time.

According to the Wiener-Khintchine theorem, the cross-spectral density function of the non-stationary field described by Eq. (4) is

$$W(z_1, z_2; \omega_1, \omega_2) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \Gamma(z_1, z_2; t_1, t_2) \exp[-i(\omega_1 t_1 - \omega_2 t_2)] dt_1 dt_2. \quad (7)$$

The spectral intensity and the complex degree of spectral coherence of the pulse are defined as

$$S(z, \omega) = W(z, z; \omega, \omega) \quad (8)$$

and

$$\mu(z_1, z_2; \omega_1, \omega_2) = \frac{W(z_1, z_2; \omega_1, \omega_2)}{[S(z_1, \omega_1)S(z_2, \omega_2)]^{1/2}}, \quad (9)$$

respectively.

3. Gaussian temporal modulation of sources with Gaussian spectra

We proceed by assuming that both the spectrum of the original stationary source and the temporal modulation profile are Gaussian, i.e.,

$$S_s(\omega) = S_0 \exp\left[-\frac{(\omega - \omega_0)^2}{\Omega_s^2}\right] \quad (10)$$

and

$$M(t) = \exp\left(-\frac{t^2}{2T_m^2}\right), \quad (11)$$

where Ω_s is the characteristic spectral width of the spectrum of the original source and T_m is a measure of the pulse duration (the subscript m refers to the modulator). The model for the spectrum is rather realistic if we consider typical light-emitting diodes, for which the spectral width Ω is of the order of tens of nanometers. While the temporal modulation function $M(t)$ implemented by real electro-optic or acousto-optics modulators is typically more like a flat-top function than a Gaussian function, the Gaussian model used here still provides useful order-of-magnitude estimates for modelling various phenomena.

Inserting Eq. (10) into Eq. (1) and performing the integration we see that

$$\Gamma_s(\tau) = \Gamma_0 \exp\left(-\frac{1}{4}\Omega_s^2\tau^2\right) \exp(-i\omega_0\tau), \quad (12)$$

where $\Gamma_0 = \sqrt{\pi}\Omega_s S_0$. We next insert Eqs. (12) and (11) into Eq. (4). In view of Eqs. (5) and (6), we see that a temporal Gaussian Schell-model pulse with an intensity distribution of the form

$$I(z, t) = \Gamma_0 \exp[-(t - z/c)^2/T^2] \quad (13)$$

and a complex degree of coherence $\gamma(z_1, z_2; t_1, t_2)$ with

$$|\gamma(z_1, z_2; t_1, t_2)| = \exp\left\{-\left[(t_1 - z_1/c) - (t_2 - z_2/c)\right]^2 / 2T_c^2\right\}, \quad (14)$$

$$\arg \gamma(z_1, z_2; t_1, t_2) = \omega_0 [(t_1 - z_1/c) - (t_2 - z_2/c)], \quad (15)$$

where

$$T = T_m \quad \text{and} \quad T_c = \sqrt{2}/\Omega_s. \quad (16)$$

Thus a temporal Gaussian Schell-model plane-wave pulse [1] is generated, with Gaussian distributions of the intensity and absolute value of the complex degree of spectral coherence. The parameters T and T_c describing the properties of the pulse, namely the pulse duration and coherence time, are simply related to the spectral width Ω_s of the stationary field and the modulation length T_m .

We proceed to insert the results of the previous calculation into Eq. (7). This yields a spectral Gaussian Schell-model planar pulse with a spectrum

$$S(z, \omega) = \exp\left[-(\omega - \omega_0)^2 / \Omega^2\right] \quad (17)$$

and a complex degree of spectral coherence $\mu(z_1, z_2; \omega_1, \omega_2)$ of the form

$$|\mu(z_1, z_2; \omega_1, \omega_2)| = \exp\left[-(\omega_1 - \omega_2)^2 / 2\Omega_c^2\right], \quad (18)$$

$$\arg \mu(z_1, z_2; \omega_1, \omega_2) = (\omega_2 z_2 - \omega_1 z_1) / c. \quad (19)$$

where

$$\Omega^2 = \frac{1}{T^2} + \frac{2}{T_c^2} = \Omega_s^2 + \frac{1}{T_m^2} \quad (20)$$

and

$$\Omega_c^2 = \frac{T_c}{T^2} \Omega^2 = \frac{2}{T_m^2} \left(1 + \frac{1}{T_m^2 \Omega_s^2}\right) \quad (21)$$

To obtain the latter forms of these expressions we have used Eqs. (16).

4. Interpretation of the results

Expressions (20) and (21) relate the parameters that define the spectral properties of the pulse, namely the spectral width Ω and the spectral coherence width Ω_c , to the parameters T_m and Ω_s that we may control, at least over certain intervals, by experimental means. These expressions may be called equivalence relations in the sense that there exists an infinite number of combinations of T_m and Ω_s that yield either the same spectrum or the same spectral degree of coherence (but not both at the same time).

It is interesting to look at certain limiting cases. Expression (20) shows that with slow modulation ($T_m \rightarrow \infty$) the spectrum of the pulse is the same as the spectrum of the stationary source, while Eq. (21) implies that $\Omega_c \rightarrow 0$, i.e., the pulse becomes spectrally uncorrelated. These are indeed the expected results in in this limit we have a stationary field, which is well known to be spectrally uncorrelated. In the opposite limit of fast modulation ($T_m \rightarrow 0$) the spectral width grows without limit, as does the spectral coherence width: a fully coherent pulse is obtained. While these limiting cases are both well understood, the Gaussian Schell model provides a continuous, quantitative link between them.

5. Discussion

We have applied the general coherence-theoretic model for temporal model of stationary light sources, presented in Section 2, to the Gaussian Schell-model for which analytical results can be obtained. We stress, however, that the model of Section 2 is generally applicable. In a future paper we plan to discuss in a less formal manner.

The main results of this paper, Eqs. (20) and (21), imply that it is not possible to reduce the spectral width of the original source by control of the spectral coherence with temporal modulation. This is rather obvious in vacuum, but the situation may become more interesting in dispersive media if chirped pulses are employed. We have shown before [1] that there exists an intimate relationship between partial spectral coherence and chirping of coherent pulses if the pulse exceeds the condition for minimum spectral–temporal product; in fact, it is not a simple matter to distinguish these effects without making explicit spectral coherence measurements. In order to perform these it appears necessary to develop new experimental techniques. Hence at least two lines of further research can be foreseen at this stage: modelling of partially spectrally coherent pulse propagation in dispersive media and experimental measurement of the spectral coherence properties of pulses.

In this paper we have assumed that the pulse is effectively a plane-wave pulse as illustrated in the geometry of Fig. 1. It would be of interest to extend the results to spatially finite, partially coherent stationary sources, which are modulated temporally as assumed here. Such an analysis appears mathematically difficult but could offer significant additional insight. For instance, it appears obvious that both the spectra and the spectral coherence properties of fields thus generated would depend on both time and position. Thus extensions to Wolf's theory of spectral shifts to pulsed fields are foreseen.

It is also interesting to compare the experimental generation of spectrally partially coherent Gaussian Schell-model pulses to their spatial, stationary, counterparts [10, 11] (see Fig. 1). The scheme of generating a spectrally Gaussian Schell-model field, illustrated in Fig. 1(a) is clear from the discussion presented above. The spatial analogy is provided in a $4f$ system with a reflective spatial filter $F(\mathbf{r})$ (see Fig. 1(b)). Here one starts with an incoherent Gaussian source in the spatial frequency domain. The intensity distribution of this source (at certain frequency) is a Gaussian function $S_s(\mathbf{f})$. The field in the spatial domain has a constant intensity distribution but a Gaussian Schell-model-type distribution of the complex degree of spatial coherence $\mu(\mathbf{r}_1 - \mathbf{r}_2)$. When truncated spatially by the (reflective) filter function $F(\mathbf{r})$, we obtain a field with a Gaussian spectrum $S(\mathbf{f})$ and a Gaussian distribution of the degree of spatial coherence $\mu(\mathbf{f}_1, \mathbf{f}_2)$ in the spatial frequency domain.

We have so far assumed that the stationary source is modulated externally. However, we do believe that there exist spectrally partially coherent primary sources that could be described by the presented model, at least approximately. The most obvious candidates are excimer lasers and other high-power lasers, whether they are modulated internally or externally. It is difficult to imagine that full (or even substantial) spectral coherence can be gained in laser cavities if the beam effectively exits the cavity without experiencing more than a few passes through it. If this is the case, the pulse must surely be spectrally partially coherent and the theory presented in this paper should be considered for the characterization of its spectral coherence properties, at least to separate the effects of chirping and partial spectral coherence. This leads directly to possible applications in, e.g., optical data transmission over relatively short distances via fiber links; however, these aspects remain to be examined further.

6. Conclusions

We have shown that spectrally partially coherent pulses are generated routinely when stationary sources of light with spectra of finite width (such as light-emitting diodes) are modulated

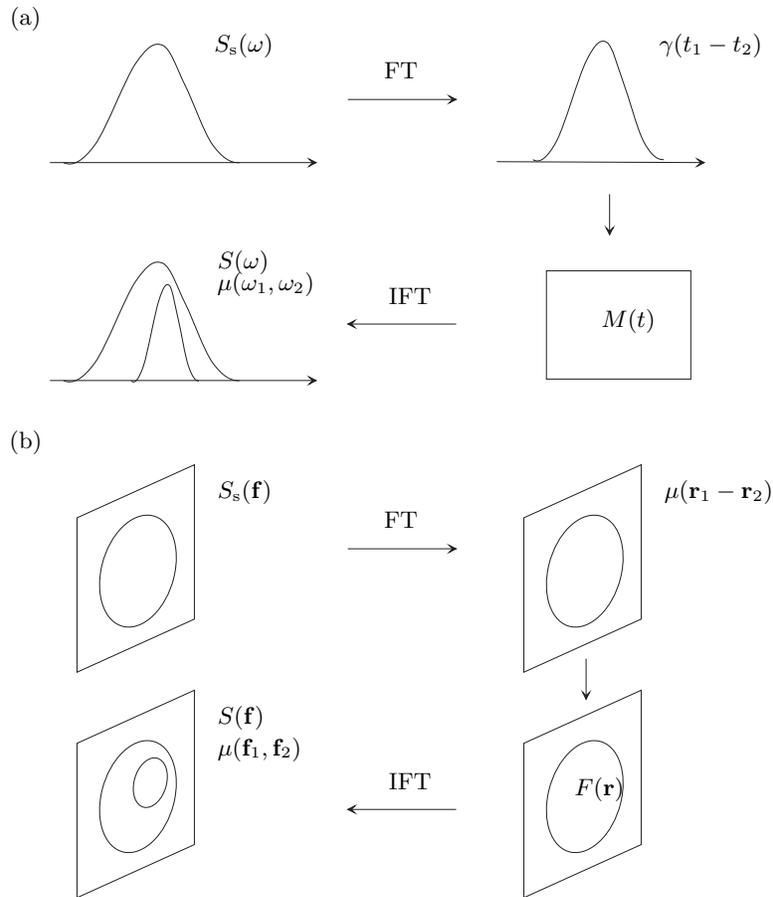


Fig. 1. (a) Pulse generation by chopping the collimated beam emitted by a stationary light source with a modulator. (b) Generation of partially coherent fields in the spatial frequency domain by spatial modulation of a field originating from an incoherent source.

temporally. In particular, we have demonstrated that useful model pulses, namely Gaussian Schell-model pulses introduced in Ref. [1], can be generated by Gaussian temporal modulation of the radiation emitted by sources with Gaussian spectra. Using the general formalism presented in Section 2 it is possible to model any real spectra and modulation functions, and to predict the spectrum and the spectral coherence function numerically. We believe that the results obtained here are of interest in the theory of nonstationary fields and that they may also have substantial practical applications in, e.g., the optimization of short-distance optical fiber communication links.

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