Defocus measurement for random self-affine fractal surfaces

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Abstract: We studied correlation between fractal dimensions and image contrast for metallic surfaces. The study has led to an interesting finding that the maximum fractal dimension of the object surface under imaging gives the best focal plane. The significant finding can be made use of to estimate the best focal plane or measure the focus error with high sensitivity of a few microns, which are well within depth of field of the microscopic imaging system.

OCIS codes: (110.0180) Microscopy; (110.2960) Image analysis; (080.1010) Aberrations (global).

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1. Introduction

In nature, a wide variety of practical surfaces are present with the roughness or height fluctuation distribution well described as random self-affine fractals, scaling from the kilometer-scale structure of mountain terrain to nanometer-scale topology of thin films [1]. Various physical processes, including milling, ion bombardment, etc, produce such kind of surface morphology [2-4]. For analysis using a microscopic imaging system, e.g. an optical microscope or a scanning electron microscope, defocus as one of the main aberrations reduces the sharpness and contrast of the image. This blurs high frequency fine structures making it difficult to resolve them [5]. The experiments also showed defocus causes variations in magnification, depending on the field curvature of the illumination system [6]. Estimation of

#91400 - $15.00 USD Received 4 Jan 2008; revised 8 Feb 2008; accepted 14 Feb 2008; published 19 Feb 2008

the defocus error is essential, espically for a microscopic imaging system with a small depth of field.

In this paper, we study correlation between fractal dimensions and contrast of the images. Our study reveals the peak of fractal dimensions from a sequence of images, having different translations along the optical axis, gives the location of the best focal plane. This is in agreement with the fact that a highest contrast of the image gives the best focal plane of an imaging system [7]. This proposed technique allows measurement of the defocus error with a sensitivity of a few microns, well within the system's depth of field. This feature makes the technique potentially useful in an optical and electronic microscopic imaging system by reducing the defocus aberration.

2. Theory

In a linear shift invariant imaging system, if the object illumination is incoherent, such as incoherent white light, normally used in an optical microscope, the imaging system is treated as linear mapping of intensity. The intensity distribution on the image plane is a convolution of the intensity point spread function $|k(r)|^2$ of the imaging system with the intensity $I_0(r')$ on the object plane [8],

$$I(r) = \iint I_0(r')\left|k(r-r')\right|^2 dr'^2 = I_0(r') * \left|k(r)\right|^2, \tag{1}$$

where $*$ denotes a two-dimensional convolution integral, and r' and r are the position vector on the object plane and image plane, respectively. Considering incremental variance of the intensity distribution, $\sigma^2(\Delta r) = \left\langle \left|I(r+\Delta r) - I(r)\right|^2 \right\rangle$, where Δr is an incremental distance over a fix direction. For an isotropic fractal surface, the incremental variance is expressed by the power function as a function of fractal dimension D_s [9]

$$\sigma^2(\Delta r) = \left\langle \left|I(r+\Delta r) - I(r)\right|^2 \right\rangle \propto \left|\Delta r\right|^{2(3-D_s)}. \tag{2}$$

Using this equation, fractal dimension deduced in Fourier domain by correlation, can be rewritten as $R(r+\Delta r, r) \propto \frac{1}{2}\left(\left|r+\Delta r\right|^{2(3-D_s)} + \left|r\right|^{2(3-D_s)} - \left|\Delta r\right|^{2(3-D_s)}\right)$.

One can also define the fractal dimension in a 3-D set $A \, \varepsilon \, \Re^3$, e.g. a set of pixels in a grey-level image [10]. The three dimensions read as a set of spatial coordinates and a set of intensity values of the image. The fractal dimension (*FD*) using box counting of a set *A* in a grey image is given as,

$$FD = \lim_{n\to\infty} \log\left(N_{(A,\varepsilon)}\right)/\log(1/\varepsilon), \tag{3}$$

where $N_{(A,\varepsilon)}$ denotes the number of boxes of side length, $\varepsilon = 2^n$, $n = 1,2,3,4\ldots$ An image of $M \times M$ pixels can be divided into box of side length $\varepsilon = (2)^n$. If the minimum grey level of the image in the grid (i, j) falls in box number k, and the maximum grey level of the $(i, j)^{th}$ grid falls in box number *1*, then n is counted as [11]

$$n = \mathrm{int}\left(Gray\_\max(i,j)/\varepsilon\right) - \mathrm{int}\left(Gray\_\min(i,j)/\varepsilon\right) + 1. \tag{4}$$

Taking contributions from all grid, $N_{(A,\varepsilon)} = \sum n(i,j)$ is counted for the different values of box dimension, ε. The FD can be estimated from the least-squares linear fit of the $\log\left(N_{(A,\varepsilon)}\right) \sim \log(1/\varepsilon)$ curve.

Either the incremental variance of intensity distribution in Eq. (2) or the difference of the maximum grey level and minimum grey level in Eq. (4) gives a measure of image contrast. Hence we report a correlation between the fractal dimensions and image contrast for random

#91400 - $15.00 USD Received 4 Jan 2008; revised 8 Feb 2008; accepted 14 Feb 2008; published 19 Feb 2008

self-affine fractal surfaces, e.g. a metallic surface. This is using the former to estimate the latter and hence to measure the defocus error, which is fundamental in a microscopic imaging instrument or an adaptive optics system.

3. Experiment, results and discussion

The experiment involved a 30 mm × 20 mm metallic (magnesium alloy AZ91D) specimen, illuminated with incoherent while light and imaged through a long-working-distance microscopic lens. The lens includes a 6.5× Optem® Macro Zoom Lens, a 2× lens attachment, and a close-up lens. The depth of focus of the imaging system is up to 20 μm. With a SONY CCD camera, the scattered light was subsequently digitalized and processed using a computer through a NI IMAQ-PCI card. The field of view is 2.38 mm × 3.17 mm corresponding to the image resolution 576 × 768 pixels. We programmed the software for calculation of fractal dimension in MATLAB and used the program by Sasaki [12] as a reference.

Figure 1(a) shows the texture of a metallic surface captured on focal plane by a human visual judgment. Using the box counting method, fractal dimension (FD) was calculated from the ln(N(ε)) ~ ln(1/ε) curve, as shown in Fig. 1(b). With the box side lengths ε of 4, 8, 16, 32, and 64 pixels (or the scale ratios, $1/\varepsilon = (1/2)^n$, n = 2, 3, 4, 5, and 6), the value is FD = 2.627. In addition, the fractal dimension D was obtained in Fourier domain by least–square linear fit of the log-log curve of magnitude and frequency using the equation, D = (6+β)/2, where β is the slope of the log-log curve [9]. The fractal dimension is D = (6+β)/2 = 2.687, as shown in Fig. 1(c).

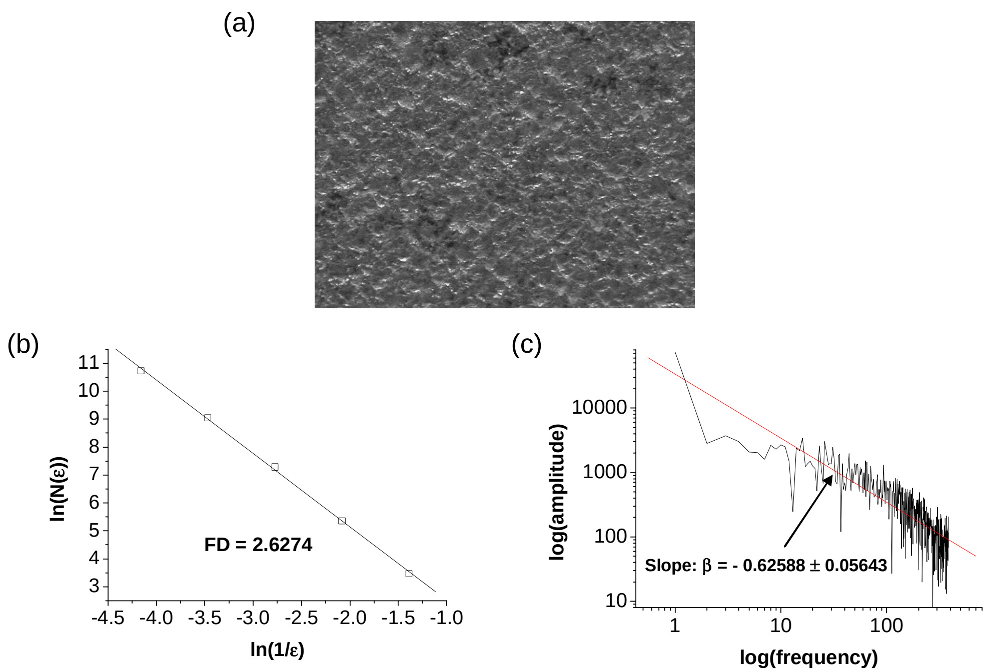

Fig. 1. (a). Intensity distribution scattered from a metallic fractal surface. (b) Fractal dimension, FD = 2.6274, measured from the ln(N(ε))~ln(1/ε) curve. (c) Fractal dimension, D = (6+β)/2 = 2.687, measured from the log-log curve of power magnitude and frequency.

Furthermore, the two fractal dimensions were calculated from sequences of images having a certain displacement (10 μm or 50 μm) in surface normal direction. We captured the image

#91400 - $15.00 USD Received 4 Jan 2008; revised 8 Feb 2008; accepted 14 Feb 2008; published 19 Feb 2008

sequences with the first of which on the focal plane by a human visual judgment. Figure 2 shows the fractal dimension estimated by box counting method and Fourier transform method for the image sequence having a relative displacement of 50 μm in the direction normal to the object surface. Standard deviation (gradient) was calculated for comparison purposes. The maximum FD at the displacement from 50 to 100 μm gives the best focus range. For a more accurate measurement, the fractal dimensions were estimated, as shown in Fig. 3, for an image sequence having a relative displacement of 10 μm within the displacement range from 0 to 200 μm. The maximum FD shows the focal plane at the displacement of 70 μm. Note the almost identical values of standard deviation at the range from 60 to 90 μm shows the box counting dimension has a better sensitivity than standard deviation.

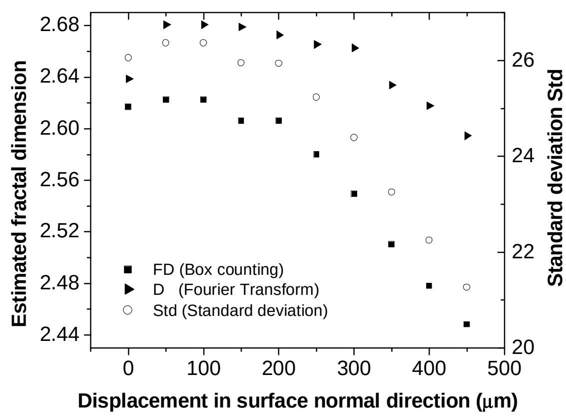

Fig. 2. Fractal dimension measured from a sequence of images, which have a relative displacement of 50 μm. The maximum fractal dimension indicates the best focal plane. The result by the different methods – Fourier power spectrum ("triangle") and box counting ("square") agrees well with that by standard deviation ("circle").

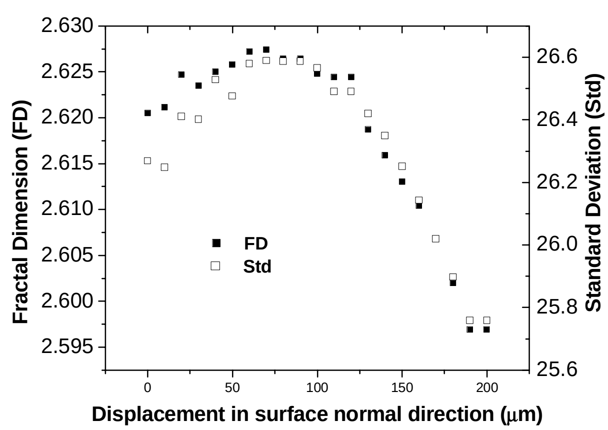

Fig. 3. Fractal dimension measured from a sequence of images, which have a relative displacement of 10 μm. The peak having a maximum fractal dimension represents the best focal plane.

#91400 - $15.00 USD Received 4 Jan 2008; revised 8 Feb 2008; accepted 14 Feb 2008; published 19 Feb 2008

4. Conclusion

We estimate the defocus error using fractal dimensions for isotropic fractal (texture) surfaces with accuracy up to 10 μm, well within the system's depth of field. In contrast, defocus error according to speckle correlation [13] is larger than the depth of field. We demonstrated that the maximum fractal dimension describes a good image contrast and hence represents the best focal plane in a microscopic imaging system.

Acknowledgments

The authors would acknowledge the financial support from A*STAR (Agency for Science, Technology and Research), Singapore, through the project on "novel optical nanoprobe for nanometrology based on surface plasmon polaritons" (SERC Grant No. 072 101 0023).