

Encryption of digital hologram of 3-D object by virtual optics

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Abstract: We present a simple technique to encrypt a digital hologram of a three-dimensional (3-D) object into a stationary white noise by use of virtual optics and then to decrypt it digitally. In this technique the digital hologram is encrypted by our attaching a computer-generated random phase key to it and then forcing them to Fresnel propagate to an arbitrary plane with an illuminating plane wave of a given wavelength. It is shown in experiments that the proposed system is robust to blind decryptions without knowing the correct propagation distance, wavelength, and phase key used in the encryption. Signal-to-noise ratio (SNR) and mean-square-error (MSE) of the reconstructed 3-D object are calculated for various decryption distances and wavelengths, and partial use of the correct phase key.

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1. Introduction

Encryption of 2- or 3-D objects has been extensively studied in recent years since Refregier and Javidi proposed the double-random phase encoding technique [1]. The double-random phase encoding technique is to convert a primary image into a stationary white noise by two independent random phase masks placed at the input plane and the Fourier plane. Recently, a lensless optical encryption system was proposed, in which the double-random phase encoding was implemented in the Fresnel domain [2]. Digital watermarking of 2-D image or 3-D object by double-random phase encoding was also successfully verified [3-5]. It was shown that 2-D images encrypted by double-random phase encoding could be recorded in digital holography, in which the original image was decrypted digitally or optically [6, 7].

Several methods of encrypting or watermarking 2-D images by virtual optics were also reported in which both the encryption and the decryption were performed digitally on a computer [8-11]. This method has an advantage over the conventional optical encryption techniques because it can digitally control the random phase mask and therefore there is no need to manufacture the random phase mask physically and no difficulty in the alignment of the random phase mask during the decryption.

Holograms of 2-D images or 3-D objects could be encrypted with phase-shifting digital holography by insertion of a phase mask in the path of the signal or the reference beam. In this case the digital holograms of the original object and the random phase mask were separately recorded at the CCD (Charge-Coupled Device) and then decryption was performed digitally [12-14]. Another encryption technique in the phase-shifting digital holography was proposed by insertion of two pure random intensity masks in the path of the signal beam [15].

In this paper we propose an encryption of digital hologram of a 3-D object by use of virtual optics. This method can overcome the skew alignment problem in double-random phase encoding because both the encryption and the decryption are performed digitally. We first obtain optically a digital hologram of a 3-D object in the phase-shifting digital holography and then modulate its phase randomly by assuming the hologram to pass a random phase mask. Next, we calculate the Fresnel propagation of this distorted hologram over an arbitrary distance carried by a plane wave of an arbitrary wavelength. We show that the encrypted hologram becomes a stationary white-noise like pattern. We calculate signal-to-noise ratio (SNR) and mean-square-error (MSE) of the reconstructed 3-D object for various decryption conditions.

2. Principle of phase-shifting digital holography

A Fresnel diffraction of a 3-D object can be recorded at CCD by an interference with a plane reference wave in on-axis phase-shifting digital holography as shown in Fig. 1. The digital hologram of the object can be obtained from four different interference patterns generated by changing the relative reference beam phase to 0, $-\pi/2$, $-\pi$, and $-3\pi/2$, respectively [16, 17].

Let the Fresnel diffraction, or the digital hologram, of a 3-D object $U_o(x_o, y_o; z)$ at the CCD plane be given as:

$$U_H(x, y) = \iint \int_{d_o-\Delta/2}^{d_o+\Delta/2} \left\{ \exp[j2\pi z/\lambda_r] / (j\lambda_r z) \right\} \exp[j\pi(x^2 + y^2)/\lambda_r z] \times \\ U_o(x_o, y_o; z) \exp[j\pi(x_o^2 + y_o^2)/\lambda_r z] \exp[-j2\pi(xx_o + yy_o)/\lambda_r z] dz dx_o dy_o \quad (1)$$

The above Fresnel transformation over a distance d_o with object depth Δ along z-axis is represented by

$$FrT\{U_o(x_o, y_o; z)\}_{z=d_o \pm \Delta/2},$$

and its value is represented by multiplication of the amplitude and the phase terms as

$$A_H(x, y) \exp[j\phi_H(x, y)]. \quad (2)$$

It is well known that $A_H(x, y)$ follows the Rayleigh distribution whereas $\phi_H(x, y)$ is uniformly distributed in $[0, 2\pi]$ and therefore the digital hologram is of the form of a complex Gaussian white noise [18].

Let the plane reference wave at the CCD plane be given as:

$$R(x, y; \alpha) = A_R \exp[j(\phi_R + \alpha)], \quad (3)$$

where A_R and ϕ_R are the constant amplitude and the phase of the reference wave, respectively, and α is a relative phase given by either $0, -\pi/2, -\pi,$ or $-3\pi/2$. Then the interference pattern recorded at the CCD plane is given as:

$$\begin{aligned} I(x, y; \alpha) &= |U_H(x, y) + R(x, y; \alpha)|^2 \\ &= A_H^2(x, y) + A_R^2 + 2A_H(x, y)A_R \cos[\phi_H(x, y) - \phi_R - \alpha]. \end{aligned} \quad (4)$$

It can be easily shown that the digital hologram of the 3-D object is obtained from the four interference patterns as:

$$U_H(x, y) = \frac{1}{4} [I(x, y; 0) - I(x, y; -\pi) + j\{I(x, y; -3\pi/2) - I(x, y; -\pi/2)\}]. \quad (5)$$

Figure 1 shows a schematic diagram of the experimental setup. Quarter-wave and half-wave plates control the relative phase of the reference beam.

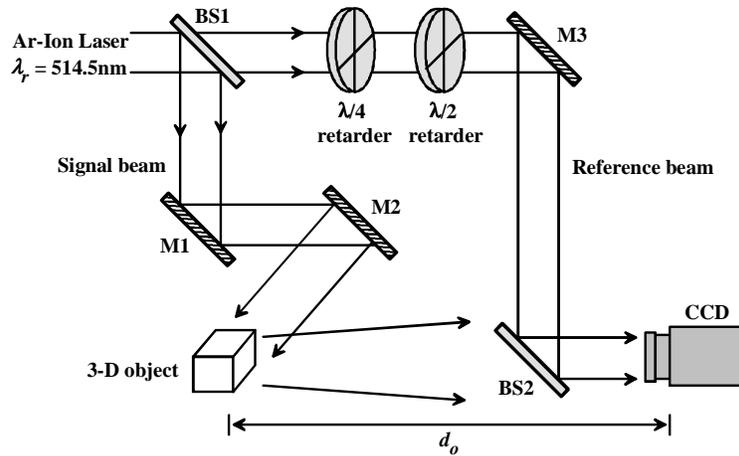


Fig. 1. Phase-shifting digital holography. BS's, Beamsplitter; M's, Mirror.

3. Encryption and decryption of digital hologram of 3-D object

3.1 Encryption of digital hologram

The digital hologram recorded in the conventional method Fig. 1 may need to be encrypted since it can be restored almost completely even if the exact system parameters such as object distance d_o , wavelength of recording beam λ_r , and pixel pitch of CCD are not known. Figure 2

shows the encryption of a digital hologram proposed in this paper. First, digital hologram of a 3-D object at the CCD, $U_H(x, y)$, is optically obtained with phase-shifting digital holography as described in section 2. Next, a computer-generated random phase key, $\phi_E(x, y)$, is attached to the digital hologram at CCD and then the Fresnel propagation of this distorted hologram to an arbitrary plane $z = d_e$ with a virtual plane wave of wavelength λ_e is obtained electronically. The encrypted hologram, $U_E(x_e, y_e)$, is given by:

$$U_E(x_e, y_e) = \left\{ \exp\left[j2\pi d_e / \lambda_e \right] / (j\lambda_e d_e) \right\} \exp\left[j\pi(x_e^2 + y_e^2) / \lambda_e d_e \right] \times \iint_{\Sigma} U_H(x, y) \exp[j\phi_E(x, y)] \exp\left[j\pi(x^2 + y^2) / \lambda_e d_e \right] \exp\left[-j2\pi(xx_e + yy_e) / \lambda_e d_e \right] dx dy, \quad (6)$$

where Σ represents the aperture size of CCD. This Fresnel diffraction integral can be calculated simply by using a fast Fourier-transform algorithm. It is noted that the complex constant $\exp[j2\pi d_e / \lambda_e] / (j\lambda_e d_e)$ can be neglected in general. One can see easily from Eq. (6) that the digital hologram of a 3-D object is encrypted by selecting secretly the random phase key $\phi_E(x, y)$, the virtual propagation distance d_e , and the wavelength λ_e .

It is required that the encrypted hologram $U_E(x_e, y_e)$ should be a stationary white noise to be robust against blind decryptions [1]. In our case this stationary whiteness can be verified by the autocorrelation function given by:

$$E\left\{ U_E^*(x_e, y_e) U_E(x_e + p, y_e + q) \right\} = \left[\frac{1}{(\lambda_e d_e)^2} \sum_{\xi=0}^{N_x-1} \sum_{\eta=0}^{N_y-1} |U_H(\xi, \eta)|^2 \right] \delta(p, q), \quad (7)$$

where $E\{ \}$ represents the ensemble average over a random phase $\phi_E(x, y)$ uniformly distributed in $[0, 2\pi]$ and the asterisk denotes the complex conjugate, and $\delta(p, q)$ is Kronecker delta function. Its derivation is shown in Appendix at the end. Therefore, the encrypted hologram $U_E(x_e, y_e)$ becomes a stationary white noise. This property will be numerically verified in the later section by calculating the autocorrelation function of the encrypted hologram.

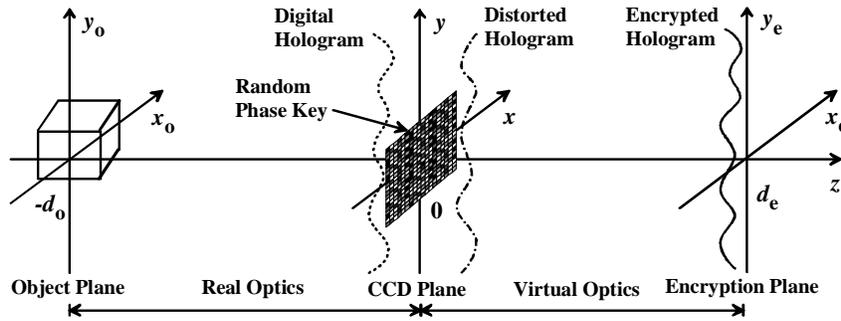


Fig. 2. Encryption of digital hologram of a 3-D object by virtual optics. Digital hologram is multiplied by a computer-generated random phase key at CCD plane and then Fresnel propagation of this distorted hologram over a distance d_e is calculated on a computer.

3.2 Decryption and reconstruction of 3-D object

Decryption of the encrypted digital hologram and reconstruction of the original 3-D object is performed digitally on a computer by virtual optics as shown in Fig. 3. First, the encrypted digital hologram, $U_E(x_e, y_e)$, is inversely Fresnel transformed over the encryption distance d_e with a virtual plane wave of wavelength λ_e . Next, it is multiplied by the conjugated random

phase key, $\exp[-j\phi_E(x, y)]$, followed by an inverse Fresnel transformation over the distance d_o with a virtual plane wave of wavelength λ_r .

In a blind decryption, where d_e , λ_e , d_o , λ_r and $\phi_E(x, y)$ are not known, the reconstructed 3-D object is generally given by:

$$U_R(x_r, y_r; z = d_{r2}) = IFrT\{IFrT\{U_E(x_e, y_e)\}_{z=d_{r1}} \times \exp[-j\phi_D(x, y)]\}_{z=d_{r2}}, \quad (8)$$

where $IFrT\{ \}_{z=d_r}$ represents inverse Fresnel transformation over a distance d_r and $\phi_D(x, y)$ represents the wrong decryption phase key. In this case we use the metrics of SNR and MSE to quantitatively analyze the quality of the 3-D object reconstructed in the presence of errors in the object distance, decryption distance, wavelength, and phase key. The SNR and MSE are defined as:

$$\text{SNR} = \frac{\sum_{x=0}^{N_x-1} \sum_{y=0}^{N_y-1} |U_O(x, y; z = d_o)|^2}{\sum_{x=0}^{N_x-1} \sum_{y=0}^{N_y-1} [|U_O(x, y; z = d_o)| - |U_R(x, y; z = d_{r2})|]^2},$$

$$\text{MSE} = \frac{\sum_{x=0}^{N_x-1} \sum_{y=0}^{N_y-1} [|U_O(x, y; z = d_o)| - |U_R(x, y; z = d_{r2})|]^2}{N_x \times N_y}, \quad (9)$$

where $U_O(x, y; z = d_o)$ and $U_R(x, y; z = d_{r2})$ represent the original object obtained from the digital hologram with no encryption and the reconstructed object from the encrypted hologram, respectively, which are normalized by the maximum of the original object, and N_x and N_y are the numbers of CCD pixels in the x and y coordinates, respectively.

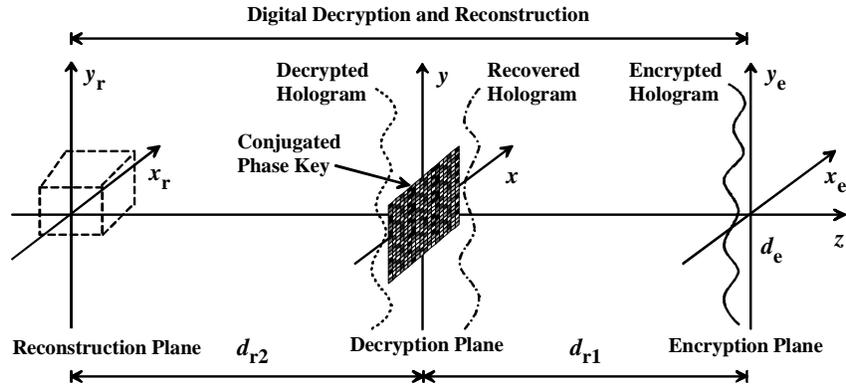


Fig. 3. Decryption of encrypted digital hologram and reconstruction of 3-D object. Both the decryption and the reconstruction are performed digitally on a computer.

4. Experimental results

4.1 Reconstruction of 3-D object from the conventional digital hologram

In the experiment a 3-D object was first recorded optically as a digital hologram by phase-shifting digital holography in the setup of Fig. 1. A toy car was used as a 3-D object with a size of 3.5cm \times 3.0cm \times 7.0cm. It was located at a distance $d_o = 250$ cm from the CCD. Four interference patterns were recorded at the CCD with a plane reference wave of wavelength $\lambda_r = 514.5$ nm by changing its relative phase to 0, $-\pi/2$, $-\pi$, and $-3\pi/2$, respectively. The CCD has pixels of 640 \times 480 at a pitch of 8.4 μ m \times 9.8 μ m.

Figure 4 shows the real and imaginary parts of the digital hologram of the 3-D object recorded without any encryption in the experiment. Figure 5 shows the 3-D objects reconstructed at the distances of 250cm and 150cm, respectively. It is noted from Fig. 5(b) that the 3-D object can be reconstructed fairly well even though there is a 40% error in the object distance in the reconstruction.

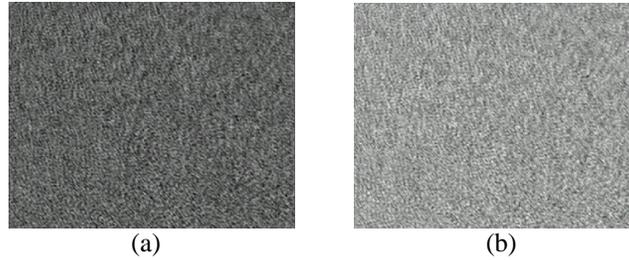


Fig. 4. Digital hologram of a 3-D object. (a) Real part, (b) imaginary part.

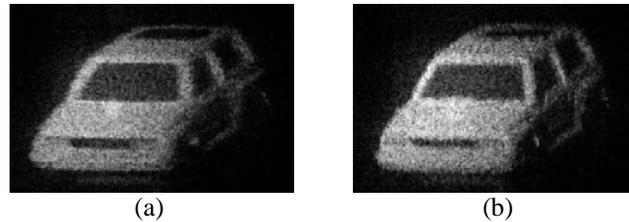


Fig. 5. Reconstructed 3-D objects at (a) $d_r = 250$ cm and (b) $d_r = 150$ cm.

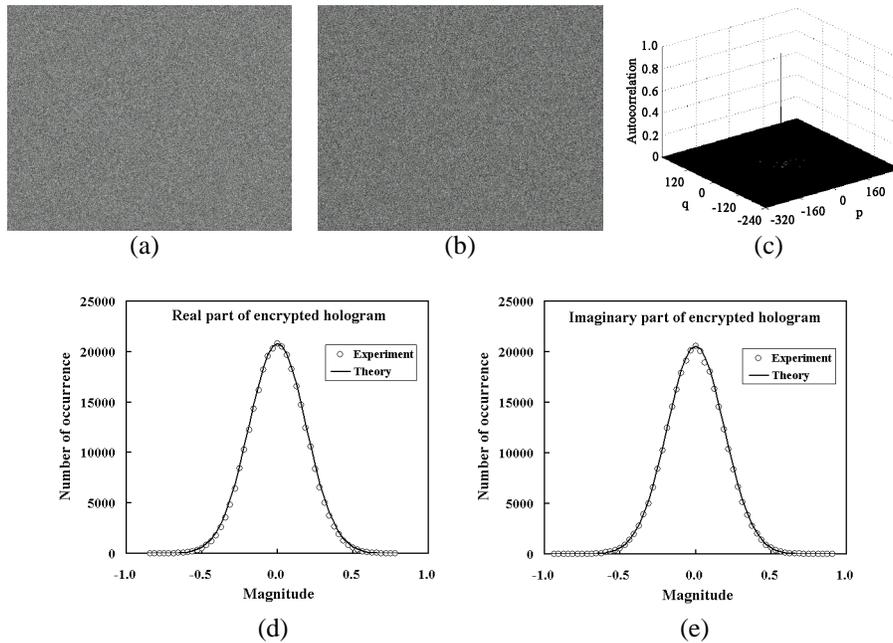


Fig. 6. (a) Real part, (b) imaginary part of the encrypted digital hologram; (c) the autocorrelation of the encrypted digital hologram; histograms of (d) real part, (e) imaginary part of the encrypted digital hologram.

4.2 Encryption of digital hologram into a stationary white noise

For encryption, the digital hologram obtained in the previous section was multiplied by a computer-generated random phase key uniformly distributed in $[0, 2\pi]$. Then it was virtually illuminated by a plane wave of wavelength $\lambda_e = 800\text{nm}$, and the Fresnel propagation was calculated using a fast Fourier-transform algorithm over a distance $d_e = 100\text{cm}$ for the encrypted digital hologram.

Figures 6(a) and 6(b) show the real and imaginary parts of the encrypted digital hologram. Figure 6(c) shows the autocorrelation function of the complex encrypted digital hologram. Note that the autocorrelation is calculated from Eq. (7) to be less than 10^{-3} except at $(p, q) = (0, 0)$, which ensures that the encrypted hologram of our system becomes a stationary white noise. Figures 6(d) and 6(e) show histograms of the real and imaginary parts of the encrypted digital hologram. The mean and variance of the encrypted digital hologram are calculated from Eq. (7) as zero and 0.072. Note that Gaussian distribution with zero mean and variance of 0.036 fits the histograms very well.

4.3 Decryption and reconstruction of 3-D object

When the decryption was performed according to the procedure of section 3.2 using the correct system parameters such as object distance $d_o = 250\text{cm}$, encryption distance $d_e = 100\text{cm}$, wavelengths of $\lambda_e = 800\text{nm}$ and $\lambda_r = 514.5\text{nm}$, and correct phase key, the reconstructed 3-D object from the encrypted hologram was the same as Fig. 5(a). In this case SNR and MSE were given 2.2×10^{26} and 9.4×10^{-30} , respectively.

One may try to reconstruct the 3-D object from our encrypted digital hologram in the conventional reconstruction method as mentioned in section 4.1. In this case the reconstructed 3-D object was given by a white-noise like pattern regardless of the reconstruction distance. SNR and MSE of the reconstructed object were calculated as functions of the reconstruction distance in Fig. 7(a) when a virtual plane wave of wavelength 514.5nm was used as the reconstruction wave.

Next, when the encrypted hologram was decrypted using the correct distances d_e and d_o , and correct wavelengths λ_e and λ_r , but using the incorrect phase key, SNR and MSE were calculated to be 0.76 and 0.0027, respectively. Similarly, SNR and MSE were calculated for the object reconstructed using the correct system parameters but using only a certain portion of the correct decryption phase key in Fig. 7(b). In this case the reconstructed object was recognizable somewhat when the portion of the correct phase key used was larger than 10%. Figure 8 shows the reconstructed 3-D objects when the portion of the correct phase key used in the decryption was 1%, 5%, and 10%, respectively.

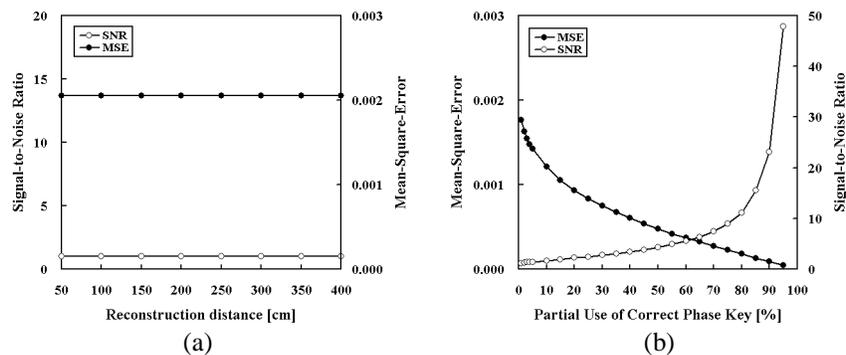


Fig. 7. (a) SNR and MSE versus reconstruction distance in blind decryptions without knowing the correct distances, wavelengths, and phase key. (b) SNR and MSE versus percentage of correct phase key used in the decryption, where the correct distances and wavelengths were known.

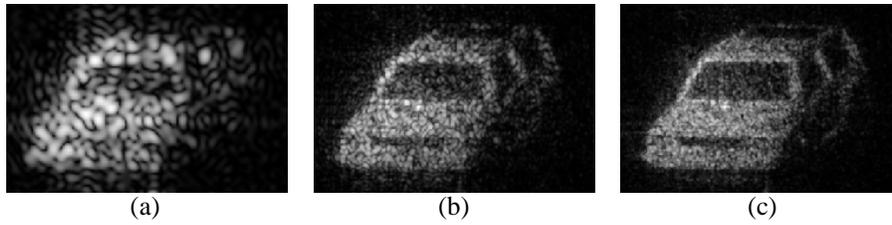


Fig. 8. Reconstructed 3-D objects when the portion of the correct phase key used in the decryption was (a) 1%, (b) 5%, and (c) 10%.

The decryption procedure can be explained by Eq. (8), which is similar to the equation for encryption, Eq. (6), with signs of the exponents reversed. Since, in this case, the first term of a constant phase can be ignored, it is noted that the product of the distance and the wavelength, λd , behaves as a single variable in the decryption. In the experiment the 3-D object was reconstructed from the encrypted hologram with the correct phase key but with a wrong value of the distance-wavelength product $\lambda_e d_e$. Figure 9 shows the 3-D objects reconstructed with errors of 0.01%, 0.05%, and 0.2% in the value of $\lambda_e d_e$ whose correct value is $0.8\mu\text{m} \times 10^6\mu\text{m}$. Figure 10(a) shows SNR and MSE of the reconstructed 3-D object calculated as functions of percent error of $\lambda_e d_e$. Next, Fig. 10(b) shows SNR and MSE of the 3-D object reconstructed with a wrong value of $\lambda_r d_o$, whose correct value is $0.5145\mu\text{m} \times (2.5 \times 10^6)\mu\text{m}$, instead of $\lambda_e d_e$ for comparison. It is noted that the product $\lambda_e d_e$ can be used as another encryption key in addition to the random phase key, and that the product $\lambda_r d_o$, which is required in the reconstruction of a 3-D object in the conventional method, does not work as an encryption key.

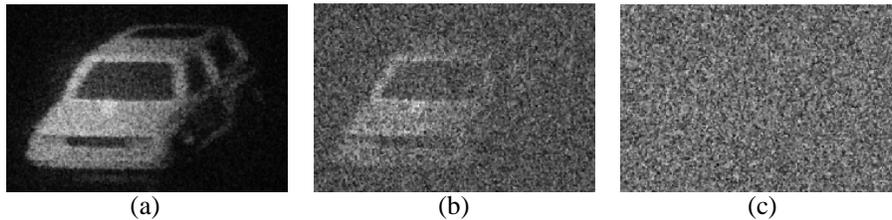


Fig. 9. Reconstructed 3-D objects when the distance-wavelength product error was (a) 0.01%, (b) 0.05%, and (c) 0.2% in the decryption.

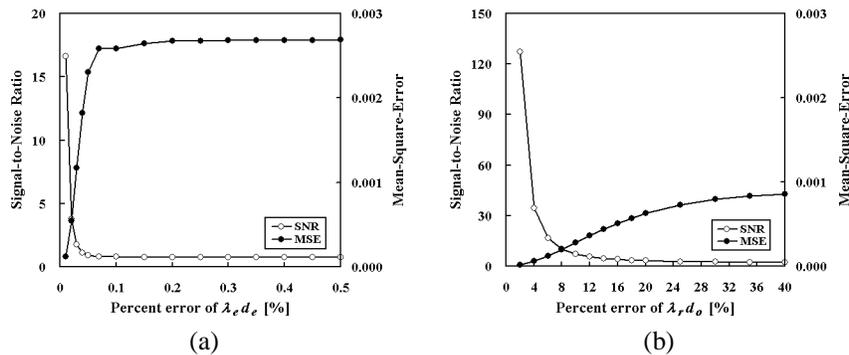


Fig. 10. SNR and MSE versus (a) percent error of $\lambda_e d_e$ and (b) percent error of $\lambda_r d_o$.

5. Conclusion

In summary, we have proposed a simple technique to encrypt a digital hologram of a 3-D object into a stationary white noise by use of virtual optics. The digital hologram obtained with the phase-shifting digital holography was encrypted by our attaching a computer-generated random phase key and then forcing them to Fresnel propagate to an arbitrary plane with a virtual plane wave of a given wavelength. We verified the robustness of the proposed system to blind decryptions and calculated SNR and MSE to analyze the quality of the 3-D object reconstructed in blind decryptions. It was shown in the experiment that the reconstructed 3-D object was seriously blurred to be unrecognized when the portion of the correct decryption phase key used in the reconstruction was less than 1%. When the error in the distance-wavelength product $\lambda_e d_e$ was larger than 0.2% in the decryption, the reconstructed 3-D object appeared as a white-noise like pattern. The proposed system was shown to be more sensitive to an error in the distance-wavelength product than the random phase key. Therefore, the virtual propagation distance and wavelength selected secretly in the encryption could be used as another key in addition to the random phase key in the decryption.

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Appendix

The encrypted digital hologram $U_E(x_e, y_e)$ of Eq. (6) can be represented by the discrete convolution of the distorted hologram $U_H(\xi, \eta)\exp[j\phi_E(\xi, \eta)]$ and the impulse response $h(x, y)$ as follows:

$$U_E(x_e, y_e) = \sum_{\xi=0}^{N_x-1} \sum_{\eta=0}^{N_y-1} U_H(\xi, \eta) \exp[j\phi_E(\xi, \eta)] h(x_e - \xi, y_e - \eta), \quad (\text{A-1})$$

where the impulse response $h(x, y)$ is defined as:

$$h(x, y) = \frac{\exp[j2\pi d_e / \lambda_e]}{j\lambda_e d_e} \exp\left[j\pi(x^2 + y^2)/\lambda_e d_e\right]. \quad (\text{A-2})$$

Then the autocorrelation function of $U_E(x_e, y_e)$ is given by:

$$E\{U_E^*(x_e, y_e)U_E(x_e + p, y_e + q)\} = \sum_{\xi=0}^{N_x-1} \sum_{\eta=0}^{N_y-1} \sum_{\alpha=0}^{N_x-1} \sum_{\beta=0}^{N_y-1} U_H^*(\xi, \eta)U_H(\alpha, \beta) \\ \times E\{\exp[j\phi_E(\alpha, \beta) - j\phi_E(\xi, \eta)]\} h^*(x_e - \xi, y_e - \eta)h(x_e + p - \alpha, y_e + q - \beta). \quad (\text{A-3})$$

Since the random phase key $\phi_E(x, y)$ is uniformly distributed in $[0, 2\pi]$, it can be easily shown that $E\{\exp[j\phi_E(\alpha, \beta) - j\phi_E(\xi, \eta)]\} = \delta(\xi - \alpha, \eta - \beta)$, where $\delta(x, y)$ is Kronecker delta function. Then Eq. (A-3) becomes

$$E\{U_E^*(x_e, y_e)U_E(x_e + p, y_e + q)\} = \sum_{\xi=0}^{N_x-1} \sum_{\eta=0}^{N_y-1} U_H^*(\xi, \eta)U_H(\xi, \eta) \\ \times h^*(x_e - \xi, y_e - \eta)h(x_e + p - \xi, y_e + q - \eta). \quad (\text{A-4})$$

Using Eq. (A-2) it can be shown that

$$h^*(x_e - \xi, y_e - \eta)h(x_e + p - \xi, y_e + q - \eta) = \frac{\exp[j\pi(p^2 + q^2)/\lambda_e d_e]}{(\lambda_e d_e)^2}$$

$$\times \exp\left[j 2\pi\{(x_e - \xi)p + (y_e - \eta)q\} / \lambda_e d_e\right]. \quad (\text{A-5})$$

Therefore, the autocorrelation function Eq. (A-4) can be rewritten as:

$$E\left\{U_E^*(x_e, y_e)U_E(x_e + p, y_e + q)\right\} = \frac{\exp[j\pi(p^2 + q^2)/\lambda_e d_e]}{(\lambda_e d_e)^2} \exp\left[j 2\pi(x_e p + y_e q)/\lambda_e d_e\right] \\ \times \sum_{\xi=0}^{N_x-1} \sum_{\eta=0}^{N_y-1} |U_H(\xi, \eta)|^2 \exp[-j 2\pi(\xi p + \eta q)/\lambda_e d_e]. \quad (\text{A-6})$$

Note that the discrete Fourier transform of the equation is given by a Kronecker delta function multiplied by a constant because the digital hologram $U_H(\xi, \eta)$ is given by a complex Gaussian white noise. Then Eq. (A-6) is rearranged as:

$$E\left\{U_E^*(x_e, y_e)U_E(x_e + p, y_e + q)\right\} = \frac{\exp[j\pi(p^2 + q^2)/\lambda_e d_e]}{(\lambda_e d_e)^2} \exp\left[j 2\pi(x_e p + y_e q)/\lambda_e d_e\right] \\ \times \left[\sum_{\xi=0}^{N_x-1} \sum_{\eta=0}^{N_y-1} |U_H(\xi, \eta)|^2 \right] \delta(p, q) \\ = \left[\frac{1}{(\lambda_e d_e)^2} \sum_{\xi=0}^{N_x-1} \sum_{\eta=0}^{N_y-1} |U_H(\xi, \eta)|^2 \right] \delta(p, q). \quad (\text{A-7})$$

It is noted from Eq. (A-7) that the encrypted digital hologram is a complex stationary white noise with a zero mean and a variance of

$$\left[\frac{1}{(\lambda_e d_e)^2} \sum_{\xi=0}^{N_x-1} \sum_{\eta=0}^{N_y-1} |U_H(\xi, \eta)|^2 \right]. \quad (\text{A-8})$$