Stable discrete surface light bullets

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Abstract: We analyze spatiotemporal light localization near the edge of a semi-infinite array of weakly coupled nonlinear optical waveguides and demonstrate the existence of a novel class of continuous-discrete spatiotemporal solitons, the so-called *discrete surface light bullets*. We show that their properties are strongly affected by the presence of the surface. To this end the crossover between surface and quasi-bulk bullets is studied by analyzing the families of solitons propagating at different distances from the edge of the waveguide array.

OCIS codes: (190.4420) Nonlinear optics, transverse effects in; (190.5530) Nonlinear optics: Pulse propagation and solitons; (190.5940) Self-action effects.

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1. Introduction

Surface modes are regarded as a special type of waves propagating along and localized near an interface separating two different media. In optics, electromagnetic surface waves are known to exist in the linear limit as the waves localized at the interface separating either *two homogeneous* (one of them has to be surface-active, i.e., exhibits a negative permittivity, [1]) or *homogeneous* and *periodic* dielectric media [2], while nonlinear dielectric media can support different types of nonlinear guided waves localized at or near the surfaces [3, 4]. Nonlinear guided waves in planar waveguides have been studied extensively about 20 years ago (see a series of the pioneering papers [5, 6, 7, 8, 9] and references therein).

Recently, the interest in the study of electromagnetic surface waves has been renewed after the first theoretical prediction [10] and subsequent experimental demonstration [11] of nonlinearity-induced self-trapping of light near the edge of a one-dimensional waveguide array with self-focusing nonlinearity that can lead to the formation of *a discrete surface soliton*. A similar effect of light localization has been predicted theoretically and observed experimentally for defocusing nonlinear media [12, 13], when the surface gap solitons can be regarded as an optical analog of nonlinear Tamm states [14].

In this paper, we suggest an important extension of the concept of discrete surface solitons and initiate the study of a rich variety of the surface-mediated effects associated with *spatiotemporal evolution* of nonlinear surface waves and surface solitons. The study of optical spatiotemporal solitons, often referred to as *light bullets* in the three-dimensional case [15, 16], has been attracted attention of many research groups as an unique opportunity to create a self-supporting fully localized object in space and time. In particular, the existence and properties of continuous-discrete spatiotemporal solitons have been extensively investigated in cubic [17, 18, 19, 20] and quadratic [21, 22] nonlinear optical media, and stable odd-symmetry spatiotemporal solitons have been shown to exist. In this work, we extend this analysis to the case of nonlinear surface waves, and consider a truncated array of weakly coupled optical waveguides taking into account the spatiotemporal evolution of light near the edge of the waveguide array. We combine the key features of both *continuous* and *discrete* nonlinear models and analyze, for the first time to our knowledge, the existence and properties of continuous-discrete soliton families describing *spatiotemporal discrete surface solitons*.

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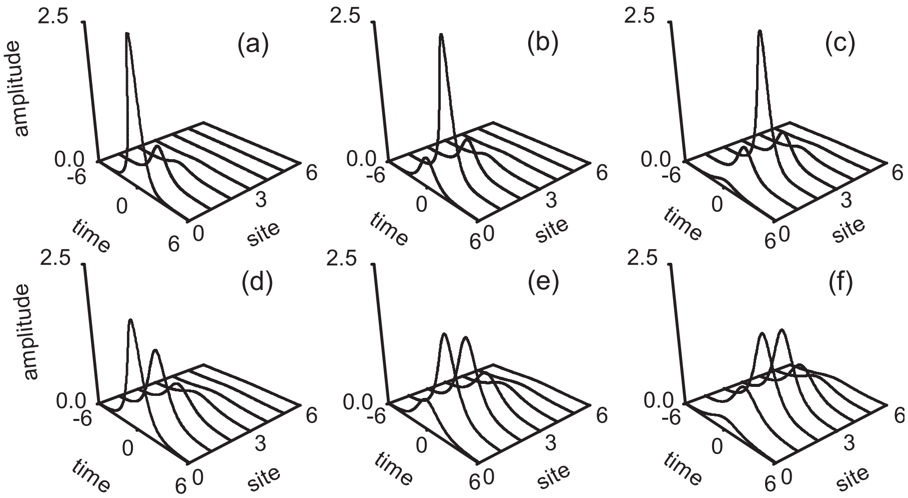

Fig. 1. Examples of *stable* spatiotemporal surface solitons localized at distances of (a) $d = 0$, (b) $d = 1$, and (c) $d = 2$ from the edge of the waveguide array (at $\beta = 3.5$), and *unstable* modes centered at distances of (d) $d = 0$, (e) $d = 1$, and (f) $d = 2$ (at $\beta = 2.5$).

2. Model

We consider an array of weakly coupled nonlinear optical waveguides described, in the tight-binding approximation, by the effective discrete nonlinear equations [23]. We take into account the spatiotemporal evolution of light, similar to the earlier studies [17, 18, 19, 20], but also assume that our waveguide array is truncated so that the light localization occurs near the edge of the waveguide array. The corresponding nonlinear model can be written in the form,

$$\begin{aligned} &i\frac{\partial E_1}{\partial z} - \gamma\frac{\partial^2 E_1}{\partial t^2} + E_2 + \sigma|E_1|^2 E_1 = 0, \\ &i\frac{dE_n}{dz} - \gamma\frac{\partial^2 E_n}{\partial t^2} + (E_{n+1} + E_{n-1}) + \sigma|E_n|^2 E_n = 0, \quad n \geq 2, \end{aligned} \tag{1}$$

where $n = 1$ designates the edge of the waveguide array. In this semi-infinite continuous-discrete model (1) the propagation coordinate z and the dispersion coefficient γ are normalized to the intersite coupling V. In deriving Eqs. (1) the actual electric field in the n^{th} guide $\mathbf{E}_n$ has been decomposed into the product of the vectorial guided mode profile of the isolated channel waveguide $\mathbf{e}(x,y)$ and the respective mode amplitude $\mathscr{E}_n$, which can be finally normalized to give $E_n = \sqrt{\chi_{\mathrm{eff}}/V}\mathscr{E}_n$, where the effective nonlinear coefficient is $\chi_{\mathrm{eff}} = \frac{\omega}{c}\frac{n_2}{A_{\mathrm{eff}}}n_2$ being the nonlinear refractive index of the material and A_{eff} the effective mode area. $\sigma = \pm 1$ defines focusing or defocusing nonlinearity of the waveguide material, respectively.

3. Spatiotemporal surface waves

First, we are looking for spatiotemporal soliton solutions of this nonlinear model in the form $E_n(t;z) = \exp(i\beta z)E_n(t)$, where β is the nonlinearity-induced shift of the waveguide propagation constant, serving likewise as a family parameter, and the envelope $E_n(t)$ describes the temporal evolution of the soliton-like pulse in the $n-$th waveguide. Although in a discrete model various combinations of the signs of dispersion and nonlinearity as well as the spatial topology (unstagged solutions - in phase solitons, staggered solutions - π- out of phase solutions) may potentially lead to spatio-temporal localized solution we restrict ourselves here for the sake of

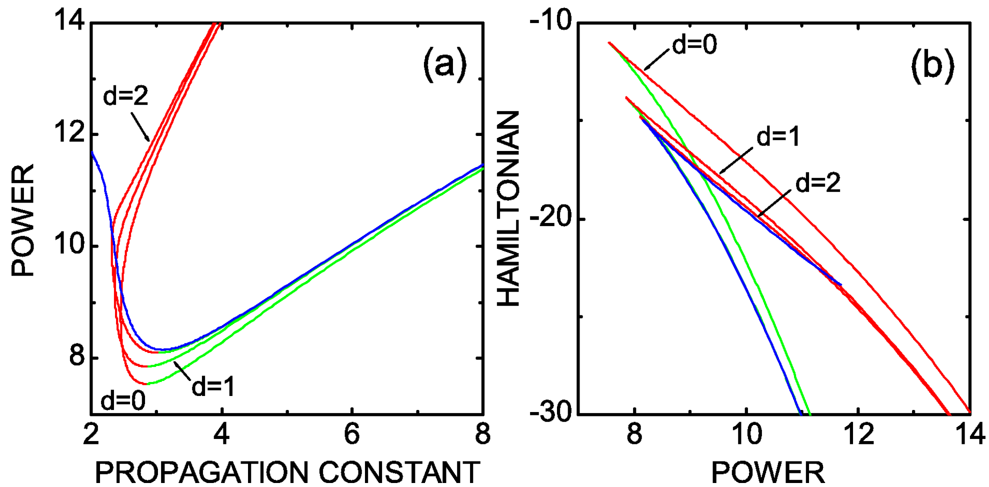

Fig. 2. Families of the spatiotemporal surface solitons. (a) Normalized power vs. propagation constant β for the surface solitons located at the distances $d = 0$, $d = 1$, and $d = 2$ from the edge of the array. (b) Hamiltonian vs. power for the localized surface modes. Stable branches are plotted by black lines whereas unstable branches– by red lines; the blue lines show the corresponding dependencies for the case of odd spatiotemporal solitons in the infinite waveguide array.

clarity to the case of anomalous dispersion $(\gamma < 0)$, focusing nonlinearity $(\sigma = +1)$ and in-phase solitons. If we scale out the dispersion parameter by the transformation $t \to \tau\sqrt{|\gamma|}$we obtain

$$\frac{d^2E_1}{d\tau^2} - \beta E_1 + E_2 + |E_1|^2 E_1 = 0,$$
$$\frac{d^2E_n}{d\tau^2} - \beta E_n + (E_{n+1} + E_{n-1}) + |E_n|^2 E_n = 0, \quad n \geq 2. \tag{2}$$

We find numerically localized solutions $E_n(t)$ of the coupled equations (2) assuming that the amplitude of the pulses in each waveguide, $\max|E_n|$, decays rapidly far from the edge of the waveguide array, so that the corresponding solution describes a mode localized near the surface. We find the localized surface solitons by solving (2) using a standard band-matrix algorithm [24] to deal with the corresponding two-point boundary-value problem.

Figures 1(a-f) show several examples of the nonlinear spatiotemporal continuous-discrete localized states (‘discrete surface light bullets’) located at different distances d from the surface for the case of the focusing nonlinearity, similar to the case of multiple surface plane waves analyzed earlier [25]. The nonlinear modes can be characterized by the total mode power

$$P(\beta) = \sum_{j=1}^{\infty} \int_{\infty}^{+\infty} |E_j(\beta)|^2 d\tau\,, \tag{3}$$

which is a conserved quantity. The localized solution centered at the edge waveguide $(n = 1)$ describes the light bullet with the maximum localized at the surface, this solution is a spatiotemporal generalization of the discrete surface solitons predicted earlier by Makris et al. [10]. However, there exist multiple localized states near the surface, and their stability is an important characteristic of an interplay between nonlinearity, dispersion and discrete diffraction in the array, on one hand, and the surface created by the lattice truncation, on the other. Therefore, we find other spatiotemporal states including both *odd* and *even* modes [23] located at the finite

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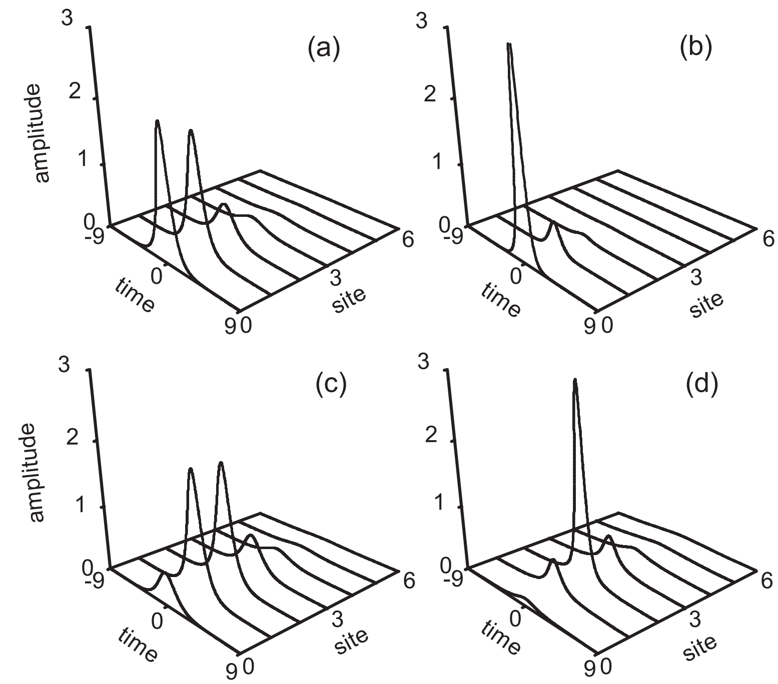

Fig. 3. Instability-driven evolution of unstable solitons corresponding to the upper branches in Fig. 2 for $\beta = 3$. (a,b) Reshaping of the unstable $d = 0$ soliton after the propagation for $z = 1200$ into a stable $d = 0$ soliton. (c,d) Hopping of the unstable $d = 1$ soliton ($z = 0$) into the neighboring site and the generation of a stable $d = 2$ soliton at $z = 1200$.

distances $d = 1, 2, \ldots$ from the edge of the waveguide array. Several such modes are shown in Figs. 1(b-f), with the corresponding power dependencies constructed in Fig. 2(a); they describe a crossover regime between the continuous-discrete surface light bullet of Fig. 1(a), with the maximum amplitude located at the surface, and their counterpart predicted to exist inside of the waveguide arrays [17, 18, 19] when the surface effects vanish.

If we compare the corresponding power curves of different surface modes including the case of a spatiotemporal soliton deep inside the array [blue curve in Fig. 2(a)], we notice that the threshold power of surface localized modes is lower than that of the bulk mode. Therefore, in sharp contrast with one-dimensional surface solitons [10, 25], the surface of a waveguide array creates an effectively attractive potential for the spatiotemporal localized modes that reduces the threshold power for the mode localization.

To analyze linear stability of the nonlinear states found numerically, we calculate not only the mode power (2) but also the second conserved quantity of the dynamical system (2), the system's Hamiltonian H

$$H = \sum_{j=1}^{\infty} \int_{-\infty}^{+\infty} \left(\left|E_j - E_{j-1}\right|^2 - 2\left|E_j\right|^2 + \left|\frac{\partial E_j}{\partial \tau}\right|^2 - \frac{1}{2}|E_j|^4 \right) d\tau \,. \qquad (4)$$

Stable spatiotemporal solitons should correspond to the lower branch of the dependence $H = H(P)$. The typical single cusp-behaviour of the dependence $H = H(P)$ is shown in Fig. 2(b) where the lower branches correspond to the stable surface modes. This observation is confirmed by direct simulations of the propagation of the stationary solitons perturbed by a white noise.

The stability results follow from the dependence $H = H(P)$ of Fig. 2(b), and they have been

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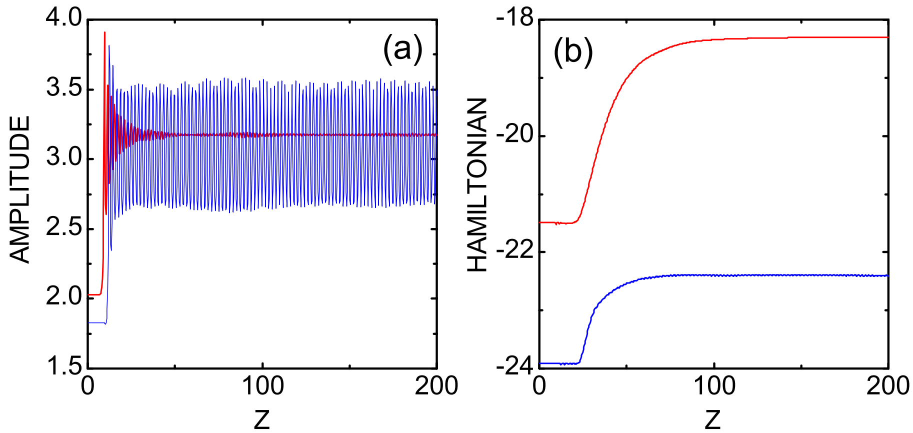

Fig. 4. Evolution of the soliton amplitude (a) and Hamiltonian (b) versus propagation distance for the tho instability scenarios shown in Fig. 3. Red lines: the reshaping of the unstable $d = 0$ solitons into the stable $d = 0$ soliton; blue lines: the flipping of the unstable $d = 1$ soliton into a stable $d = 2$ one.

checked in direct simulations of the dynamical equations (1) carried out by means of the Crank-Nicholson scheme with transparent boundary conditions to account for an escape of radiation from the computation window. The system of nonlinear finite-difference equations was solved first by means of the Picard iteration method [26], and the resulting linear system was treated using the Gauss-Seidel iterative scheme. For a good convergence, five Picard iterations and six Gauss-Seidel iterations were required. We have employed a transverse grid with the step-length $\Delta\tau = 0.02$, and used a typical longitudinal step-size of $\Delta z = 2\times 10^{-4}$.

Figures 3(a-d) demonstrate two different scenarios of the evolution of unstable high-power spatiotemporal solitons located at the distances $d = 0$ and $d = 1$ from the surface and corresponding to the upper unstable branches in Fig. 2(b). Typically, we observe either reshaping of an unstable soliton after its propagation into a stable soliton of the same family [see Fig. 3(a,b)] or hopping of the surface mode into the neighboring site and the the formation of a stable soliton of another family with the center position shifted away from the surface [see Figs. 3(c,d)]. These instability-driven scenarios are confirmed by a direct study of the evolution of the soliton amplitude and the corresponding Hamiltonian, as shown in Figs. 4(a,b), that indicate clearly both switching and hopping mechanisms of the mode instability.

4. Conclusions

We have analyzed the spatiotemporal light localization near the edge of a semi-infinite one-dimensional array of weakly coupled nonlinear waveguides. We have revealed the existence of a novel class of continuous-discrete spatiotemporal surface solitons (discrete surface light bullets) and described their unique properties. Our results can be easily extended for describing spatiotemporal localization effects for staggered solitons such as surface gap solitons [12, 13] in defocusing nonlinear media with normal dispersion.

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