

Fast approximation of transfer cross coefficient for optical proximity correction

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Abstract: Model Based Optical Proximity Correction (MBOPC) is since a decade a widely used technique that permits to achieve resolutions on silicon layout smaller than the wavelength used in commercially-available photolithography tools. This is an important point, because patterns dimensions on masks are continuously shrinking. Commonly-used algorithms, involving Transfer Cross Coefficients (TCC) drawn from Hopkins formulation to compute aerial images during MBOPC treatment are based on TCC decomposition into its eigenvectors using matricization and the well known Singular Value Decomposition (SVD) tool. This technique remains highly runtime consuming. We propose in this paper to extend a fast fixed point algorithm to estimate an *a priori* fixed number of leading eigenvectors required to obtain a good approximation while ensuring a low information loss for computing aerial images.

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References and links

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1. Introduction

Model Based Optical Proximity Correction (OPC) can treat layouts by deforming mask pattern to improve resolution on silicon. As for current masks, several billions of segments have to be moved during several iterations necessary to reach convergence, fast and accurate algorithms

are mandatory to perform OPC on a mask in a reasonable time for industry. To overcome this limitation, mathematical simplifications have to be done. As imaging with a lithography system (see Fig. 1) is analogous to microscopy, the theory used in Model Based OPC is drawn from works originally designed for microscopy theory. Fourier optics was first used to describe the image formed by a microscope [1, 2]. In 1951, Hopkins developed an other formulation or method that has been afterward used in other works involving image formation [3, 4, 5, 6]. This formulation has the advantage of a four way transmission function independent of mask layout. However, this method remains highly runtime consuming and needs a numerical approximation in order to reach production constraints.

In most lithography simulation tools, Sum Of Coherent Systems (SOCS) approximation to Hopkins partial coherent system model is used [7]. This approximation consisting in a discretization whose terms are called Transfer Cross Coefficients (TCC) and a decomposition into its eigenvectors describes the illumination and projection pupils. It can be thus calculated only once for a given imaging system. Nevertheless, runtime needed by existing algorithm SOCS can be improved using an other decomposition method.

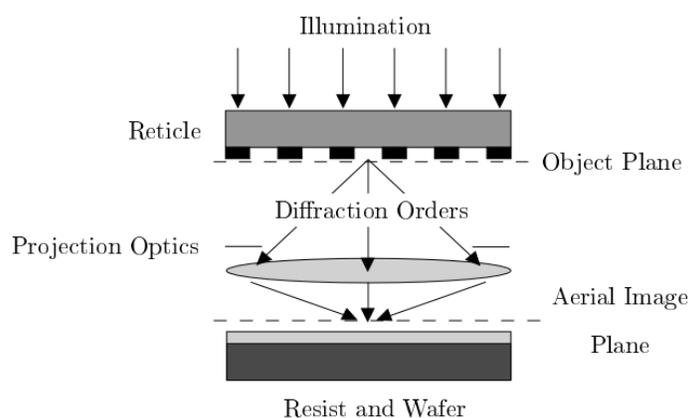


Fig. 1. General optical lithography process diagram.

The remainder of the paper is organized as follows : Section 2 reviews optical equations decomposition in lithography. Section 3 presents a new algorithm to estimate leading eigenvectors by using fixed point algorithm. Experimental results are presented in Section 4. Finally, Section 5 concludes this paper.

2. Overview on existing methods

2.1. Hopkins Formulation

Diffraction theory has always been a domain where equations formulation are complex and heavy to compute. Hopkins formulation is an original theory of image formation[4] in terms of a phase coherence factor introduced in [3]. It leads to a four-fold integral in the frequency domain, involving the phase coherence function, Γ , the complex transmission of the optical system, F , and the complex transmission of the object, E . Therefore, the optical intensity in the image plane is written as follows:

$$I(u, v) = \left(\frac{1}{2\pi} \right)^2 \sum_{i_1}^{I_1} \sum_{i_2}^{I_2} \sum_{i_3}^{I_3} \sum_{i_4}^{I_4} TCC(i_1, i_2, i_3, i_4) E(i_1, i_2) E^*(i_3, i_4) e^{i\{(i_1-i_3)u+(i_2-i_4)v\}} \quad (1)$$

where

$$TCC(i_1, i_2, i_3, i_4) = 2\pi \iint_{-\infty}^{+\infty} \Gamma(x, y) F(x + i_1, y + i_2) F^*(x + i_3, y + i_4) dx dy \quad (2)$$

Typically, $I_1 = I_2 = I_3 = I_4 = I$ for non-astigmatic systems for symmetry reasons.

2.2. Hopkins model Decomposition

Commonly used Hopkins model for aerial image calculation [5] provides a general, parametric scalar imaging formulation. In [8] a modal expansion using Fourier decomposition of Hopkins model is proposed. Cobb [7] also developed an imaging system model involving a decomposition of Hopkins. Eigenvector decomposition of TCC obtained in the Hopkins imaging formulation is used. First, in 1D case TCC is represented as a matrix. Singular Value Decomposition (SVD) of TCC is performed to obtain eigenvectors also referred as *optical kernels*. In 2D case, TCC is represented as a four indices array, consequently SVD can not directly be applied. To circumvent this restriction, Cobb proposed to unfold this multiway array as a matrix \mathbf{T} owing to a column stacking function.

Usual SVD algorithm is then applied to $I^2 \times I^2$ matrix \mathbf{T} yielding the decomposition

$$\mathbf{T} = \sum_{k=1}^{I^2} \lambda_k \mathbf{u}_k \mathbf{u}_k^H \quad (3)$$

the inverse column stacking operation leads to the desired functions, $\Phi_k = \mathcal{S}^{-1}(\mathbf{u}_k)$. This algorithm may become highly runtime consuming as I increases and computes all eigenvectors as only few of them are used in practical cases. The new algorithm proposed in this paper computes only the required leading eigenvectors and reduces therefore computational load.

3. Runtime improvement: Proposed algorithm

To compute the required eigenvectors instead of SVD in SOCS algorithm we propose an original adaptation of a Fixed Point algorithm [9]. It allows to spare a high amount of computational time while preserving a high level of accuracy. One way to compute the K orthonormal basis vectors is to use Gram-Schmidt method (see **Algorithm 1**).

The eigenvectors associated with largest eigenvalues will be calculated first. Similarly, all the remaining $K - 1$ basis vectors (orthonormal to the previously measured basis vectors) will be measured one by one in a reducing order of dominance. The previously measured $(p - 1)^{\text{th}}$ basis vectors will be utilized to find the p^{th} basis vector. The algorithm for p^{th} basis vector will converge when the new value \mathbf{u}_p^+ and old value \mathbf{u}_p are such that $\mathbf{u}_p^{+T} \mathbf{u}_p = 1$. It is usually economical to use a finite tolerance error to satisfy the convergence criterion $|\|\mathbf{u}_p^{+T} \mathbf{u}_p - 1\|| < \eta$ where η is a prior fixed threshold. Let $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_K]$ be the matrix whose columns are the K orthonormal basis vectors. Then $\mathbf{U}\mathbf{U}^T$ is the projector onto the subspace spanned by the K eigenvectors associated with the K largest eigenvalues. This subspace is also called "signal subspace".

Algorithm 1 Fixed-point

1. Choose K , the number of principal axes or eigenvectors required to estimate. Consider matrix \mathbf{T} and set $p \leftarrow 1$.
 2. Initialize eigenvector \mathbf{u}_p of size $d \times 1$, *e. g.* randomly;
 3. Update \mathbf{u}_p as $\mathbf{u}_p \leftarrow \mathbf{T}\mathbf{u}_p$;
 4. Do the Gram-Schmidt orthogonalization process $\mathbf{u}_p \leftarrow \mathbf{u}_p - \sum_{j=1}^{p-1} (\mathbf{u}_p^T \mathbf{u}_j) \mathbf{u}_j$;
 5. Normalize \mathbf{u}_p by dividing it by its norm: $\mathbf{u}_p \leftarrow \frac{\mathbf{u}_p}{\|\mathbf{u}_p\|}$.
 6. If \mathbf{u}_p has not converged, go back to step 3.
 7. Increment counter $p \leftarrow p + 1$ and go to step 2 until p equals K .
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4. Experimental results

The proposed methods can be applied to any multidimensional data set such as image, multicomponent seismic signals, hyperspectral images. In this paper we focus only on TCC matrixization approximation. We wish to compare Fixed Point algorithm benefits over SVD. We use square size matrices with increasing rows and columns number as data set to compare algorithms runtime and accuracy.

4.1. Runtime improvement

We first focus on Fixed Point algorithm runtime gain toward SVD algorithm. Figure 2 shows runtime curves of increasing size matrices decomposition. Matlab[®] SVD algorithm is used. We chose to compute 10 eigenvectors for each matrix with fixed point algorithm. This value corresponds to an average value of eigenvectors number used in classical OPC models. However, as Fixed Point algorithm runtime is linear with eigenvectors number to compute, Table 1 shows that this number can be increased up to around 300 eigenvectors for 3000×3000 matrices and up to around 100 eigenvectors for 120×120 matrices.

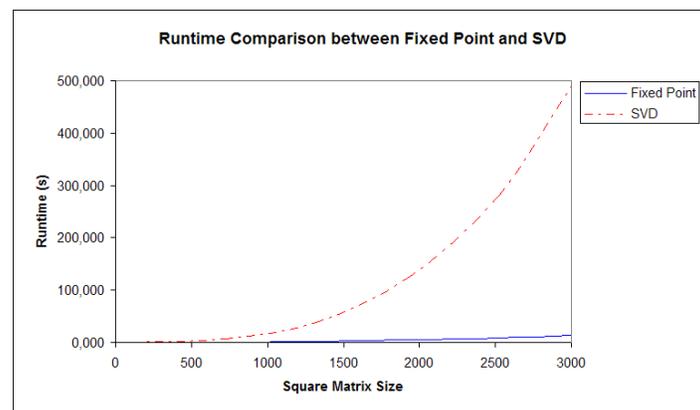


Fig. 2. Computational times (in sec.) as a function of the number of rows and columns.

Table 1. Some numerical values corresponding with Fig. 2.

Matrix size	120	720	1320	1920	2520	3000
SVD runtime (sec)	0,109	6,262	38,911	122,810	279,762	489,808
Fixed point runtime (sec)	0,010	0,406	1,762	4,536	9,220	14,522
SVD/FP rate	10,629	15,441	22,083	27,073	30,343	33,728

4.2. Fixed Point algorithm reconstruction error

In order to assess our algorithm precision toward SVD decomposition, Fig. 3 shows reconstruction error of both Fixed Point and SVD algorithms, $\frac{\|T-T'\|}{\|T\|}$, where T denotes the original matrix and T' the reconstructed matrix. For constancy purposes with Fixed Point algorithm, only the 10 first eigenvectors computed with SVD are used for reconstruction.

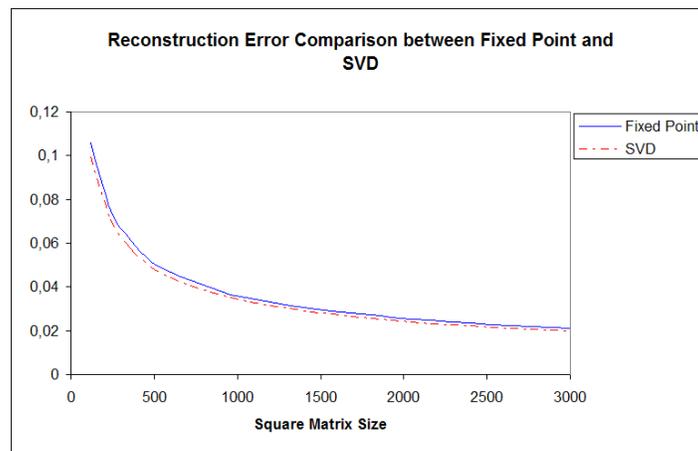


Fig. 3. Reconstruction error comparison between SVD and Fixed Point Algorithm.

The computations have been run with Matlab[®] on a 2.4GHz dual core Pentium with 4Go RAM under Windows XP[®].

5. Conclusion

In this paper, we have proposed a fast method to Transfer Cross Coefficient approximation in diffraction theory of optical images. The main objective of this work is to improve runtime by using fixed point algorithm. Promising results have been given concerning the estimation of leading eigenvectors. With an analogous error to SVD, the proposed method allows to spare huge amount of computer load.