

# X – waves in nonlinear normally dispersive waveguide arrays

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**Abstract:** We theoretically demonstrate that optical discrete X-waves are possible in normally dispersive nonlinear waveguide arrays. We show that such X-waves can be effectively excited for a wide range of initial conditions and in certain occasions can be generated in cascade. The possibility of observing this family of waves in AlGaAs array systems is investigated in terms of pertinent examples.

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## References and links

1. D. N. Christodoulides, F. Lederer, and Y. Silberberg, "Discretizing light behaviour in linear and nonlinear waveguide lattices," *Nature* **424**, 817-823 (2003).
2. H. S. Eisenberg, Y. Silberberg, R. Morandotti, and J. S. Aitchison, "Diffraction management," *Phys. Rev. Lett.* **85**, 1863-1866 (2000).
3. D. N. Christodoulides and R. I. Joseph, "Discrete self-focusing in nonlinear arrays of coupled waveguides," *Opt. Lett.* **13**, 794-796 (1988).
4. R. Morandotti, U. Peschel, J. S. Aitchison, H. S. Eisenberg, and Y. Silberberg, "Dynamics of discrete solitons in optical waveguide arrays," *Phys. Rev. Lett.* **83**, 2726-2729 (1999).
5. J. Meier, J. Hudock, D. N. Christodoulides, G. Stegeman, Y. Silberberg, R. Morandotti, and J. S. Aitchison, "Discrete vector solitons in Kerr nonlinear waveguide arrays," *Phys. Rev. Lett.* **91**, 143907 (2003).
6. J. W. Fleischer, M. Segev, N. K. Efremidis, and D. N. Christodoulides, "Observation of two-dimensional discrete solitons in optically induced nonlinear photonic lattices," *Nature* **422**, 147-150 (2003).
7. N. K. Efremidis, S. Sears, D. N. Christodoulides, J. W. Fleischer, and M. Segev, "Discrete solitons in photorefractive optically induced photonic lattices," *Phys. Rev. E* **66**, 046602 (2002).
8. H. Martin, E. D. Eugenieva, Z. Chen, and D. N. Christodoulides, "Discrete solitons and soliton-induced dislocations in partially coherent photonic lattices," *Phys. Rev. Lett.* **92**, 123902 (2004).
9. R. Iwanow, R. Schiek, G. I. Stegeman, T. Pertsch, F. Lederer, Y. Min, and W. Sohler, "Observation of discrete quadratic solitons," *Phys. Rev. Lett.* **93**, 113902 (2004).
10. J. Meier, G. I. Stegeman, D. N. Christodoulides, Y. Silberberg, R. Morandotti, H. Yang, G. Salamo, M. Sorel, and J. S. Aitchison, "Experimental observation of discrete modulational instability," *Phys. Rev. Lett.* **92**, 163902 (2004).
11. J. W. Fleischer, T. Carmon, M. Segev, N. K. Efremidis, and D. N. Christodoulides, "Observation of discrete solitons in optically induced real time waveguide arrays," *Phys. Rev. Lett.* **90**, 023902 (2003).
12. D. Mandelik, R. Morandotti, J. S. Aitchison, and Y. Silberberg, "Gap solitons in waveguide arrays," *Phys. Rev. Lett.* **92**, 093904 (2004).
13. J. W. Fleischer, G. Bartal, O. Cohen, O. Manela, M. Segev, J. Hudock, and D. N. Christodoulides, "Observation of vortex-ring "discrete" solitons in 2D photonic lattices," *Phys. Rev. Lett.* **92**, 123904 (2004).
14. D. N. Neshev, T. J. Alexander, E. A. Ostrovskaya, and Y. S. Kivshar, "Observation of discrete vortex solitons in optically induced photonic lattices," *Phys. Rev. Lett.* **92**, 123903 (2004).
15. J. Y. Lu and J. F. Greenleaf, "Nondiffracting X waves-exact solutions to free-space scalar wave equation and their finite aperture realizations," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **39**, 19 (1992).
16. D. N. Christodoulides, N. K. Efremidis, P. Di Trapani, and B. A. Malomed, "Bessel X waves in two- and three- dimensional bidispersive optical systems," *Opt. Lett.* **29**, 1446-1448 (2004).

17. P. Di Trapani, G. Valiulis, A. Piskarskas, O. Jedrkiewicz, J. Trull, C. Conti, and S. Trillo, "Spontaneously Generated X-Shaped Light Bullets," *Phys. Rev. Lett.* **91**, 093904 (2003).
18. C. Conti, S. Trillo, P. Di Trapani, G. Valiulis, A. Piskarskas, O. Jedrkiewicz, and J. Trull, "Nonlinear Electromagnetic X Waves," *Phys. Rev. Lett.* **90**, 093904 (2003).
19. U. Peschel, R. Morandotti, J. M. Arnold, J. S. Aitchison, H. S. Eisenberg, and Y. Silberberg, T. Pertsch and F. Lederer, "Optical discrete solitons in waveguide arrays. 2. Dynamic properties," *J. Opt. Soc. Am. B* **19**, 11, 2637-2644 (2002).

## 1. Introduction

Discrete optical systems exhibit several intriguing properties that have no analog whatsoever in the bulk [1]. This is primarily due to the fact that the linear diffraction properties of such discrete configurations are significantly altered as a result of structural periodicity. This itself has several interesting consequences in both the linear and nonlinear regime [1, 2, 3]. For example, anomalous diffraction [2] is possible in the linear domain whereas discrete optical solitons can form under nonlinear conditions [3]. Thus far, discrete solitons have been observed in several physical systems such as for example in Kerr nonlinear AlGaAs waveguide arrays [2, 4, 5], in two-dimensional biased photorefractive lattices [6, 7, 8], as well as in quadratic ( $\chi^{(2)}$ ) arrays [9]. In addition, other processes like discrete modulational instability [10], staggered or gap solitons [11, 12], and discrete vortex solitons [13, 14] have also been reported in these settings.

Quite recently another class of waves, the so-called X-waves has attracted considerable attention. In general X-waves represent stationary or invariant field configurations that can propagate free of diffraction and or dispersion effects. These waves involve infinite energy and are possible in bidispersive systems, e.g. physical arrangements that exhibit both normal and anomalous dispersive/diffracting behavior in different spatial or spatio-temporal coordinates. 3-D X-wave solutions were first obtained within the context of ultrasonics [15] whereas their 2-D analogues and 3-D generalizations have only recently been found [16]. The generation of nonlinearly induced X-waves has been experimentally and theoretically considered in both quadratic and Kerr normally dispersive media [17, 18]. In particular, spontaneously generated X-light bullets were experimentally observed in lithium triborate crystals [17].

An important aspect associated with AlGaAs is that this material system, besides from being highly nonlinear, exhibits appreciable normal dispersion around 1.5 $\mu$ m. In essence in AlGaAs arrays two basic linear processes are in play: discrete diffraction and normal dispersion. As a result, for in phase waveguide excitation, the array behaves in a bidispersive fashion, e.g., the sign of the dispersion term in the evolution equation is opposite to that of diffraction. Following the previous discussion, it is natural for one to ask whether X-resembling wave configurations are possible in these arrays. In this study, we show that such arrays can support a novel class of waves: discrete X-waves. Even though these waves are closely related to the X-family encountered in the bulk, as we will see their behavior is very particular to discrete systems. Even more importantly, in space-time they exhibit features that can not be observed in either bulk or single-waveguide configurations.

## 2. System analysis

To analyze discrete X-wave excitation, consider an array of uniformly spaced weakly-coupled single-mode waveguides. From coupled-mode theory, the modal field amplitude  $U_n$  in the  $n$ -th waveguide evolves according to [3, 19]:

$$i \frac{\partial U_n}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 U_n}{\partial t^2} + C[U_{n+1} + U_{n-1}] + k_0 n_2 |U_n|^2 U_n = 0, \quad (1)$$

where  $\beta_2$  is the group velocity dispersion,  $C$  the linear coupling strength,  $n_2 = (\hat{n}_2 n)/(2\eta_0)$ ,  $\hat{n}_2 = 2.47 \times 10^{-13} \text{ cm}^2 / W$  is the AlGaAs self-focusing Kerr coefficient and  $k_0 = 2\pi / \lambda_0$ .  $n$  is the

linear refractive index of AlGaAs and  $\eta_0 = (\mu_0 / \epsilon_0)^{1/2}$ . We consider a  $1\text{cm}$  long array consisting of  $41$  waveguide elements with an effective cross-sectional area of  $5\mu\text{m}^2$ . The distance between waveguides is taken to be  $10\mu\text{m}$ . This array is used at  $\lambda_0 = 1.55\mu\text{m}$  where the AlGaAs normal dispersion is  $\beta_2 = +1.3\text{ps}^2/\text{m}$ . The coupling constant is assumed to be  $C = 728\text{m}^{-1}$  which is close to that encountered in previous experiments [2, 4, 5]. In all the examples, Eq. (1) is solved numerically by means of a symmetrized split-step Fourier scheme, with the nonlinear part treated on the basis of a fourth order Runge-Kutta method. The accuracy of the simulations is checked against the invariants of this equation during propagation. These correspond to the total power  $P$  and the Hamiltonian  $H$  of the system, i.e.,

$$P = \sum_n \int_{-\infty}^{+\infty} |U_n|^2 dt, \quad H = \sum_n \int_{-\infty}^{+\infty} \left\{ -\frac{\beta_2}{2} \left| \frac{\partial U_n}{\partial t} \right|^2 - \frac{\gamma}{2} |U_n|^4 - C_0 (U_n^* U_{n+1} + U_n U_{n+1}^*) \right\} dt \quad (2)$$

The array is excited in time with hyperbolic secant-pulses. In the transverse plane (waveguide sites), we use either single or multiple waveguide excitation as initial conditions.

### 3. Discrete X-wave formation

X-wave formation in such a system is investigated over a wide range of input conditions. This is done in terms of the initial peak-power and temporal width. In this section, we varied the initial peak-power from  $0.5\text{kW}$  to  $1.5\text{kW}$ , where the 2- and 3-photon absorption is negligible compared to the Kerr-effect in AlGaAs waveguides. The initial pulse duration was taken to be either  $1\text{ps}$  or  $200\text{fs}$ . In all the figures,  $|U_n|^2 = 1$  represents the peak-power initially used.

#### 3.1 Single waveguide excitation

We will first examine X-wave formation by keeping the temporal width of the initial pulse at  $1\text{ps}$  and by varying only the input peak-power. For a low input peak-power ( $0.5\text{kW}$ ) the coupling among waveguides dominates the process and the initial pulse spreads in the transverse direction, as shown in Fig. 1. It is evident that the process of X-wave formation has already begun even at this power level. Clearly the nonlinearity is not strong enough to prevent the initial pulse from spreading in space and as a result it discretely diffracts.

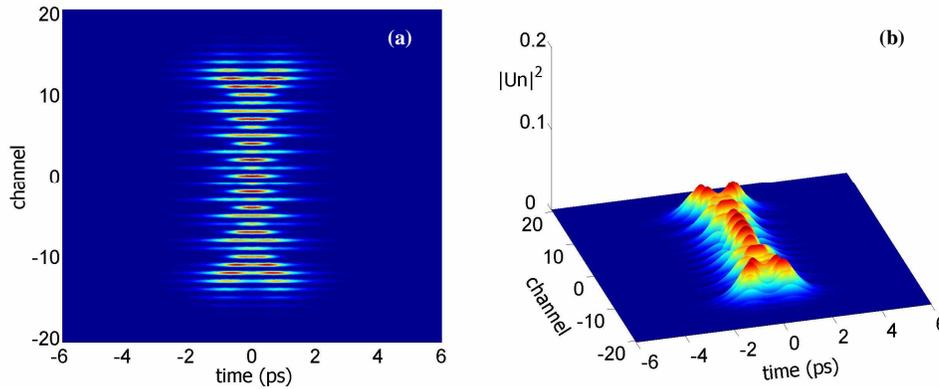


Fig. 1. Output intensity profile at  $z=1\text{cm}$  in an AlGaAs array, when the central channel is excited with a sech-pulse of  $1\text{ps}$  duration ( $1.76\text{ps}$  FWHM) and  $0.5\text{kW}$  peak-power; (a) contour plot and (b) three-dimensional depiction.

At an intermediate input peak-power ( $0.7\text{kW}$ ), discrete X-waves are more effectively excited. The interplay between diffraction and nonlinearity causes this pattern to arise in the following way. During propagation the initial pulse reaches a stage of maximum time-compression and edge-steepening occurs, leading to an X-like (or H-like) pattern. This pattern owes its shape to

the confinement of the highly nonlinear part of the pulse in the central waveguide and to the linear diffraction of the low-power pulse edges. After this compression, both the linear edges (that diffract far from the central waveguide) as well as the main part of the pulse continue to broaden due to normal dispersion. The broadening of the main body of the pulse reduces the level of nonlinearity and it is therefore more amenable to diffraction. As a result, the central part of the remaining pulse diffracts, giving birth to two ultrashort spikes that keep moving apart. The X-like pattern that is generated by this procedure is depicted in Fig. 2.

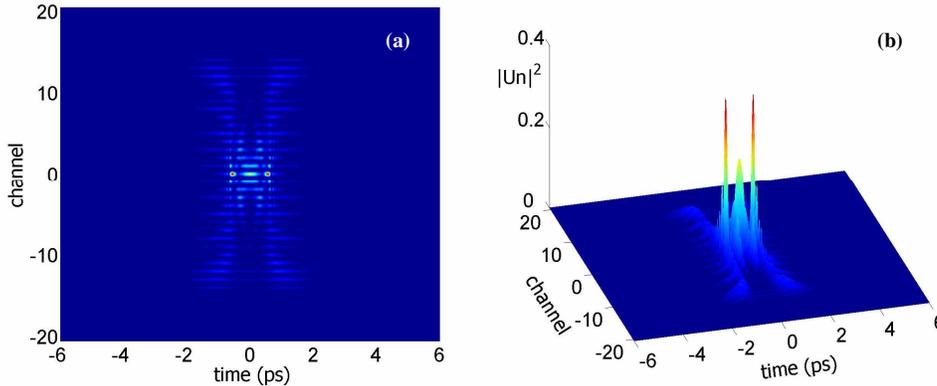


Fig. 2. Output intensity profile at  $z=1\text{cm}$  in an AlGaAs array, when the central channel is excited with a sech-pulse of  $1\text{ps}$  duration ( $1.76\text{ps}$  FWHM) and  $0.7\text{kW}$  peak-power; (a) contour plot movie (336 KB) and (b) three-dimensional depiction.

At a high input peak-power ( $1.5\text{kW}$ ) nonlinearity dominates over coupling, thus confining the major part of the initial pulse within the central waveguide, as shown in Fig. 3. The side lobes again diffract in a manner similar to that described previously, but they are now weaker. In a way, the high nonlinearity in the central waveguide decouples it from the rest of the array and the evolution of the initial pulse resembles that of pulse broadening in normal-dispersive nonlinear fibers. In the array however, this pulse profile is considerably steeper at the edges, due to the discrete diffraction of the low power lobes in the transverse direction. This process shows promise in terms of generating square-like pulses.

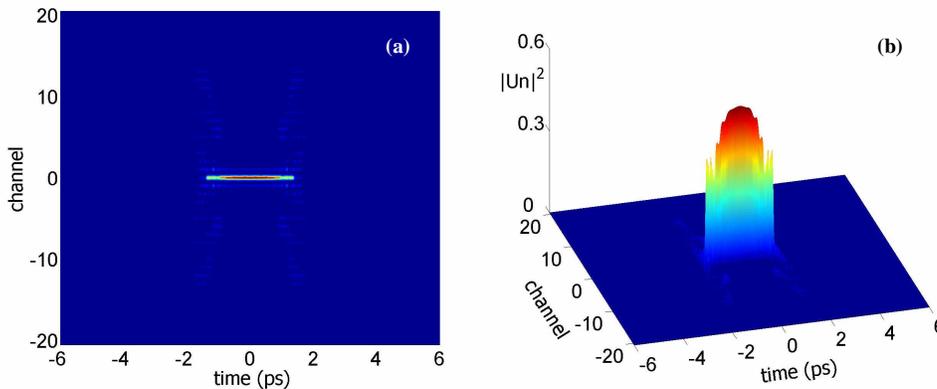


Fig. 3. Output intensity profile at  $z=1\text{cm}$  in an AlGaAs array, when the central channel is excited with a sech-pulse of  $1\text{ps}$  duration ( $1.76\text{ps}$  FWHM) and  $1.5\text{kW}$  peak-power; (a) contour plot and (b) three-dimensional depiction.

By keeping the input peak-power at  $1.5\text{kW}$  and by using a shorter  $200\text{fs}$  pulse at the input, the effect of group velocity dispersion becomes even more pronounced. As depicted in Fig. 4,

appreciable pulse broadening occurs and this fact leads to a totally different scenario. In particular, this leads to an X-formation and after that high-intensity spikes are generated sequentially. These spikes in turn produce other new X-waves in cascade.

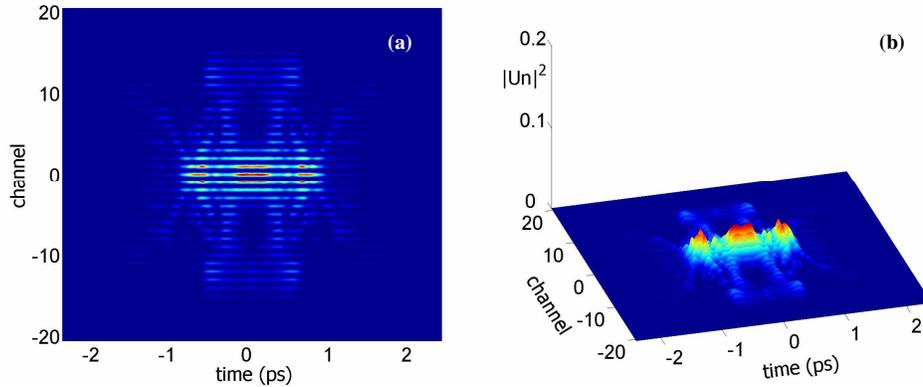


Fig. 4. Output intensity profile at  $z=1cm$  in an AlGaAs array, when the central channel is excited with a sech-pulse of  $200fs$  duration ( $352fs$  FWHM) and  $1.5kW$  peak-power; (a) contour plot and (b) three-dimensional depiction.

### 3.2 Multiple waveguide excitation

We next investigate the behavior of this array when a Gaussian beam is used to excite the waveguides at the input. In this case, the diffraction properties of the system depend on the excitation angle, that is on the initial phase difference  $\Delta\phi$  among adjacent waveguides [2]. For in-phase excitation ( $\Delta\phi = 0$ ), the X-pattern is again observed as shown in Fig. 5 when the peak power in the central waveguide is  $0.5kW$  and the initial temporal width of the pulse is set to be  $1ps$ .

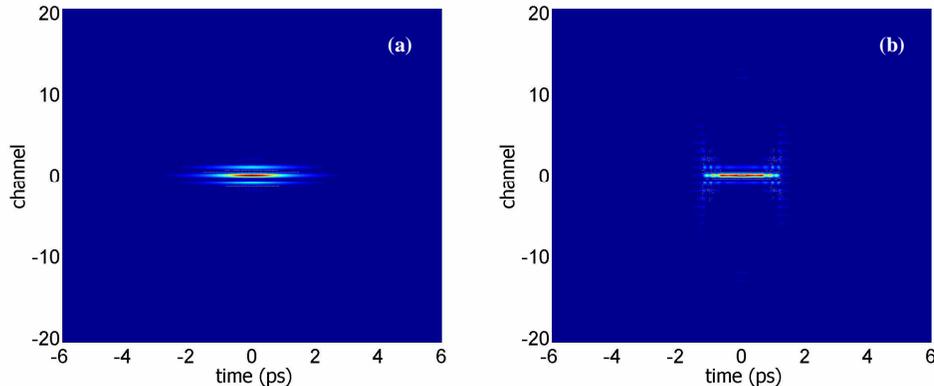


Fig. 5. Input (a) and output (b) intensity profile at  $z=1cm$  in an AlGaAs array, when the central channel is excited with a sech-type Gaussian beam of  $1ps$  and  $0.5kW$  peak-power at the central waveguide. The initial phase difference among excited waveguides is  $0$ .

When the initial phase difference is set to values other than zero the system response changes. For the same initial power and temporal conditions but for  $\Delta\phi = \pi$ , where the diffraction in the system is anomalous, the beam discretely diffracts (Fig. 6(a)), being unable to support any kind of X-like formation. For  $\Delta\phi = \pi/2$ , where the diffraction in the system is zero, a more complicated V-like pattern is observed (Fig. 6(b)) due to higher-order diffraction effects. Again, the non-bidispersive nature of this configuration inhibits any X-like pattern.

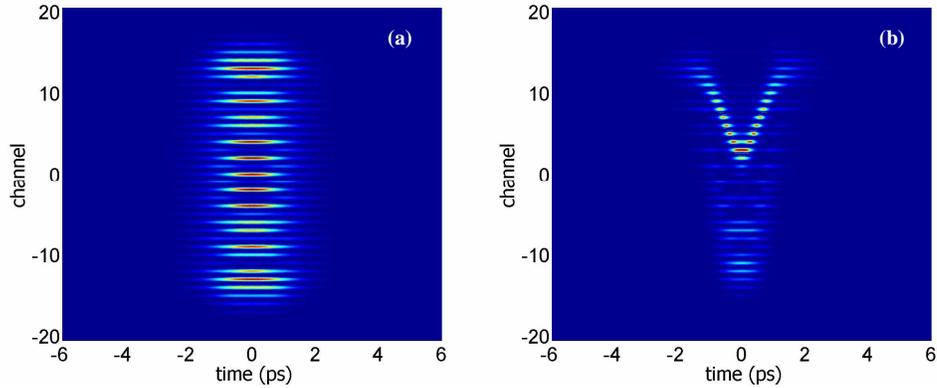


Fig. 6. (a) Output intensity profile at  $z=1\text{cm}$  in an AlGaAs array, when the central channel is excited with a sech-type Gaussian beam of  $1\text{ps}$  and  $0.5\text{kW}$  peak-power at the central waveguide. The initial phase difference among excited waveguides is  $\pi$ . (b) Output intensity profile at  $z=1\text{cm}$  in an AlGaAs array, when the central channel is excited with a sech-type Gaussian beam of  $1\text{ps}$  and  $0.5\text{kW}$  peak-power at the central waveguide. The initial phase difference among excited waveguides is  $\pi/2$ .

#### 4. Conclusions

In conclusion, we have demonstrated that X- wave formation is possible in nonlinear normally dispersive waveguide arrays. We have shown that the interplay between diffraction and nonlinearity leads to features that can not be otherwise observed in either bulk or single-waveguide configurations. The possibility to excite such X-waves in AlGaAs arrays was also considered in terms of relevant examples. The derivation of the dispersion relations of spatial-temporal waves and the study of stationary solutions are of great importance and will be considered in our future work.

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