

Highly nonparaxial (1 + 1)-D subwavelength optical fields

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Abstract: A general approach for describing (1 + 1)-D subwavelength optical field whose waist is much smaller than the wavelength is presented. Exploiting the vectorial Rayleigh-Sommerfeld diffraction theory, a suitable expansion in the ratio between the beam waist and the wavelength allows us to prove the a (1+1)D highly nonparaxial field is generally the product of a cylindrical wave carrier and an envelope which is angularly slowly varying. We apply our general approach to the case of highly nonparaxial Hermite-Gaussian beams whose description is fully analytical.

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OCIS codes: (050.1960) Diffraction theory; (050.6624) Subwavelength structures.

References and links

1. J. B. Pendry, "Negative Refraction Makes a Perfect Lens," *Phys. Rev. Lett.* **85**, 3966 (2000).
2. J. Takahara, S. Yamagishi, H. Taki, A. Morimoto, and T. Kobayashi, "Guiding of a one-dimensional optical beam with nanometer diameter," *Opt. Lett.* **22**, 475 (1997).
3. H. J. Lezec, A. Degiron, E. Devaux, R. A. Linke, L. Martin-Moreno, F. J. Garcia-Vidal, and T. W. Ebbesen, "Beaming Light from a Subwavelength Aperture," *Science* **297**, 820 (2002).
4. L. Verslegers, P.B. Catrysse, Z. Yu, J. S. White, E. S. Barnard, M. L. Brongersma, and S. Fan, "Planar Lenses Based on Nanoscale Slit Arrays in a Metallic Film," *Nano Lett.* **9**, 235 (2009).
5. A. Ciattoni, B. Crosignani, and P. Di Porto, "Vectorial free-space optical propagation: a simple approach for generating all-order nonparaxial corrections," *Opt. Commun.* **177** 9 (2000).
6. A. Yu. Savchenko and B. Ya. Zel'dovich, "Wave propagation in a guiding structure: one step beyond the paraxial approximation," *J. Opt. Soc. Am. B* **13**, 273 (1996).
7. A. Ciattoni, B. Crosignani, and P. Di Porto, "Vectorial analytical description of propagation of a highly nonparaxial beam," *Opt. Commun.* **202**, 17 (2002).
8. Z. Mei and D. Zhao, "Nonparaxial analysis of vectorial Laguerre-Bessel-Gaussian beams," *Opt. Express* **15**, 11942 (2007).
9. R. Martinez-Herrero, P. M. Mejias, and A. Carnicer, "Evanescence field of vectorial highly non-paraxial beams," *Opt. Express*, **16**, 2845 (2008).
10. P. Liu, B. Lü, and K. Duan, "Propagation of vectorial nonparaxial Gaussian beams through an annular aperture," *Opt. Laser Technol.* **38**, 133 (2006).
11. E. Moreno, F. J. García-Vidal, and L. Martín-Moreno, "Enhanced transmission and beaming of light via photonic crystal surface modes," *Phys. Rev. B* **69**, 121402 (2004).
12. S. K. Morrison and Y. S. Kivshar, "Engineering of directional emission from photonic-crystal waveguides," *Appl. Phys. Lett.* **86**, 081110 (2005).
13. C. Liu, N. Chen, and C. Sheppard, "Nanoillumination based on self-focus and field enhancement inside a sub-wavelength metallic structure," *Appl. Phys. Lett.* **90**, 011501 (2007).
14. V. I. Smirnov, *A Course of Higher Mathematics*, Vol. 4 (Pergamon, Oxford, 1975).

1. Introduction

The demand of controlling light behavior in subwavelength range has stimulated an intense research effort aimed at reducing the size of integrated device components and increasing the achievable spatial resolution. The main issue is the diffraction limit, generally detrimental for any nano- and integrated optical applications. However, in the last decade, many researches have found several solutions to overcoming the resolution limit imposed by evanescent waves and remarkable examples are the perfect lenses based on negative refraction [1], the guiding of a beam with nanometer diameter in a negative dielectric waveguide [2] and the beaming from a subwavelength aperture in a metallic slab due to the excitation of the surface plasmon polaritons [3, 4]. In all these situations, the optical structure has dimensions smaller than the wavelength and evanescent waves generally play an important role. Therefore the description of optical propagation cannot be achieved by exploiting the well-known paraxial approach and generally, the full vectorial electromagnetic model has to be numerically solved. As a consequence, the developing of an analytical approach for dealing with highly nonparaxial fields (i.e. whose waist is much smaller than the wavelength) is crucial to grasp the main features of light propagation and to explore novel nano-optical applications. Analytical approaches for describing (1 + 2)-D monochromatic optical propagation have been considered both in nonparaxial [5, 6] and highly nonparaxial regimes [7–10]. On the other hand, research interest has also been focused on (1 + 1)-D optical propagation of sub-wavelength-sized fields such as guided waves propagating in a slab waveguide or light emitted from a rectangular aperture [11–13].

In this paper, we consider an approach for describing the propagation of (1 + 1)-D monochromatic optical field in the highly nonparaxial regime where the beam waist w is much smaller than the wavelength λ and it is based on the expansion of the exact vectorial Rayleigh-Sommerfeld diffraction formulas in the parameter $w/\lambda \ll 1$. By its general derivation, our scheme is such that the smaller the waist, the higher the accuracy of the description so that it proves to be suitable for describing field configurations occurring in nano-optical applications. We show that (1 + 1)-D highly nonparaxial field is the product between a cylindrical wave carrier and a slowly varying envelope as opposed to the (1 + 2)-D situation [7] where the carrier is a spherical wave. We obtain an integral representation for the envelope whose propagation kernel is the exponential of a quadratic form, a situation particularly suitable for handling gaussian fields. Therefore, as an application of the general scheme, we consider highly nonparaxial Hermite-Gaussian fields whose description is fully analytical. It is worth stressing that the (1 + 1)-D highly nonparaxial description cannot be obtained from its (1 + 2)-D counterpart [7] since the transition should have to be performed by letting the subwavelength transversely two-dimensional field to have an infinite width along one transverse direction.

2. One-dimensional highly nonparaxial fields

Propagation of a (1 + 1)-D monochromatic optical field $Re[\mathbf{E}(x, z)e^{-i\omega t}]$ (of frequency ω) through a homogenous medium whose refractive index is n , is described by the Rayleigh-Sommerfeld diffraction formulas

$$\begin{aligned} \mathbf{E}_{\perp}(x, z) &= -\frac{i}{2} \int_{-\infty}^{+\infty} dx' \frac{\partial}{\partial z} \left(H_0^{(1)}(kR) \right) \mathbf{E}_{\perp}(x', 0), \\ E_z(x, z) &= \frac{i}{2} \int_{-\infty}^{+\infty} dx' \frac{\partial}{\partial x} \left(H_0^{(1)}(kR) \right) E_x(x', 0), \end{aligned} \quad (1)$$

where $R = \sqrt{(x-x')^2 + z^2}$, $k = \omega n/c$, $H_0^{(1)}$ is the Hankel function of the first kind and order 0 and $\mathbf{E}_\perp = E_x \hat{\mathbf{e}}_x + E_y \hat{\mathbf{e}}_y$ [14]. Consider the field [7]

$$\mathbf{F}_\perp(x, z) = \frac{i}{2} \int_{-\infty}^{+\infty} dx' H_0^{(1)}(kR) \mathbf{E}_\perp(x', 0), \quad (2)$$

so that electric field components can be expressed as

$$\mathbf{E}_\perp = -\frac{\partial \mathbf{F}_\perp}{\partial z}, \quad E_z = \frac{\partial F_x}{\partial x}. \quad (3)$$

Note that, from Maxwell equations, it is easy to prove that y-component of the magnetic field is $B_y = (i/\omega)(\partial_x^2 + \partial_z^2)F_x$ so that, exploiting the fact that \mathbf{F}_\perp satisfies the 2D Helmholtz equation $(\partial_x^2 + \partial_z^2 + k^2)\mathbf{F}_\perp = 0$, we have

$$B_y = \frac{k^2}{i\omega} F_x, \quad (4)$$

which is a relation clarifying the physical meaning of F_x .

In order to investigate the highly nonparaxial regime, we follow the approach of Ref. [7] and we consider optical fields such that $\mathbf{E}_\perp(x, 0)$ has a width w much smaller than the wavelength λ . It is convenient to replace in the field of Eq. (2) the Hankel function with its asymptotic expansion thus getting

$$\mathbf{F}_\perp = \int_{-\infty}^{+\infty} dx' \sqrt{\frac{1}{2\pi kR}} \exp\left[i\left(kR + \frac{\pi}{4}\right)\right] \mathbf{E}_\perp(x', 0). \quad (5)$$

and this is allowed if $kR > 1$. Since the integrand of Eq. (5) is not negligible only for $|x'|/\lambda < w/\lambda \ll 1$, we expand R in the smallness parameter x'/λ , thus obtaining

$$\mathbf{F}_\perp = \exp\left[i\left(kr + \frac{\pi}{4}\right)\right] \sqrt{\frac{1}{2\pi kr}} \mathbf{G}_\perp(x, z), \quad (6)$$

where

$$\mathbf{G}_\perp(x, z) = \int_{-\infty}^{+\infty} dx' \exp\left(-\frac{ikxx'}{r} + \frac{ikz^2x'^2}{2r^3}\right) \mathbf{E}_\perp(x', 0) \quad (7)$$

and $r = \sqrt{x^2 + z^2}$. The truncation of the series expansion of the exponent within the integral of Eq. (5) is assured by requiring that the third term [not reported in Eq. (5)] is much smaller both than the second term and than one, requirements assured by the condition $z \gg w$. On the other hand, since $w < 1/k$ for the fields we are considering, we conclude that our approach is valid for $z > 1/k$ [condition allowing both the series truncation and the asymptotic expansion of the Hankel function in Eq. (2)]. Note that the cylindrical wave $\sqrt{1/(2\pi kr)} \exp(ikr)$ plays a crucial role in the highly nonparaxial regime and it is the (1+1)D counterpart of spherical wave of (1+2)-D approach [7, 8]. The field \mathbf{F}_\perp of Eq. (6) is the product of the cylindrical wave carrier and an envelope \mathbf{G}_\perp which is a slowly varying function of variables x/r and $(1/kr)(z/r)^2$, i.e. it is angularly slowly varying. In addition, it is evident from Eq. (7) that the field \mathbf{G}_\perp asymptotically (i.e. for $r \rightarrow \infty$) approaches a function of x/r so that the field \mathbf{F}_\perp asymptotically reproduces an angular modulated cylindrical wave.

A situation admitting analytical description is the extreme nonparaxial situation where the boundary electric field is $\mathbf{E}_\perp(x, 0) = E_0 w \delta(x) \hat{\mathbf{e}}_x$ where $\delta(x)$ is the Dirac delta function, E_0 is a complex constant and w is an arbitrary length. Substituting this boundary profile into Eq. (6), we obtain $\mathbf{F}_\perp = E_0 w \sqrt{1/(2\pi kr)} \exp[i(kr + \pi/4)] \hat{\mathbf{e}}_x$, which is a pure cylindrical wave where $\mathbf{G}_\perp = E_0 w \hat{\mathbf{e}}_x$ is constant as expected for an infinitely narrow source.

3. Highly nonparaxial Hermite-Gaussian fields

A relevant example of vectorial highly nonparaxial (1 + 1)-D fields admitting fully analytical description is the class of Hermite-Gaussian beams whose profile at $z = 0$ is

$$\mathbf{E}_\perp(x, 0) = E_0 H_m \left(\frac{x - x_0}{w} \right) \exp \left[-\frac{(x - x_0)^2}{2w^2} \right] \hat{\mathbf{e}}_x \quad (8)$$

where x_0 is a real constant, H_m is the Hermite polynomial of order m defined by the Rodrigues' formula $H_m(\xi) = (-1)^m \exp(\xi^2/2) \frac{d^m}{d\xi^m} \exp(-\xi^2/2)$. Substituting Eq. (8) into Eq. (7) and exploiting the Rodrigues' formula, we get

$$\mathbf{G}_\perp(x, z) = E_0 \hat{\mathbf{e}}_x w^m \frac{\partial^m}{\partial x_0^m} \int_{-\infty}^{+\infty} dx' \exp \left[-\frac{ikxx'}{r} + \frac{ikz^2x'^2}{2r^3} - \frac{(x' - x_0)^2}{2w^2} \right]. \quad (9)$$

Evaluating the Gaussian integral appearing in this expression we obtain

$$\mathbf{G}_\perp(x, z) = E_0 \hat{\mathbf{e}}_x w^{m+1} \sqrt{\frac{2\pi}{P(x, z)}} \frac{\partial^m g(x, z|x_0)}{\partial x_0^m}, \quad (10)$$

where

$$g = \exp \left[\frac{w^2}{2r^2 P} \left(ikx - \frac{rx_0}{w^2} \right)^2 - \frac{x_0^2}{2w^2} \right], \quad P = 1 - \frac{ikw^2 z^2}{r^3}. \quad (11)$$

Substituting this explicit expression of \mathbf{G}_\perp into Eq. (6) and exploiting Eqs. (3) it is straightforward to deduce the highly nonparaxial Hermite-Gaussian electric field. For the fundamental Hermite-Gaussian field ($m = 0$) we obtain (for $x_0 = 0$)

$$\begin{aligned} \mathbf{G}_\perp(r, \theta) &= \hat{\mathbf{e}}_x E_0 w \sqrt{\frac{2\pi}{P}} \exp \left[-\frac{k^2 w^2}{2P} \sin^2 \theta \right], \\ E_x(r, \theta) &= E_0 \frac{kw \cos \theta}{\sqrt{Pkr}} \exp \left[ikr + i\frac{\pi}{4} - \frac{k^2 w^2}{2P} \sin^2 \theta \right] \times \\ &\quad \times \left\{ -\left(i - \frac{1}{2kr} \right) - \frac{kw^2}{Pr} \left[\sin^2 \theta + \frac{i}{2kr} \left(1 - \frac{k^2 w^2}{P} \sin^2 \theta \right) (3 \sin^2 \theta - 1) \right] \right\}, \\ E_z(r, \theta) &= E_0 \frac{kw \sin \theta}{\sqrt{Pkr}} \exp \left[ikr + i\frac{\pi}{4} - \frac{k^2 w^2}{2P} \sin^2 \theta \right] \times \\ &\quad \times \left\{ \left(i - \frac{1}{2kr} \right) - \frac{kw^2}{Pr} \left[\cos^2 \theta + \frac{i}{2kr} \left(1 - \frac{k^2 w^2}{P} \sin^2 \theta \right) (3 \cos^2 \theta) \right] \right\} \quad (12) \end{aligned}$$

where polar coordinates $x = r \sin \theta$ and $z = r \cos \theta$ have been introduced and, from the second of Eqs. (11), $P = 1 - (ikw^2/r) \cos^2 \theta$. From the first of Eqs. (12), we note that, in agreement with the general discussion of Section 2, the envelope \mathbf{G}_\perp is a slowly varying function of the variables $\sin \theta = x/r$ and $(1/kr) \cos^2 \theta = (1/kr)(z/r)^2$ (since $kw \ll 1$). In addition \mathbf{G}_\perp asymptotically (i.e. for $r \rightarrow \infty$) contributes to the total optical field as pure angular distribution (since $P \rightarrow 1$). From the second and the third of Eqs. (12) we note that E_x and E_z have definite spatial parity (even and odd respectively) and that their orders of magnitude are comparable, a distinct feature of the nonparaxial (in this case highly nonparaxial) regime.

In Fig. 1 we report the modulus of the normalized field F_x for the $m = 0$ [Fig. 1(a)] and $m = 1$ [Fig. 1(b)] Hermite-Gaussian fields and the plots show the main features of (1 + 1)D highly nonparaxial fields. Note that F_x [and hence the magnetic field, see Eq. (4)], vanishes for $x \rightarrow \infty$

as $x^{-1/2}$ in agreement with the general behavior of these fields. In Fig. 2 and Fig. 3 we plot the moduli of the x - and z -component of the electric field in the case of the fundamental Gaussian field and the first order Hermite-Gaussian field, respectively. It is worth noting that the x and z - components of the electric field, for $|x| \rightarrow \infty$, vanish as $x^{-3/2}$ and $x^{-1/2}$, as opposed to the (1 + 2)-D situation where E_x and E_z vanish as x^{-2} and as x^{-1} , respectively.

In the considered situation where the magnetic field is polarized along the y -axis (transverse magnetic case), the average Poynting vector is

$$\mathbf{S}(x, z) = \frac{1}{2\mu_0} \text{Re}(\mathbf{E} \times \mathbf{B}^*) = \frac{k^2}{2\omega\mu_0} \text{Im} \left(F_x^* \frac{\partial F_x}{\partial x} \hat{\mathbf{e}}_x + F_x^* \frac{\partial F_x}{\partial z} \hat{\mathbf{e}}_z \right). \quad (13)$$

where Eq. (4) has been used for eliminating the magnetic field. In Figs. 4 and 5, we report the spatial profile of the Poynting vector components for the $m = 0$ and $m = 1$ Hermite-Gaussian fields, respectively. Note that presence of a large x - component of the Poynting vector (which is spatially odd along the x - axis) corresponds to an energy flow diverging from the origin. This is also consistent with the fact that the field asymptotically coincides with an angularly modulated cylindrical wave so that the field lines of the Poynting vector are asymptotically radial straight lines.

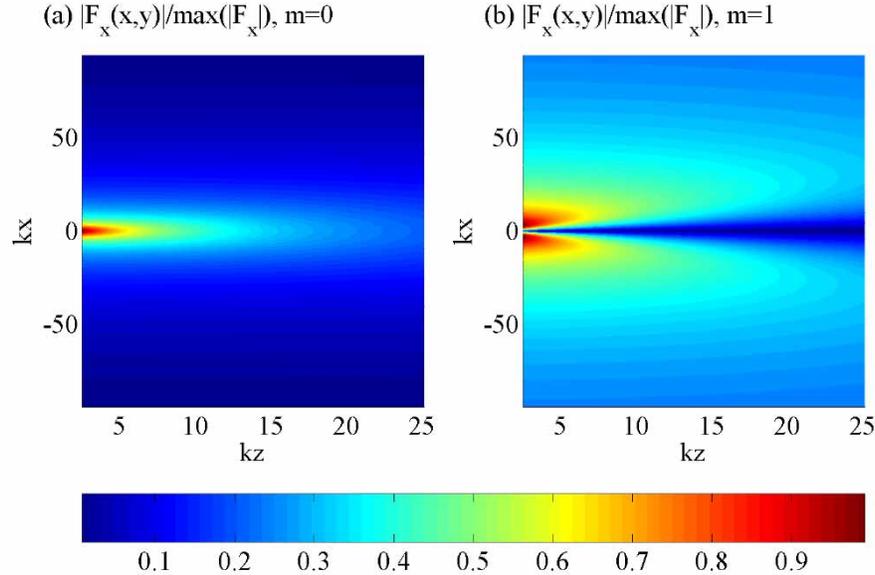


Fig. 1. Modulus of the normalized field $F_x(x, z)/\max(|F_x|)$ of the fundamental Gaussian field ($m = 0$ subplot (a)) and of the first order Hermite-Gaussian field ($m = 1$ subplot (b)) where $\lambda = 0.5\mu\text{m}$, $x_0 = 0$, $w = \lambda/10$. The refractive index is $n = 1$.

4. Conclusions

In conclusion, we have presented an approach for describing (1 + 1)-D optical fields in the highly nonparaxial regime where the initial beam waist is much smaller than the wavelength. The description of the radiated field is analytical so that our method is of particular relevance for theoretically dealing with nano- and integrated optical applications. Exploiting the general

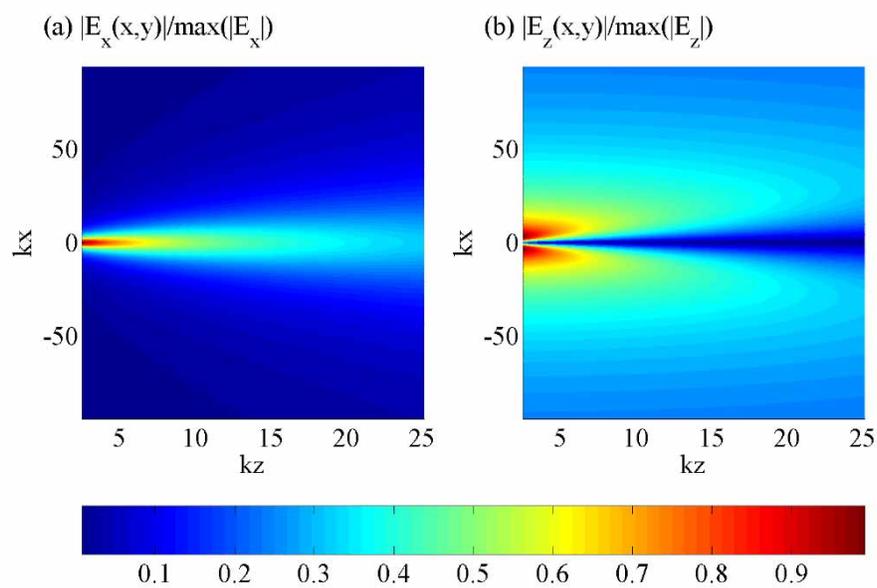


Fig. 2. Modulus of the normalized field $E_x(x,z)/\max(|E_x|)$ (subplot (a)) and $E_z(x,z)/\max(|E_z|)$ (subplot (b)) of the fundamental Gaussian field ($m = 0$) for $\lambda = 0.5\mu\text{m}$, $x_0 = 0$, $w = \lambda/10$ and $n = 1$.

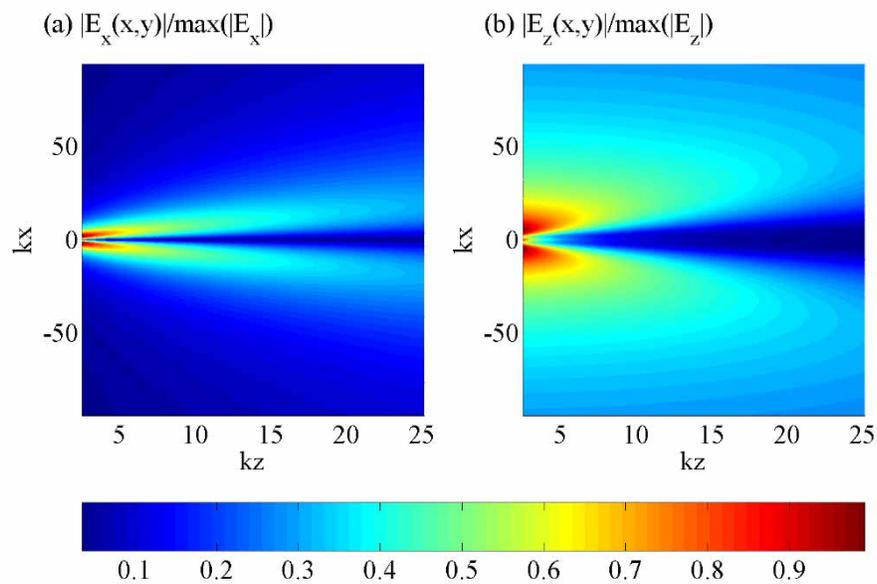


Fig. 3. Modulus of the normalized field $E_x(x,z)/\max(|E_x|)$ (subplot (a)) and $E_z(x,z)/\max(|E_z|)$ (subplot (b)) of the first order Hermite-Gaussian field ($m = 1$) for $\lambda = 0.5\mu\text{m}$, $x_0 = 0$, $w = \lambda/10$ and $n = 1$.

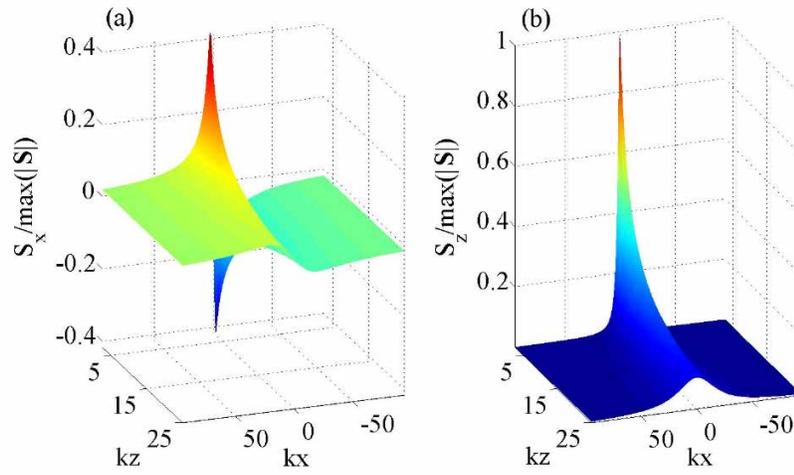


Fig. 4. Normalized Poynting vector components $S_x(x,z)/\max(|\mathbf{S}|)$ (subplot (a)) and $S_z(x,z)/\max(|\mathbf{S}|)$ (subplot (b)) of the Gaussian field ($m = 0$) for $\lambda = 0.5\mu\text{m}$, $x_0 = 0$, $w = \lambda/10$ and $n = 1$.

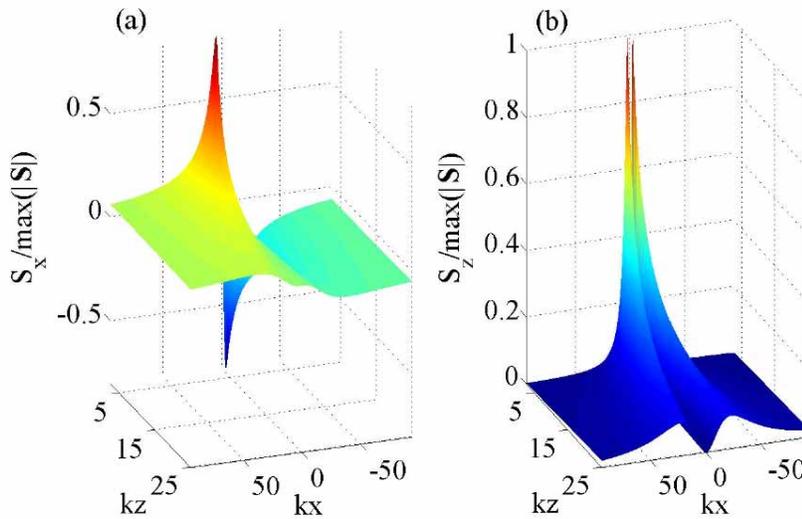


Fig. 5. Normalized Poynting vector components $S_x(x,z)/\max(|\mathbf{S}|)$ (subplot (a)) and $S_z(x,z)/\max(|\mathbf{S}|)$ (subplot (b)) of the first order Hermite-Gaussian field ($m = 1$) for $\lambda = 0.5\mu\text{m}$, $x_0 = 0$, $w = \lambda/10$ and $n = 1$.

approach we have derived analytical expressions for describing highly nonparaxial Hermite-Gaussian fields.