Effect of two-photon absorption on cavity soliton: 
stability and perturbation analysis

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Abstract: We study the stability criteria and propagation dynamics of cavity soliton under two photon absorption by solving Lugiato–Lefever equation numerically. A semi-analytical variational method is further adopted to explain the complete dynamics.

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Cavity solitons (CSs) are stable localised pulses that persist in a driven passive cavity system and they are subset of the dissipative solitons. The CS dynamics is governed by Lugiato–Lefever equation (LLE) [1]. In semiconductor based microresonator, the presence of two-photon absorption (TPA) is relevant which modifies the stability criteria of the system and leads to the change in amplitude and pulse width of the CS. Using CW bistability analysis [2] we find an upper limit of TPA coefficient (Kc) beyond which the formation of CS ceases to exist. Further, we adopt a semi-analytical variational technique to study the complex dynamics of perturbed CS that is governed by LLE. We derive a set of coupled ordinary differential equations that predicts the change in amplitude and pulse width of CS under TPA. The results obtained from direct solution of LLE agree with the variational predictions. Both confirm that, under TPA, the pulse amplitude oscillates initially before reaching a stable state. The pulse amplitude and width at stable state are well predicted by the variational method.

The nonlinear passive cavity dynamics under TPA with intracavity field amplitude u(t, τ) is modeled through mean-field normalised Lugiato-Lefever (LLE) equation [1] as,

\[ \frac{\partial u}{\partial t} = -1 + i \left\{ (1 + iK)|u|^2 - \Delta \right\} - i \text{sgn}(\beta_2) \frac{\partial^2}{\partial \tau^2} u + S, \]  

(1)

where the TPA loss is included through the dimensionless TPA coefficient K. The standard split-step Fourier method is used to solve this inhomogeneous nonlinear equation by launching standard sech pulse in anomalous dispersion domain (sgn(β2) = −1).

In Fig. 1(a) we have plotted the output profile of the generated CS in presence (for K=0.03) and in absence of TPA. In the three dimensional plot in Fig. 1(b) we capture the entire evolution of the CS. In case of standard Kerr soliton, amplitude decays adiabatically due to TPA. On the contrary in case of CS, there is no decay after attaining an initial stability. It is also evident from the simulation that for the input parameters \( \Delta = 3; \ X = |S|^2 = 3.5 \) there is a critical value of \( K = K_c = 0.04 \), beyond which no CS will form. We try to understand this critical phenomenon by using CW bistable analysis. From Eq. (1) we obtain the cubic equation describing optical bistability using the steady-state cw response \( u_s \) with intracavity power \( Y = |u_s|^2 \) as,

\[ X = (1 + K^2)Y^3 - 2(\Delta - K)Y^2 + (\Delta^2 + 1)Y. \]

(2)

Now along with \( \Delta \), we have an additional parameter \( K \) which can control the bistability. In Fig. 1(c) and (d) the Eq. (2) is plotted in two different sets of parameter space \((\Delta, Y)\) for fixed \( X = 3.5 \) and \((X, Y)\) for fixed \( \Delta = 3 \) respectively. From this two figures it is clear that the stability region is defined by \( K < K_c \) for a fixed \( X \) and \( \Delta \), where the vertical dashed lines touch the bistable curve (red line). Also we can tune the \( K_c \) by changing \( X \) and \( \Delta \).

To grasp the effects of TPA on the formation and stability of CS, we adopt the standard soliton perturbative analysis [3, 4] by approximating the CGLE as a perturbed nonlinear Schrödinger equation (NLSE): \[ \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u - \Delta u = i \epsilon(u), \] where we explicitly consider the dispersion to be anomalous \( (\beta_2 < 0) \) and \( \epsilon(u) \) includes the inhomogeneous term S, TPA term, and linear loss: \( \epsilon(u) = [S - K|u|^2 u - u] \). Here we use the complete form of standard sech ansatz,

\[ u(t, \tau) = \left( \frac{E(t)\eta(t)}{2} \right)^{1/2} \text{sech} \{ \eta(t)\tau \} \exp [i\phi(t)], \]

(3)
where energy $E$, inverse pulse width $\eta$ and phase $\phi$ are now assumed to depend on $t$ and $\varepsilon(u)$ is considered as a small perturbation depending on $u$ and $u^*$. The perturbation technique is used to evaluate the dynamics of each individual parameter as a function of the round trip time $t$. Introducing the Lagrangian density ($L$) [3] and integrating it over $\tau$, we get the total Lagrangian ($L = \int_{-\infty}^{\infty} L \, d\tau$). Using the Lagrangian one can obtain the reduced Euler-Lagrange equation for the pulse parameters as variables, which leads to a set of two coupled ordinary differential equations with one self-consistent equation for the three parameters that describe the dynamics of a perturbed CS.

\[
\frac{dE}{dt} = -2E - \frac{2}{3}K\eta E^2 + 2S \left( \frac{E}{2\eta} \right)^{1/2} \pi \cos \phi, \quad \frac{d\phi}{dt} = \frac{1}{3} \eta (E - \eta) - \Delta - S \left( \frac{1}{2E\eta} \right)^{1/2} \pi \sin \phi, \quad \eta = \frac{E}{4} + \frac{3S}{2\eta} \left( \frac{1}{2E\eta} \right)^{1/2} \pi \sin \phi. 
\]  

In Fig. 1(e) and (f) we plot the evolution of temporal width and peak intensity of CS which is generated from sech input pulse. The red dotted lines give the direct simulation results from LLE, whereas the solid blue lines give the variational results obtained by solving the set of equations (Eq. (4)). The input conditions are: $\Delta = 3$, $X = 3.5$ and the initial intensity and temporal width are taken to be $|u_0|^2 = E\eta/2$ and $\tau_w = 2\eta^{-1} = 2$, respectively. The direct simulation shows that before attending stabilization, the propagating pulse breathes which leads to the initial oscillation in amplitude and pulse width. The variational calculations also predict the initial oscillation that matches well with the numerical results.

References