Detection method of nonlinearity errors by statistical signal analysis in heterodyne Michelson interferometer

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Abstract: Periodic nonlinearity that ranges from tens of nanometers to a few nanometers in heterodyne interferometer limits its use in high accuracy measurement. A novel method is studied to detect the nonlinearity errors based on the electrical subdivision and the analysis method of statistical signal in heterodyne Michelson interferometer. Under the movement of micropositioning platform with the uniform velocity, the method can detect the nonlinearity errors by using the regression analysis and Jackknife estimation. Based on the analysis of the simulations, the method can estimate the influence of nonlinearity errors and other noises for the dimensions measurement in heterodyne Michelson interferometer.

OCIS codes: (120.3180) Interferometry; (120.3930) Metrological instrumentation; (120.1880) Detection; (220.4560) Optical data processing.

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1. Introduction

In precision engineering, heterodyne interferometer has become an important instrument for the dimensions measurement. However, periodic nonlinearity that ranges from tens of nanometers to a few nanometers in heterodyne interferometer limits its use in high accuracy measurement [1,2]. Many researchers have analyzed the behavior of the nonlinearity [1–7,10] and have presented the various methods of detection of the nonlinearity errors [1,8,9,23]. Furthermore, some methods have been studied to eliminate the nonlinearity in heterodyne interferometer [1,10–16,24–28]. Of the above methods, most of them use the optical technique while other methods use the electrical signal processing to detect or eliminate the nonlinearity. In the detection methods that use the electrical signal processing, a frequency domain method is proposed by Badami and Patterson [9] while Monte Carlo evaluation method is proposed by Schmitz and Kim [23]. In the compensation methods that use the electrical signal processing, a digital first-order periodic error reduction scheme is described by Chu and Ray [24], and the experimental validation of the scheme is provided by Schmitz, Chu, Houch III and Kalem [27]. Moreover, some improved methods are proposed to eliminate the nonlinearity by Schmitz, Chu, Houch III and Kim [25,26] based on Chu and Ray's scheme [24]. In this paper a novel method is proposed to detect the nonlinearity based on the electrical subdivision and the analysis method of statistical signal in heterodyne Michelson interferometer. Under the movement of micropositioning platform with the uniform velocity, the proposed method can detect the nonlinearity errors by using the regression analysis and Jackknife estimation [17]. The method is easy to be realized, and can also detect other noises in heterodyne Michelson interferometers.

The paper is organized as follows: in Section 2 we explain the principle of electrical subdivision algorithm. In Section 3 we introduce the nonlinearity of heterodyne Michelson interferometers. In section 4 we interpret the proposed method. In section 5 we present the simulation and numerical analysis, and we summarize our conclusions in section 6.

2. The electrical subdivision algorithm

Before introducing the detection method of the nonlinearity errors, we explain the principle of electrical subdivision algorithm. Several subdivision principles and realizations in heterodyne interferometer are proposed [18–22,29]. Based on Doppler frequency shift, the displacement of the micropositioning platform can be calculated in heterodyne Michelson interferometer with a double optical path as follows:

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$$L = \frac{\lambda}{2}\int_0^T \Delta f dt, \quad (1)$$

where L is the displacement of the micropositioning platform, λ is the wavelength of laser, and Δf is the Doppler frequency shift.

We also can use $\Delta\varphi$ to calculate L:

$$L = \frac{\lambda}{4\pi}\Delta\phi, \quad (2)$$

where $\Delta\varphi$ is the phase shift that is caused by Doppler frequency shift.

We use a typical scheme of heterodyne Michelson interferometer and extra circuit to realize the electrical subdivision algorithm in Fig. 1. We can get I_{m1}, I_{m2} and I_r by photodiodes D_{M1}, D_{M2} and D_R in the scheme under the ideal conditions:

$$I_{m1} = I_{m0}\cos\left[2\pi(f_2 - f_1)t + \Delta\phi + \phi_{m0}\right], \quad (3)$$

$$I_{m2} = I_{m0}\sin\left[2\pi(f_2 - f_1)t + \Delta\phi + \phi_{m0}\right], \quad (4)$$

$$I_r = I_{r0}\cos\left[2\pi(f_2 - f_1)t + \phi_{r0}\right], \quad (5)$$

where I_{m0} and I_{r0} are the amplitude of signals, f_1 and f_2 are the frequencies of incident beams, φ_{m0} and φ_{r0} are the initial and constant phase shift, $\Delta\varphi$ is the phase shift that is caused by Doppler frequency shift.

Then we use the multipliers to calculate I_{mr1} and I_{mr2}:

$$\begin{aligned} I_{mr1} &= I_{m1}I_r \\ &= \frac{1}{2}I_{m0}I_{r0}\left\{\cos\left[2\pi(2f_2 - 2f_1)t + \Delta\phi + \phi_{m0} + \phi_{r0}\right] + \cos\left(\Delta\phi + \phi_{m0} - \phi_{r0}\right)\right\}, \end{aligned} \quad (6)$$

$$\begin{aligned} I_{mr2} &= I_{m2}I_r \\ &= \frac{1}{2}I_{m0}I_{r0}\left\{\sin\left[2\pi(2f_2 - 2f_1)t + \Delta\phi + \phi_{m0} + \phi_{r0}\right] + \sin\left(\Delta\phi + \phi_{m0} - \phi_{r0}\right)\right\}. \end{aligned} \quad (7)$$

After the signal I_{mr1} and I_{mr2} pass by the low-pass filter, we can obtain:

$$I_1 = \frac{1}{2}I_{m0}I_{r0}\cos\left(\Delta\phi + \phi_{mr0}\right), \quad (8)$$

$$I_2 = \frac{1}{2}I_{m0}I_{r0}\sin\left(\Delta\phi + \phi_{mr0}\right), \quad (9)$$

where $\varphi_{mr0} = \varphi_{m0} - \varphi_{r0}$.

At last, we can get $\Delta\varphi$ as follows:

$$\Delta\phi = \arctan\frac{I_2}{I_1} - \phi_{mr0}. \quad (10)$$

When the micropositioning platform stops, we can calculate φ_{mr0} as follows:

$$\phi_{mr0} = \arctan\frac{I_{2s}}{I_{1s}}, \quad (11)$$

where I_{1s} and I_{2s} are the value of I_1 and I_2 when the micropositioning platform stops.

In the real system, the ADCs are used to sample I_1, I_2. The FPGA receives the sampled signals and outputs $\Delta\varphi$ to PC by Eq. (10) and Eq. (11). In contrast to f_1-f_2, Δf is small if the

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speed of the micropositioning platform is not fast. So we can obtain more sampling points in a period by ADC. In other words, the method can get higher resolution than sampling I_m and I_r directly.

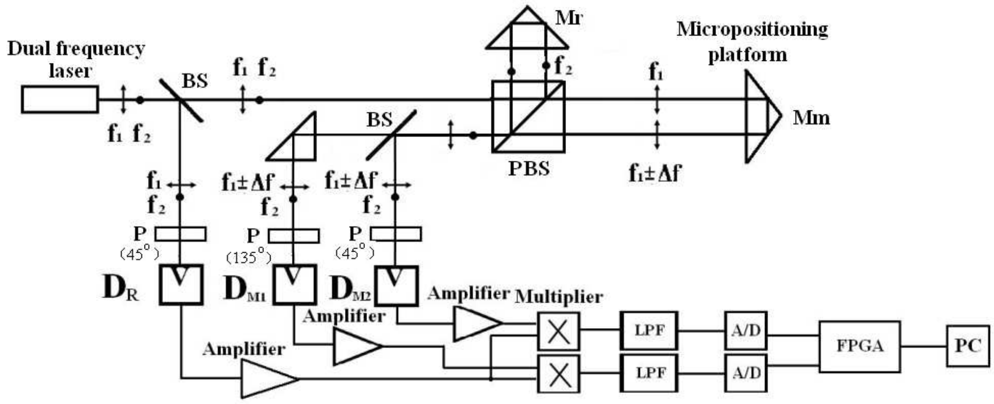

Fig. 1. a typical scheme of heterodyne interferometer: BS, beam splitter; PBS, polarizing beam splitter; P, polarization analyzer; D, photodiode detector; LPF, low-pass filter; A/D, analogue-digital converter; FPGA, field programmable gate array; PC, personal computer.

3. The nonlinearity of heterodyne interferometers

However, the nonlinearity errors exist in all heterodyne interferometers because of the nonorthogonality and ellipticity of the linearly polarized partial beams of the laser and the limited extinction capability of the polarizing beam splitters. Ref. 1 analyzed all kinds of reasons that lead to the nonlinearity errors. Ref. 5 and Ref. 10 summarize a common equation of nonlinearity errors as follows:

$$\gamma = \arctan\frac{\frac{b}{a}\sin(\Delta\phi+\theta_a)}{1+\frac{b}{a}\cos(\Delta\phi+\theta_a)} + \arctan\frac{\frac{d}{c}\sin(\Delta\phi+\theta_c)}{1+\frac{d}{c}\cos(\Delta\phi+\theta_c)}, \tag{12}$$

where a, b, c, d, θ_a, and θ_c are the influence factors that are determined by the incident beams with all possible polarizing imperfections and the attenuation and the phase shift while the beams travel through the diverse optical parts. b/a and d/c are the frequency mix ratios in two interferometer arms. θ_a and θ_c are the initial phase shifts of frequency f_1 and f_2 respectively. The derivation process of the Eq. (12) and the detailed definition of the influence factors are described in Ref.5 and Ref.10.

Therefore, the measured value of phase shift $\Delta\varphi_m$ in the heterodyne Michelson interferometer is shown as follows:

$$\Delta\phi_m = \Delta\phi + \gamma. \tag{13}$$

Figure 2 shows the images of the ideal $\Delta\varphi$ and the measured $\Delta\varphi_m$. In Fig. 2, we can see that the nonlinearity errors γ change with $\Delta\varphi$ periodically.

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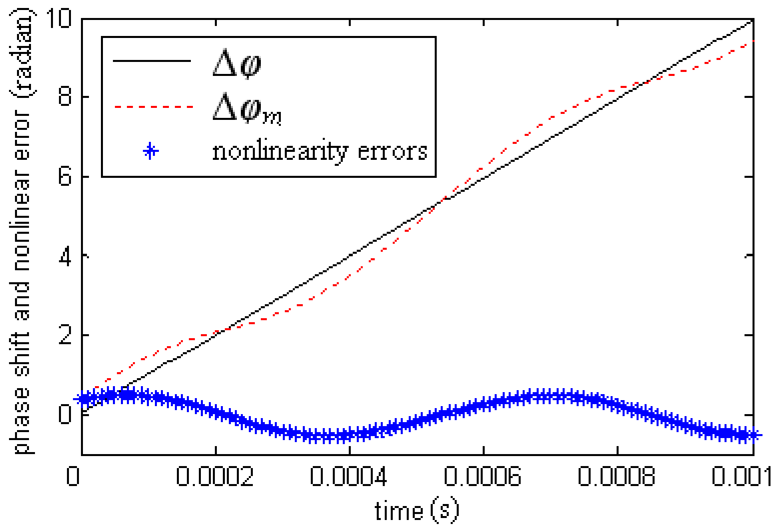

Fig. 2. The nonlinearity of heterodyne interferometer

4. Detection of nonlinearity errors

4.1 Phase shift under uniform velocity of the micropositioning platform

According to Doppler frequency shift:

$$\Delta f = \frac{v}{c} f, \tag{14}$$

where Δf is the frequency shift, v is the instant velocity of the micropositioning platform, c is the velocity of light, and f is the frequency of beam, respectively.

The relation of the frequency shift and the phase shift is:

$$\Delta\phi = 2\pi\Delta f t. \tag{15}$$

Therefore, we can obtain:

$$\Delta\phi = \frac{2\pi v f}{c} t. \tag{16}$$

In Eq. (16), we can know that the phase shift $\Delta\varphi$ is a constant value in a constant period of time when the micropositioning platform moves with the uniform velocity. So we can get:

$$\begin{aligned} \Delta\phi &= \arctan\frac{I_2}{I_1} - \phi_{mr0} \\ &= kt, \end{aligned} \tag{17}$$

where k is the parameter that is defined by the velocity v of the micropositioning platform and the frequency f of the beam as follows:

$$k = \frac{2\pi v f}{c}. \tag{18}$$

We also can get the measured value of phase shift $\Delta\varphi_m$ under the uniform velocity of the micropositioning platform as follows:

$$\Delta\phi_m = kt + \gamma. \tag{19}$$

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From Eq. (12), Eq. (19), and Fig. 2, we know that not only the image of the nonlinearity errors γ is smooth, but also the probability density function of the nonlinearity errors γ is not normal distribution. Therefore, we cannot use the traditional regression analysis to estimate k of Eq. (19) because the traditional regression analysis requires that the distribution of error is normal distribution. Therefore, we use Jackknife method to estimate k because the method has not the limitation of error distribution that is normal distribution, and has the excellent estimation performance.

4.2 Jackknife method for detection of nonlinearity errors

In the process of ADC sampling, every sampling interval is constant. From Eq. (19), we can get a discrete series as follows:

$$\Delta\phi_{mi} = kn_i + \gamma_i, i = 1, 2, 3, \cdots, N, \tag{20}$$

where n_i is the time of sampling point i and is determined by the sampling period of the ADC.

At first, we use least square method to estimate k:

$$k^* = \frac{\sum_{i=1}^{N}(n_i - \overline{n})(\Delta\phi_{mi} - \Delta\overline{\phi}_m)}{\sum_{i=1}^{N}(n_i - \overline{n})^2}. \tag{21}$$

Then we discard the data n_j and $\Delta\varphi_{mj}$ where $j \leqq N$ in the series and obtain a new series whose name is series j and the length of series j is $N-1$. We also use least square method to estimate $k^{(j)}$ in series j. We repeat the above method for N times and get a new series of $k^{(j)}$ where $j = 1,2,3,\ldots,N$. Therefore, we can obtain the unbiased estimators of k as follows:

$$\hat{k} = Nk^* - \frac{N-1}{N}\sum_{j=1}^{N}k^{(j)}. \tag{22}$$

We also can get the estimators of variance of k as follows:

$$\sigma_k^2 = \frac{N-1}{N}\sum_{j=1}^{N}\left(k^{(j)} - \frac{1}{N}\sum_{j=1}^{N}k^{(j)}\right)^2. \tag{23}$$

After getting the unbiased estimator of k, we can use the unbiased estimator to fit the image of phase shift that does not include the nonlinearity errors. By comparing the fitting phase shift with the measured phase shift, we can achieve the detection of the nonlinearity errors. The estimated nonlinearity errors are calculated as follows:

$$\hat{\gamma}_i = \Delta\phi_{mi} - \hat{k}n_i. \tag{24}$$

5. Simulations

To test the validity of the proposed method, we use Matlab to set up three simulation schemes because of the limitation of our simulation conditions. For the simulations, we use three groups of parameters to simulate three different measurement system based on heterodyne Michelson interferometer. Three groups of parameters are shown in Table 1. In Table 1, f is the optical frequency, v is the velocity of the micropositioning platform, and other parameters a, b, c, d, θ_a, and θ_c are the influence factors of nonlinearity errors, respectively.

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Table 1. Parameters of the simulations

	Group 1	Group 2	Group 3
f	4.74×10^{14}Hz	4.74×10^{14}Hz	4.74×10^{14}Hz
v	0.5mm/s	1mm/s	10mm/s
b/a	1/100	1/500	1/200
d/c	1/50	1/200	1/300
θ_a	$\pi/3$	$\pi/6$	$\pi/4$
θ_c	$\pi/4$	$\pi/3$	$5\pi/6$

5.1 Simulation 1

In the first simulation, we test the proposed method in a simple model. We neglect other noise and suppose that the micropositioning platform moves with the uniform velocity. Before the simulation, the sampling data array with a million data is created by Eq. (20). The step length of n is 0.00001. Then we use Jackknife method to estimate k. The results of the simulation are shown in Table 2. The function image of $\Delta\varphi_m$, estimated kn and nonlinearity errors in group 1 is shown in Fig. 3.

Table 2. Results in simulation 1

	Group 1	Group 2	Group 3
ideal k	4.963716×10^3	9.927433×10^3	9.927433×10^4
estimated k	4.892221×10^3	9.923737×10^3	9.927388×10^4
σ_k^2	13.785383	2.108159	1.667053

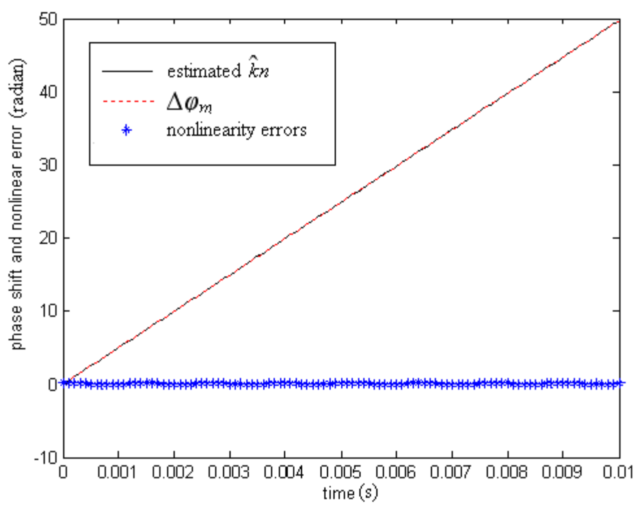

Fig. 3. Results of group 1 in simulation 1

5.2 Simulation 2

Besides the nonlinearity errors, in the simulation we add an additional noise source which simulates the electrical noise and defines a white noise with mean 0 and variance $10^{-6}k$. Although the electrical signals from the photosensors pass the filters and other electrical elements in order to remove the noises, we cannot avoid the condition that the electrical signals carry the tiny noises. So, we can get the measured value of phase shift $\Delta\varphi_{me}$ as follows:

$$\Delta\phi_{mei} = kn_i + \gamma_i + \eta_i, i = 1, 2, 3, \cdots, N, \tag{25}$$

where η is the tiny electrical noise that is often defined as a white Gaussian noise.

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Similarly, before the simulation, the sampling data array with a million data is also created by Eq. (25). The step length of n is 0.00001. Then we use Jackknife method to estimate k. The results of the simulation are shown in Table 3. The function image of $\Delta\varphi_{me}$, estimated kn and nonlinearity errors in group 1 is shown in Fig. 4.

Table 3. Results in simulation 2

	Group 1	Group 2	Group 3
ideal k	4.963716×10^3	9.927433×10^3	9.927433×10^4
estimated k	4.963114×10^3	9.927335×10^3	9.927439×10^4
σ_k^2	0.524324	0.030968	0.138890

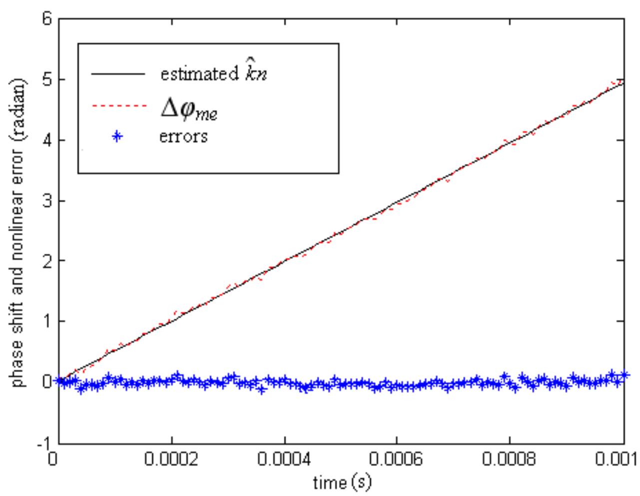

Fig. 4. Result of group 1 in simulation 2

5.3 Simulation 3

In simulation 3 we not only analyze the influence of nonlinearity errors and electrical noise, but also study the effect that is caused by the nonuniform velocity movement of the micropositioning platform. In the simulation, we suppose that the speed of the micropositioning platform has a small bias which follows normal distribution with mean v and variance $10^{-4}v$. We can get the measured value of phase shift $\Delta\varphi_{mes}$ as follows:

$$\Delta\phi_{mesi} = (k+\varepsilon_i)n_i + \gamma_i + \eta_i, i = 1,2,3,\cdots,N, \quad (26)$$

where ε is the bias value that is caused by the nonuniform velocity movement of the micropositioning platform.

Similarly, before the simulation, the sampling data array with a million data is also created by Eq. (26). The step length of n is 0.00001. Then we use Jackknife method to estimate k. The results of the simulation are shown in Table 4. The function image of $\Delta\varphi_{mes}$, estimated kn and nonlinearity errors in group 1 is shown in Fig. 5.

Table 4. Results in simulation 3

	Group 1	Group 2	Group 3
ideal k	4.963716×10^3	9.927433×10^3	9.927433×10^4
estimated k	4.963093×10^3	9.926830×10^3	9.927294×10^4
σ_k^2	0.530471	0.052653	5.614443

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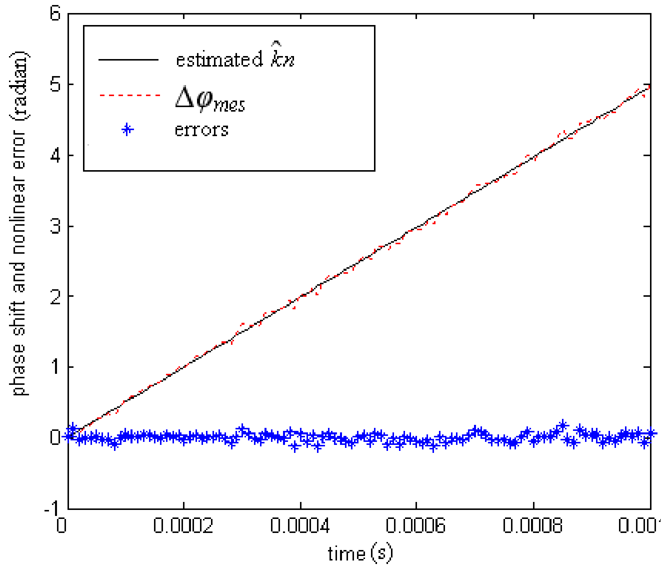

Fig. 5. Result of group 1 in simulation 3

From the above simulations, we can know that the unbiased estimator is equal to the pre-setting value approximately in all 3 cases. Therefore, we can use the analysis method of statistical signal to fit the image of phase shift without the nonlinearity errors, and detect the nonlinearity errors and other noises from the measured phase shift and the unbiased estimated phase shift.

6. Conclusion

The nonlinearity is the main error source in heterodyne interferometer. Many scientists and researchers often use the optical methods and electrical methods to detect and eliminate the nonlinearity errors. We use the analysis method of statistical signal to detect the nonlinearity errors in heterodyne Michelson interferometer. Because of nonnormal distribution of the nonlinearity errors, we use regression analysis and Jackknife method to estimate the characteristic parameter of the fitting curve. After we obtain the estimated parameter, we can extract the estimated nonlinearity errors for the detection. In the paper we also analyze other interference sources, such as the electrical noises and the nonuniform velocity movement of the micropositioning platform. To a great extent the method can detect the influence of nonlinearity errors and other noises for the dimensions measurement in heterodyne Michelson interferometer.

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