Two-dimensional photonic crystals constructed with a portion of photonic quasicrystals

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Abstract: Photonic quasicrystals (PQs) can produce interesting photonic properties. However, the exact prediction on photonic band structures (PBSs) of the PQs is currently a fundamental challenge due to the lack of periodicity of the structures. Here, we propose a kind of complex periodic two-dimensional photonic crystal (PC) structures constructed with a small portion of different PQs for the purposes of overcoming the difficulty of numerical calculations on the PBSs but maintaining the photonic properties of the original PQs owned. Theoretically calculated results on PBSs of the complex PCs with a local feature consistent with 12-fold rotational symmetry show that, in the cases of dielectric cylinders in air, air-holes in a dielectric, and metal cylinders in air, respectively, the complex PCs can indeed produce similar photonic properties of the original 12-fold PQs such as the uniform or isotropic PBGs under much lower dielectric contrast etc. Because the complex PCs can be constructed with the local parts of any high symmetric PQs, we believe that the PCs presented in this article may provide a way for creating novel photonic functional materials.

OCIS codes: (999.9999) photonic crystals; (160.4670) Optical materials

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1. Introduction

Photonic quasicrystals (PQs) are demonstrated to be more beneficial for getting interesting photonic bandgap (PBG) features (such as, for example, more uniform or isotropic PBGs under lower dielectric contrast etc.) than the conventional photonic crystals (PCs) due to their high level of rotational symmetry [1-8]. A series of experimental technologies have been successfully employed to fabricate PQs [4, 5, 10-12]. However, the exact theoretical prediction on the photonic band structures (PBSs) of PQs is currently a numerical challenge because of the lack of periodicity of the structures. Although most of previous studies have focused on the characteristics of two-dimensional (2D) PQs, only approximate PBSs can be calculated by defining periodically repeated supercells [7-9], which may cover some interesting effects behind the PQs. To exactly grasp the optical properties of PQs, one has generally to construct a real model for experimental measurements [3-5]. Obviously, this is time-consuming, money waste and even no sense sometimes. Since short-range features play the dominating role on PBSs of the PCs [1], here, we propose a kind of complex 2D PC structures constructed with a small portion of different PQs. The motivation to introduce such novel PCs is to reduce the orientation order of PCs in short range for achieving similar PBGs of the original PQs while maintain periodic scattering of the PCs to light in long range for calculating the PBSs of the PCs exactly.

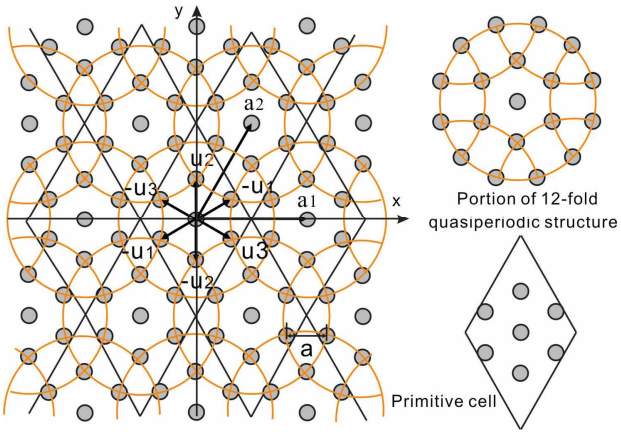

Fig. 1. (color online) Scheme of the proposed 2D PC structures constructed with a portion of 12-fold symmetric PQ (orange circles). The primitive cell (dark diamonds) included seven cylinders is defined by two primitive translation vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ .Six cylinders surrounded the central one are at positions $\mathbf{u}_1$, $-\mathbf{u}_1$, $\mathbf{u}_2$, $-\mathbf{u}_2$, $\mathbf{u}_3$ and $-\mathbf{u}_3$, respectively.

2. Model of the complex 2D PC structures

Figure 1 schematically shows the proposed 2D PCs constructed with a local part of 12-fold symmetric PQs (orange circles). The primitive cell (dark diamonds) of the structure is formed

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through two primitive translation vectors $\mathbf{a}1 = (1+\sqrt{3})a(1,0)$ and $\mathbf{a}2 = (1+\sqrt{3})a(1/2,\sqrt{3}/2)$, of which, the central cylinder is surrounded by six ones at the positions $\mathbf{u}1$ and $-\mathbf{u}1$, $\mathbf{u}2$ and $-\mathbf{u}2$, and $\mathbf{u}3$ and $-\mathbf{u}3$ with $\mathbf{u}1 = a(\sqrt{3}/2,1/2)$, $\mathbf{u}2 = a(-\sqrt{3}/2,1/2)$, and $\mathbf{u}3 = a(0,1)$, respectively (a is the lattice constant). The whole structure is a perfect periodic triangular lattice. Hence dielectric constants of the PC structures can be written as $\varepsilon(\mathbf{r}+\mathbf{R}_l)=\varepsilon(\mathbf{r})$, where $\mathbf{r}$ is the position vector. For any integers $l_{1,2}$, $\mathbf{R}_l = l_1\,\mathbf{a}1 + l_2\,\mathbf{a}2$ defines the Bravais lattice. Thus, PBSs of the PCs can exactly be calculated.

3. PBSs of the complex 2D PCs under the situations of dielectric cylinders in air, air-hole in dielectric, and metal cylinders in air, respectively

In this paper, we will use the plane wave method (PWM) [13] to calculate PBSs of the proposed PCs for getting a systemic and exact insight into their interesting optical properties. In the framework of PWM, band structure problem can be reduced to the eigenvalue problem

$$\det(\kappa - \frac{\omega^2}{c^2}) = 0 \tag{1}$$

Where ω is the frequency, c is the velocity of light in vacuum. In 2D case, we can get $\kappa(\mathbf{G},\mathbf{G}') = |\mathbf{k}+\mathbf{G}||\mathbf{k}+\mathbf{G}'|\eta(\mathbf{G}-\mathbf{G}')$ for E- polarized modes (electric field parallel to the cylinders) and $\kappa(\mathbf{G},\mathbf{G}') = (\mathbf{k}+\mathbf{G})(\mathbf{k}+\mathbf{G}')\eta(\mathbf{G}-\mathbf{G}')$ for H- polarized modes (magnetic field parallel to the cylinders), where $\mathbf{k}$ is the reciprocal vector lying inside the Brillouin zone, $\mathbf{G}$ is the translation vector of reciprocal lattice, and $\eta(\mathbf{G})$ is a matrix found by inverting the Fourier transform of the dielectric constant [14-16]. The Fourier transform is written as: $\varepsilon(\mathbf{G}) = (\int_{|\mathbf{a}1\times\mathbf{a}2|} \varepsilon(\mathbf{R}_l)\exp(-i\cdot(\mathbf{G}\cdot\mathbf{R}_l))d\mathbf{R}_l)/|\mathbf{a}1\times\mathbf{a}2|$, where the integration is taken over the primitive cell with area $|\mathbf{a}1\times\mathbf{a}2|$. We see that $\varepsilon(\mathbf{G})$ plays a central role on the formation of PBSs for both polarizations.

In the cases of dielectrics constructed PCs as shown in Fig. 1, we perform

$$\varepsilon(\mathbf{G}) = f/\varepsilon_a + (1-f)/\varepsilon_b,\ G = 0 \tag{2a}$$

$$\varepsilon(\mathbf{G}) = 2(\varepsilon_a - \varepsilon_b)(1+2\sum_{i=1}^{3}\cos(\mathbf{G}\cdot\mathbf{u}_i))\cdot f_0 \cdot J_1(GR)/(GR),\ G \neq 0 \tag{2b}$$

Where $G = |\mathbf{G}|$, ε_a and ε_b are the dielectric constants of cylinders and background, respectively. R is the radii of the cylinders, and $f = 7f_0$ is the filling fraction defined as the fraction of the primary cell area occupied by the cylinders ($f_0 = \pi\cdot R^2/|\mathbf{a}1\times\mathbf{a}2|$). $J_1(GR)$ is the first-order Bessel function. Substituting Eq. (2) into Eq. (1), we can calculate out the PBSs of the proposed PCs [14, 15].

Figure 2 displays the PBSs of a proposed PC with dielectric cylinders (ε_a =13 and R/a =0.35) embedded in air (ε_b =1) for both polarizations. One can see that, for H-polarization, six band gaps appear in the PBSs in the normalized frequency ranges ($\omega a/2\pi c$) between 0.291 and 0.318 (H-1), 0.355 and 0.373 (H-2), 0.426 and 0.433 (H-3), 0.553 and 0.574 (H-4), 0.578 and 0.598 (H-5), and 0.608 and 0.622 (H-6), respectively [Fig. 2(a)]. For E-polarization, there exist three PBGs between 0.201and 0.262 (E-1), 0.357 and 0.437 (E-2), and 0.536 and 0.617 (E-3), respectively [Fig. 2(b)]. Furthermore, some PBGs for both polarizations can overlap with each other to form a series of complete PBGs. For instance, gaps H-2 and E-2 overlapped with each other around the central frequency ω_g =0.365 can produce a complete PBG with width $\Delta\omega = 0.016$ and a gap-width ratio $\Delta\omega/\omega_g$ =4.33%. Other

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complete gaps such as formed by gaps H-3 and E-2 around ω_g =0.429 with $\Delta\omega = 0.007$ and $\Delta\omega/\omega_g$ =1.58%, H-4 and E-3 at ω_g =0.563 with $\Delta\omega = 0.021$ and $\Delta\omega/\omega_g$ =3.73%, H-5 and E-3 at ω_g =0.588 with $\Delta\omega = 0.021$ and $\Delta\omega/\omega_g$ =3.49%, and H-6 and E-3 at ω_g =0.613 with $\Delta\omega = 0.009$ and $\Delta\omega/\omega_g$ =1.53%, respectively, are also observed in the PBSs of the PC.

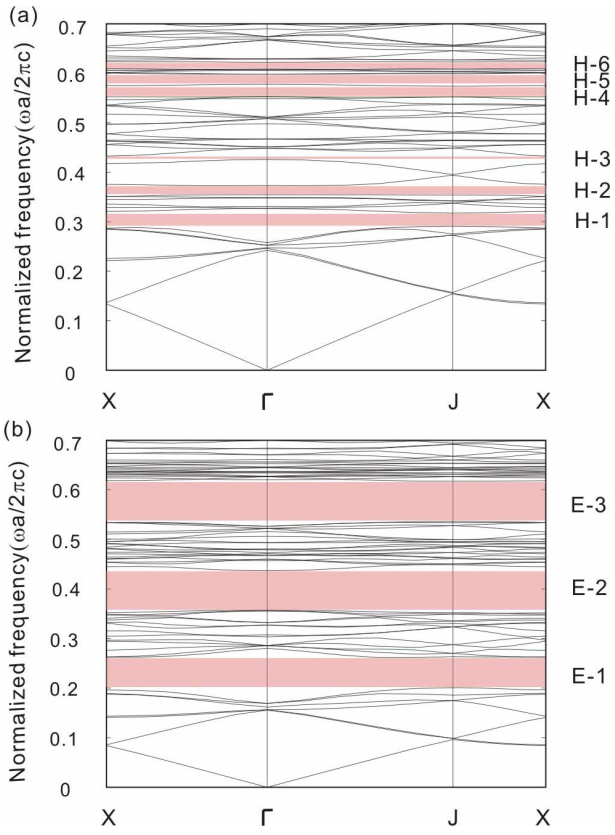

Fig. 2. (color online) Calculated PBSs of the proposed 2D PCs constructed with a portion of 12-fold symmetric PQ with dielectric cylinders (ε_a =13 and R/a =0.35) in air (ε_b =1) for (a) H- and (b) E- polarized modes, respectively. The PBGs are indicated with pink shadow.

By searching for the dependence of PBGs of the proposed PCs on dielectric contrast, we find that, for H-polarization, to open up a PBG, only ε_a >4.5 is needed. However, for the simple triangular structures (one cylinder in a primitive cell), to open up a H-polarized PBG, at least ε_a >9 is needed. Furthermore, the present PC can even produce a complete gap around ω_g =0.890 as ε_a is equal to 4.5 (not shown here). While for the simple triangular structures, no any complete gap will appear even if the structure is with any high dielectric contrast [14, 17, 18]. These are attributed to the fact that our proposed PCs are with a short-range feature consistent with local 12-fold rotational symmetry.

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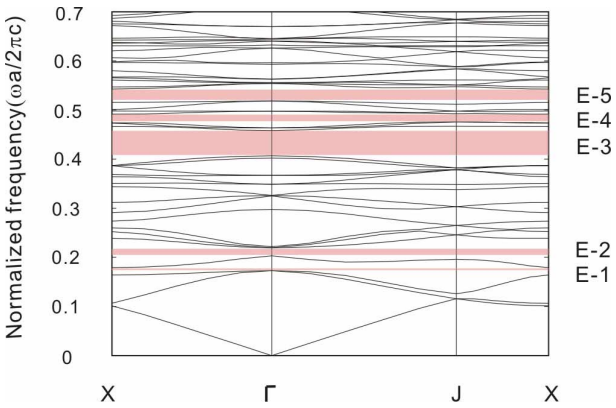

Fig. 3. (color online) Calculated PBSs of the proposed 2D PCs constructed with a portion of 12-fold symmetric PQ with air-holes (ε_a =1 and R/a =0.47) in a dielectric background (ε_b =13) for E- polarization. The PBGs are indicated with pink shadow.

In the following, we will consider the case that air-holes are embedded in a dielectric background. Figure 3 shows the calculated PBSs as the proposed PC is with ε_a =1, ε_b =13, and R/a =0.47 for E-polarization. One sees that there exist five PBGs in the frequency regions between 0.173 and 0.179 (E-1), 0.203 and 0.220 (E-2), 0.407 and 0.458 (E-3), 0.476 and 0.492 (E-4), and 0.518 and 0.543 (E-5), respectively. The gap widths $\Delta\omega$ and the corresponding gap-width ratios $\Delta\omega/\omega_g$ are 0.007 and 3.81%, 0.017 and 7.86%, 0.052 and 11.93%, 0.016 and 3.29%, and 0.024 and 4.58%, respectively. The lowest dielectric contrast for opening up a PBG is just ε_b >3.5. For a simple triangular structure, however, to open up a PBG, around ε_b >6 should be satisfied for Ref. [14]. Even with a contrast as ε_b =13, only one gap similar to E-3 of the present PCs is opened up for the simple triangular structures [14].

From above results, it is obvious that, similar to PQs [5, 6], the present PCs show significant advantages over the simple triangular structures in PBG properties. The calculated PBSs are confirmed well by previous reports on the transmission spectrum of the 12-fold PQs as the same parameters are used [6]. However, our PCs are a perfect periodic structure and hence their PBSs can exactly be predicted.

Different from the dielectrics, metal materials show frequency dispersion and their dielectric functions can be expressed by the Drude formula: $\varepsilon(\omega) = 1 - (\omega_p^2/\omega^2)$, where ω_p is the plasma frequency of the conduction electrons. Metals constructed PCs have attracted tremendous interests because metals can not only effectively enhance the dielectric contrast of the PCs, but also make optical properties of the PCs tunable by external fields [19, 20]. In the following, we will investigate into the PBSs of the proposed PCs constructed by embedding metal cylinders into air. The Fourier transforms of dielectric functions of the metallic PCs can be read as [15]:

$$\varepsilon(\mathbf{G}) = 1 - \mathrm{f}\cdot\omega_p^2/\omega^2,\ \mathrm{G} = 0 \tag{3a}$$

$$\varepsilon(\mathbf{G}) = 2(\omega_p^2/\omega^2)\cdot(1+2\sum_{i=1}^{3}\cos(\mathbf{G}\cdot\mathbf{u}_i))\cdot \mathrm{f}_0\cdot \mathrm{J}_1(\mathrm{GR})/(\mathrm{GR}),\ \mathrm{G}\neq 0 \tag{3b}$$

By combining Eqs. (1) and (3), we can calculate out the PBSs of the PCs for E-polarization (Fig. 4). The filling fraction is assumed as R/a =0.45 and the frequency range is from $\omega\,\mathrm{a}/2\pi\,\mathrm{c}$ = 0.6 to 1.1 (the lowest frequency band does not tend to zero [16]). We see that two large PBGs between $\omega\,\mathrm{a}/2\pi\,\mathrm{c}$ = 0.710 and 0.816 (E-1), and 0.835 and 0.866 (E-2)

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appear in the band structures. The corresponding gap widths and gap-width ratios are $\Delta\omega = 0.101$ and $\Delta\omega/\omega_g$ =13.17% and $\Delta\omega = 0.031$ and $\Delta\omega/\omega_g$ =3.64%, respectively. In metals constructed simple triangular lattices, however, there exists no E-polarized PBG in the PBSs [16] even if the same parameters are used.

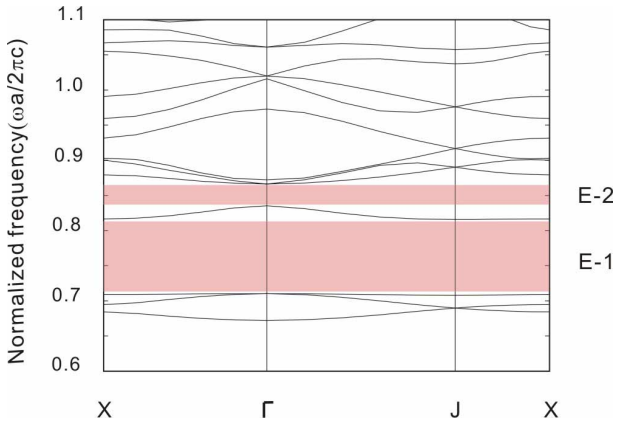

Fig. 4. (color online) Calculated PBSs of the proposed 2D PCs constructed with a portion of 12-fold symmetric PQ with metal cylinders (R/a =0.45) in air for E- polarization. Two large PBGs are indicated with pink shadow.

4. Conclusion

In conclusion, we have proposed a kind of complex 2D PCs constructed with a portion of different PQs. As an example, we have demonstrated theoretically that the proposed PCs with a local feature consistent with 12-fold rotational symmetry can indeed maintain the similar photonic properties of the original PQs. Additionally, such PCs can be formed with the local parts of any high symmetric PQs. Therefore, we believe that the complex PCs proposed in this paper may offer a big potential for creating novel photonic functional materials.

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