

# Switching of synchronized chaotic oscillations in a modulated solid-state ring laser

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**Abstract:** We study synchronization in chaotic oscillations of counterpropagating waves in a solid-state ring laser with a periodically modulated pump. The new phenomenon of spontaneous switching of in-phase- and anti-phase chaotic synchronization has been discovered in a numerical experiment.

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## 1. Introduction

In the last decade, much attention has been paid to a problem of synchronization of chaotic coupled oscillations and possible applications of this phenomenon for communication purposes (see Refs. [1-14] and the literature quoted in these Refs.). Synchronized chaotic behavior of coupled lasers has been found in several physical and numerical experiments [4-10].

In most cases, identical (in-phase) synchronization has been dealt with. In this regime the corresponding variables in coupled systems coincide. In the case of anti-phase synchronization, the corresponding variables have equal moduli and opposite signs. In both these cases one can speak about "full synchronization" of chaotic oscillations. In Refs. [11-13] the phenomenon of phase synchronization of chaotic oscillations was found. This effect was defined as the appearance of entrainment between the phases of oscillators, while their amplitudes remain chaotic and non-correlated.

This paper concerns chaotic dynamics of modulated single-mode class-B lasers. A response of such lasers to low frequency modulation of parameters was studied in many publications. In particular, the formation and evolution of strange chaotic attractors was considered in Refs. [15-21]. It was shown that parametric excitation of relaxation oscillations reduces the threshold for onset of dynamic chaos in Fabry-Perot [15-17] and ring lasers [20, 21]. Synchronization of dynamic chaos in counterpropagating ring-laser waves was observed in Ref. [14]. In this Ref. regions of existence of strange attractors of two types, synchronized and nonsynchronized, were found in the plane of the system parameters (depth and frequency of pump modulation). The present paper is devoted to a more detailed consideration of properties of the synchronized attractor. The main purpose of this paper is to demonstrate in a numerical experiment with a model of a solid-state ring laser (SSRL) the new phenomenon of spontaneous switching of in-phase and anti-phase chaotic synchronization.

## 2. Model of a bidirectional ring laser

The counterpropagating waves traveling in opposite directions inside a ring cavity interact with one another and may have different amplitudes and frequencies. We restrict our consideration to the case of single mode generation in each direction. The counterpropagating waves are coupled due to the backscattering by optical inhomogeneities in the cavity. Interference between the counterpropagating waves causes periodic spatial variation (along the cavity axis) of the energy density of the optical field. If the population inversion is saturated by the radiation field in a ring cavity, periodic structures (gratings) are induced in active medium. The population inversion  $N$  can be represented in the form of a sum of the spatial harmonics. Self-diffraction of the counterpropagating waves by the induced gratings causes the nonlinear coupling of these waves.

For a description of bidirectional lasing we use a set of equations derived in semiclassical theory of a SSRL for the complex field amplitudes  $E_{1,2}$  and the spatial Fourier components of the population inversion  $N_0$  and  $N_1 = N_{-1}^*$ :

$$\frac{dE_{1,2}}{dt} = -\frac{1}{2} \frac{\omega}{Q} E_{1,2} + \frac{im}{2} E_{2,1} + \frac{\sigma l}{2T} (N_0 E_{1,2} + N_{\mp} E_{2,1}),$$

$$T_1 \frac{dN_0}{dt} = N_{th} (1 + \eta_{eff}) - N_0 (1 + a(|E_1|^2 + |E_2|^2)) - \dot{a} N_+ E_1 E_2^* - \dot{a} N_- E_1^* E_2,$$

$$T_1 \frac{dN_{\pm}}{dt} = -N_{\pm} (1 + a(|E_1|^2 + |E_2|^2)) - \dot{a} N_0 E_1 E_2^*,$$

$$N_0 = \frac{1}{L} \int_0^L N dz, \quad N_{\pm} = \frac{1}{L} \int_0^L N e^{\pm i2kz} dz, \quad N_- = N_+^*, \quad (1)$$

where  $T_1$  is the relaxation time of the population inversion  $N$ ,  $N_{th}(1+\eta_{eff})/T_1$  is the pumping rate,  $N_{th}$  is the threshold value of  $N$ ,  $\eta_{eff}$  is the pump excess over the threshold,  $T$  is the cavity round trip time,  $\sigma$  is the laser transition cross section, and  $a$  is the saturation parameter.

The losses for counterpropagating waves are taken to be equal,  $Q$  is the quality factor of the cavity. The terms  $imE_{2,1}/2$  describe coupling between counterpropagating waves due to backscattering inside the cavity. We neglect the detuning of the optical frequency  $\omega$  from line center, assuming it is small with respect to the linewidth of atomic transition.

We report here theoretical investigations of dynamical chaos in an SSRL with periodic pump modulation. In the presence of pump modulation, the effective pump intensity  $\eta_{eff}$  is given by the expression

$$\eta_{eff} = \eta + h \cos(\omega_p t), \quad (2)$$

where  $\eta$  is the excess of the pump intensity above the threshold in the absence of pump modulation,  $h$  is the modulation depth.

The system of equations was solved numerically by the Runge-Cutta method of the eighth order with double precision. The laser parameters were the same as for the experimentally investigated Nd: YAG SSRL with a monolithic cavity [14,15]: the relaxation time of the population difference is  $T_1=240$ ms, the round-trip time of the cavity is  $T=1.66 \times 10^{-10}$  s, the losses in a single trip through the cavity are  $1-R=3.2\%$ , the moduli of the coupling coefficients are  $m/2p=170$  kHz, and the excess above the threshold is  $\eta=0.21$ . Values of parameters  $h$  and  $\omega_p$  varied in the numerical experiment.

For these parameters, in the absence of pump modulation ( $h=0$ ) intensities of the counterpropagating waves  $I_{1,2}=a|E_{1,2}|^2$  undergo antiphase sinusoidal self-modulation with the frequency  $\omega_m/2\pi=170$  kHz, and the frequency of a relaxation oscillations is  $\omega_r/2\pi=(\omega_c \eta / QT_1)^{1/2}=65$  kHz.

### 3. Chaotic regimes

In the region of the pump modulation frequencies  $\omega_p$  close to the relaxation frequency  $\omega_r$  ( $50\text{kHz} < \omega_p/2\pi < 90\text{kHz}$ ) at the modulation depths  $h > 0.12$  the lasing dynamics may become chaotic. For the values of  $h$  in the range  $0.12 < h < 0.23$ , there is a bistability in the laser behavior: regime of dynamic chaos coexists with the regime of periodic pulse modulation (spiking mode). For the spiking mode, the counterpropagating waves have equal intensities ( $I_1=I_2$ ). For the chaotic regime, pulsations of intensities  $I_{1,2}$  are not synchronized.

At the modulation depths  $h > 0.23$ , synchronization in chaotic intensities of counterpropagating waves could be observed. For synchronized chaotic oscillations, the laser

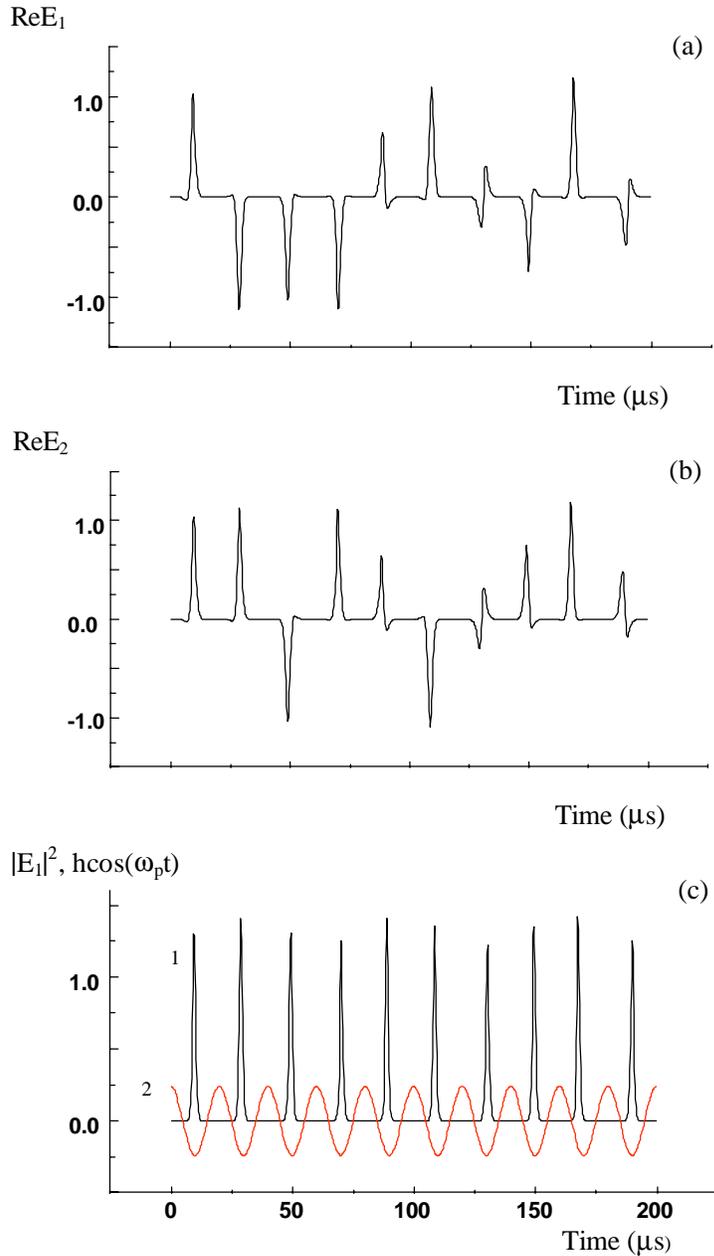


Fig.1 Time-dependence of  $\text{Re}E_1$  (a),  $\text{Re}E_2$  (b), and  $|E_1|^2$  - (1),  $h\cos(\omega_p t)$  - (2) (c) for the chaotic oscillations in a modulated SSRL. The results were obtained by numerically integrating the system of equations (1) with  $\omega_p/2\pi=50$  kHz,  $h=0.24$ .

radiation in both directions consists of a nearly periodic sequence of pulses with chaotic amplitudes. The typical time domain behavior of the normalized intensity  $I_1=|E_1|^2$  is shown in Fig.1c. Here the pump modulation signal  $h\cos(\omega_p t)$  is also shown. This chaotic regime was studied theoretically and experimentally in Ref.[14]. In this paper, we study the behavior of a real ( $\text{Re}E_1, \text{Re}E_2$ ) and imaginary ( $\text{Im}E_1, \text{Im}E_2$ ) parts of complex fields  $E_1, E_2$ .

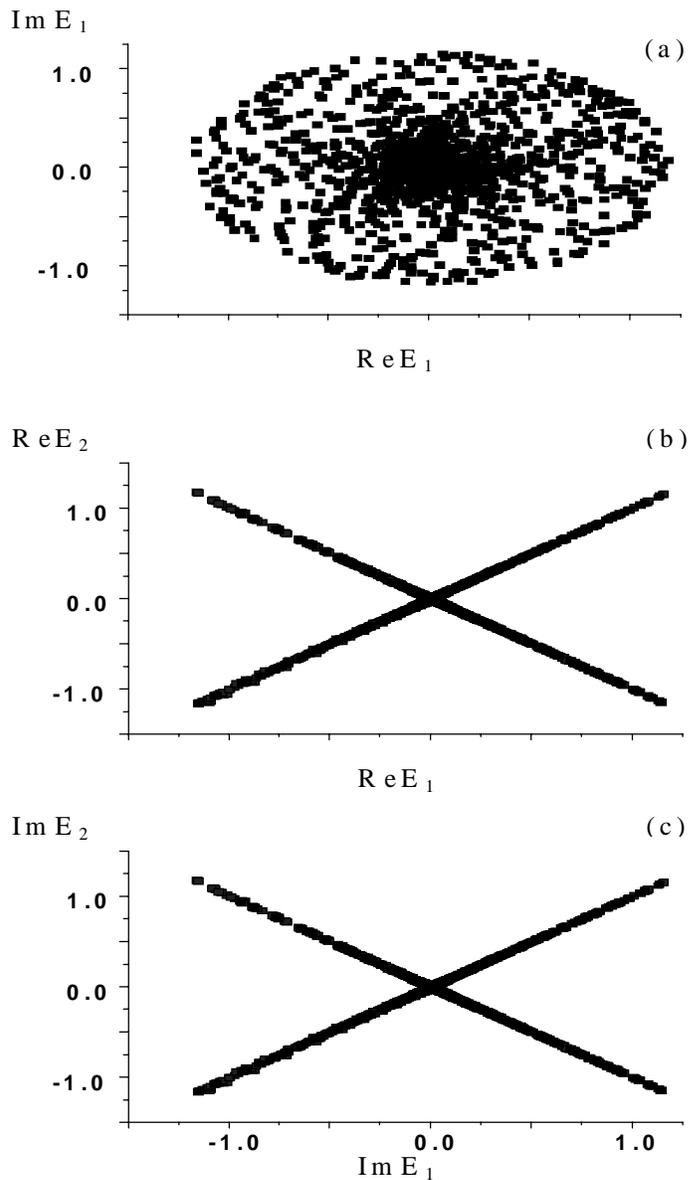


Fig.2. Projections of the chaotic pulsations on the planes of variables  $(\text{Re}E_1, \text{Im}E_1)$  (a),  $(\text{Re}E_1, \text{Re}E_2)$  (b), and  $(\text{Im}E_1, \text{Im}E_2)$  (c). The results were obtained by numerically integrating the system of equations (1) with  $\omega_p/2\pi=50$  kHz,  $h=0.24$ .

Fig. 2 shows projections of the phase space portrait for this regime onto the planes of variables  $(\text{Re}E_1, \text{Im}E_1)$  (a),  $(\text{Re}E_1, \text{Re}E_2)$  (b), and  $(\text{Im}E_1, \text{Im}E_2)$  (c). One can see from Fig.2b, 2c that the points of the phase space portrait are on two straight lines:  $E_1=E_2$  (in-phase chaotic synchronization) and  $E_1=-E_2$  (anti-phase chaotic synchronization). The typical time domain behavior of  $\text{Re}E_1$  and  $\text{Re}E_2$  is shown in Fig.1a and Fig.1b, respectively. One can see from Fig.1 that during each period of modulation the switch is observed from in-phase (anti-phase) chaotic synchronization to anti-phase (in-phase) chaotic synchronization.

We also studied the behavior of the phase difference  $\Phi = \phi_1 - \phi_2$  for counterpropagating waves. The phase difference  $\Phi$  is constant during the pulses of radiation and jumps on the value  $\pi$  between the neighboring pulses. The electric field inside the cavity represents a standing wave in each pulse. Nodes of a standing wave in a previous pulse shift to positions of antinodes in the next one.

A spectrum of the Lyapunov exponents and the information dimension of the strange attractor have been found. For the regime of identical synchronization at  $\omega_p/2\pi = 50$  kHz,  $h = 0.24$ , the Lyapunov exponents  $\lambda_i$  multiplied by  $T_1$  are equal:  $\{\lambda_i T_1\} = \{2.8; 0.92; 0; 0; -1.2; -4.1; -8.7; -3.8\}$ . There are five Lyapunov exponents for which their sum  $s = \lambda_1 + \lambda_2 + \dots + \lambda_5 > 0$ . Assuming that the Kaplan-Yorke conjecture holds for this attractor, the information dimension  $D_1$  is equal to  $D_1 = 5 + s/|\lambda_6| = 5.61$ .

#### 4. Conclusion

In conclusion, we would like to discuss possible ways of experimental observation of the considered effect. As it was shown above, the behavior of intensities of the counterpropagating waves in the regime of synchronized chaos was studied experimentally in Ref. [14]. To observe switchings between in-phase and anti-phase states of chaotic synchronization, one can compare the interference fringes formed by the counterpropagating waves in different chaotic pulses. Another possibility for observing this effect may be realized by heterodyne detection. One must mix the output waves from the ring laser with a local oscillator wave. In such an experiment, one can detect "in-phase" and "quadrature-phase" field components of the counterpropagating waves which correspond to  $\text{Re}E_{1,2}$  and  $\text{Im}E_{1,2}$ , respectively.

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