

Two-photon four-qubit cluster state generation based on a polarization-entangled photon pair

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Abstract: We propose and experimentally demonstrate a two-photon four-qubit cluster state generator using linear optics and a single polarization-entangled photon-pair source based on spontaneous parametric down-conversion (SPDC). Our novel scheme provides greater design flexibility compared to previous schemes that rely on hyperentanglement.

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1. Introduction

Realization of photonic multiple-qubit entangled states has been a key element of recent progress in quantum information (QI) science as shown in demonstrations of quantum algorithms [1-6] and non-classicality tests of quantum-mechanical states [7,8]. Whereas using photons as quantum information carriers has the advantage of low intrinsic decoherence [1], the low efficiency of photon sources has so far limited the number of available photons to six or less [3]. This has in practice set a limit on the size of quantum mechanical systems that can be built using photons.

Efforts to overcome this practical limit have led to the development of techniques to encode more than one qubit per photon [9,10]. For example, two-photon four-qubit (2P4Q) linear cluster states that encode both a polarization qubit and a momentum (i.e., path) qubit in each photon have recently been realized and successfully applied to performing one-way quantum computation [4,5]. The common feature of these 2P4Q cluster states lies in the use of hyperentanglement composed of a doublet of two-qubit entanglements [11]. Here, the two photons have their polarizations entangled with respect to each other, and likewise for their paths. A four-qubit entanglement is generated through a path-dependent polarization flip operation that entangles the polarization qubit of one photon with the path qubit of the other photon.

The hyperentanglement-based 2P4Q generators essentially demand the coherent combination of two distinct polarization-entangled photon-pair sources. The first experimental demonstration of a 2P4Q cluster state [10] realizes this coherent combination by means of a three-dimensional optical path configuration in which two photon-pair output ports are selected from a single SPDC emission cone. The four qubits are generated at a rate of 1 kHz, which is a significant improvement on the four-qubit generation rates (less than 1 Hz) obtained in previous four-photon four-qubit (4P4Q) experiments [2, 8]. Another 2P4Q demonstration [5] is based on a double-pass-pumped SPDC scheme that generates two photon pairs propagating in opposite directions. Here, the generation rate of the four qubits has been further improved to 12 kHz. Both of the 2P4Q experiments achieved a fidelity with the ideal cluster state greater than 0.84, which are comparable to the highest level achieved to date with various four-qubit cluster state generators [2, 4, 6, 8].

The 2P4Q generators described above require matching of the spectral/temporal modes and the generation efficiencies between the two photon-pair sources. In contrast to these schemes based on hyperentanglement, we propose a novel scheme to generate a 2P4Q linear cluster state based on a single polarization-entangled photon-pair source and linear optics elements. Hence the mode-matching of distinct photon pairs is not a requirement in our 2P4Q scheme. Furthermore, any recently developed structure for photon-pair generation [12-14] can be incorporated into our 2P4Q scheme in a straightforward manner. This allows greater flexibility for developing photon-based QI technology.

Section 2 of this paper describes our 2P4Q scheme. Experimental details and results are shown in Section 3. In particular, quantum correlation measurements on different pairs of qubits demonstrate the feasibility of addressing individual qubits of the 2P4Q cluster state. At present, the fidelity and the generation rate in our experiment have not yet reached the level of previous 2P4Q schemes. However, the value for the entanglement witness shows that genuine four-qubit entanglement has been achieved. A summary of our work is given in Section 4.

2. Scheme

As shown in Fig. 1(a), the proposed scheme consists of a polarization-entangled photon-pair ($|\Phi^{(+)}\rangle = 1/2^{1/2}(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2)$) source, a polarizing beam splitter (PBS), a non-polarizing beam splitter (NPBS) and a half-wave plate (HWP). Here, H (V) denotes horizontal (vertical) polarization, and the subscripts 1, 2 refer to the photons. The photon-pair source sends a

photon to the PBS and the NPBS, respectively, and the PBS transmits (reflects) a horizontally (vertically) polarized photon. These two elements transform the two-photon state to

$$|\Phi\rangle_{\text{trans}} = 1/2(|H, a\rangle_1 |H, c\rangle_2 + |H, a\rangle_1 |H, d\rangle_2 + |V, b\rangle_1 |V, c\rangle_2 + |V, b\rangle_1 |V, d\rangle_2), \quad (1)$$

where a , b , c , and d refer to the paths shown in Fig. 1(a).

The photon in path d that has been reflected by the NPBS passes through the HWP whose principal axes aligned along $\pm 45^\circ$ with respect to the horizontal axis. The common phase shift caused by the HWP can be defined to be the value by which the polarization state $|H\rangle$ changes to $-|V\rangle$ and vice versa. This definition sets the reference point from which the phase of a path qubit is measured. If the coordinate system for polarization measurement is rotated by -45° , the two-photon state after rotation can be written as

$$|\Psi\rangle = 1/2(|D, a\rangle_1 |D, c\rangle_2 + |D, a\rangle_1 |A, d\rangle_2 + |A, b\rangle_1 |A, c\rangle_2 + |A, b\rangle_1 |D, d\rangle_2), \quad (2)$$

where $|D\rangle$ and $|A\rangle$ correspond to the diagonal (45° , $|H\rangle+|V\rangle$) and anti-diagonal (-45° , $|H\rangle-|V\rangle$) polarization states, respectively.

We use the polarization and path states of photon 1 (photon 2) as qubits 1 and 2 (3 and 4), respectively, and encode qubit $|0\rangle$, $|1\rangle$ into polarization H (V), path a (b), and path c (d). By following this encoding rule, the state in Eq. (2) can be expressed as

$$|\Psi\rangle = 1/2(|+0+0\rangle + |+0-1\rangle + |-1-0\rangle + |-1+1\rangle), \quad (3)$$

where $|\pm\rangle$ corresponds to $1/2^{1/2}(|0\rangle \pm |1\rangle)$. The above state is a linear cluster state in standard form, which can be generated by applying controlled-Z operations on the adjacent qubits of a $|+\rangle|+\rangle|+\rangle|+\rangle$ state as shown in Fig. 1(b) [15].

A notable feature of the scheme in Fig. 1(a) is its use of only a single photon-pair source. In contrast, previous schemes based on hyperentanglement use two photon-pair sources having identical spectral/temporal characteristics that are realized with a four-hole structure [10] or a double-pass configuration [5]. Our feature contributes to the design flexibility that facilitates the realization of a larger-scale cluster state based on a set of 2P4Q cluster states.

3. Experiment

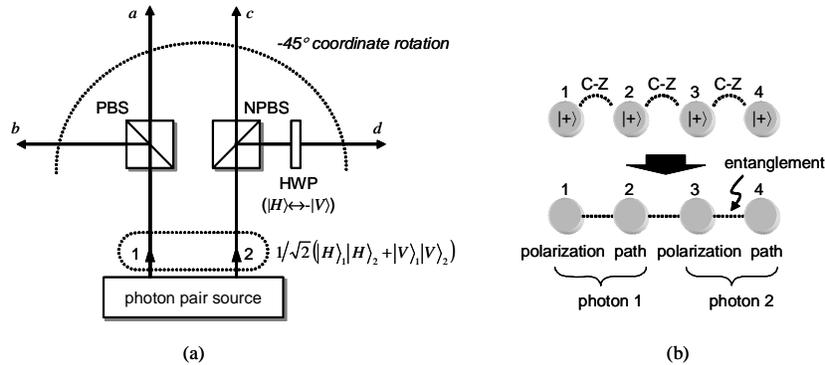


Fig. 1. Schematic of the two-photon four-qubit linear cluster state: (a) generation scheme, (b) configuration of the state. PBS, polarizing beam splitter; NPBS, non-polarizing beam splitter; HWP, half-wave plate; C-Z, controlled-Z operation.

The experimental setup to realize the proposed scheme is shown in Fig. 2. The Bell-state photon pair source is composed of a frequency-doubled mode-locked Ti:sapphire laser (wavelength 390 nm, 45° linear polarization, pulse width 90 fs, repetition rate 93 MHz) as the pump beam for SPDC, a walk-off compensating crystal, and two cascaded BBO crystals (thickness 1 mm each) whose optic axes are aligned perpendicular to each other. A non-collinear type-I SPDC process in the crystals generates a train of polarization-entangled photon pairs that diverge at a half-angle of 3° with respect to the pump beam direction. The walk-off compensating crystal is also made of BBO and compensates for the temporal delay between horizontally-polarized and vertically-polarized photon pairs [16]. The next stage comprises a PBS, an NPBS, and an HWP for generating the cluster state of Eq. (3). A birefringence compensator consisting of two BBO crystals is inserted in path *d* to compensate for the phase difference arising at mirrors and beam splitters between horizontal and vertical polarizations.

The measurement stage is identical for either photon from a photon pair. For each photon, the path qubit is measured by combining the two paths through an interferometer that comprises an NPBS, two PZT-controlled mirrors, and two electronically driven beam shutters. Adjustment of the PZT position and the shutter status projects the path qubit to one of the following: $|0\rangle$, $|1\rangle$, or $|0\rangle + e^{i\phi}|1\rangle$. The photon in the combined path passes through a 10-nm-bandwidth interference filter, an HWP, and a polarizer before being collected by a single-mode fiber (NA 0.12) with an aspheric lens (focal length 15.4 mm). The HWP and the polarizer are used for measuring the polarization qubit. The total distance between the cascaded BBO crystals and the input face of the fiber is 1 m, and the optical path lengths of all the interfering paths are matched within the coherence length of the photons ($\sim 50 \mu\text{m}$).

After the second set of NPBSs in Fig. 2, the cluster state $|\Psi\rangle$ of Eq. (3) has transformed into

$$|\Psi\rangle_{\text{meas}} = 1/2 \left(|D\rangle_1 \left(|D\rangle_2 + e^{i\phi_2} |A\rangle_2 \right) + e^{i\phi_1} |A\rangle_1 \left(|A\rangle_2 + e^{i\phi_2} |D\rangle_2 \right) \right). \quad (4)$$

Here, ϕ_1 (ϕ_2) is the phase difference between the two optical paths in the upper (lower) arm. The values for ϕ_1 and ϕ_2 affect the following count rates: (i) the coincidence count rate when the polarizer for photon 1 is *H*-polarized, the polarizer for photon 2 is *V*-polarized, and shutters 1, 2, 3 are open; (ii) the single count rate of photon counter 2 when the polarizer for

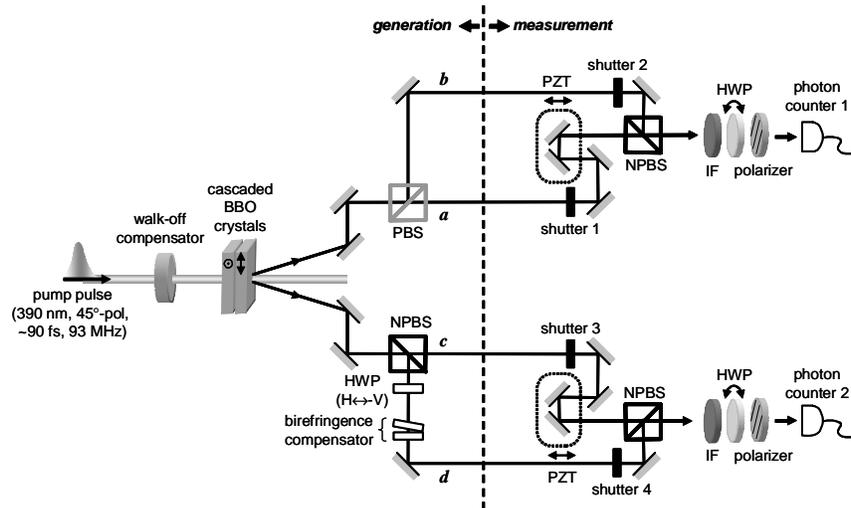


Fig. 2. Experimental setup for generation and measurement of the two-photon four-qubit cluster state. BBO, beta-barium borate; PBS, polarizing beam splitter; NPBS, non-polarizing beam splitter; HWP, half-wave plate; PZT, piezo-electric transducer; IF, interference filter.

photon 2 is V-polarized and shutters 3, 4 are open. For photons prepared in the state $|\Psi\rangle$ given in Eq. (3), the measured count rates ideally should be zero for both (i) and (ii). So the reference value of zero for the phases in path-qubit projections is defined by the following procedure: the fringe visibility for (ii) is maximized by adjusting the birefringence compensator, after which both count rates are minimized by moving the PZT actuators. The resulting settings are taken to correspond to absolute zeros for ϕ_1 and ϕ_2 . Figures 3(a) and 3(b) show typical measurement results for (i) and (ii) as ϕ_1 and ϕ_2 , respectively, are varied. The visibilities are less than unity due to a suspected spectral/temporal mode mismatch between different paths. The data in Fig. 3(b) include a dark count rate of 500 Hz, whereas the accidental coincidence count rate in Fig. 3(a) is negligible.

A cluster state with more than one qubit encoded per photon is useful to the extent that each qubit is addressable. In particular, the individual qubits should be accessible to manipulation and control that result in quantum phenomena. On a basic level, it should be possible to measure the quantum-mechanical correlations of any two qubits. Our cluster state is accessible to measurements of such correlations between the polarization qubits, between a polarization qubit and a path qubit, and between the path qubits, as shown in Fig. 4. The qubits not included in the correlation measurements are projected to either the $|+\rangle$ state or the $|0\rangle$ state to directly entangle the adjacent qubits on either side or to be removed from the cluster state, respectively [17]. The result is an entangled state of the remaining two qubits. For example, during the measurement for Fig. 4(a), qubit 2 is projected to the $|+\rangle$ state by opening shutters 1 and 2 and setting ϕ_1 to zero, and qubit 4 is projected to the $|0\rangle$ state by closing shutter 4, resulting in the entangled state $1/2^{1/2}(|0\rangle_1|0\rangle_3+|1\rangle_1|1\rangle_3)$. The expected quantum-mechanical correlations are clearly seen in the experimental results, which show

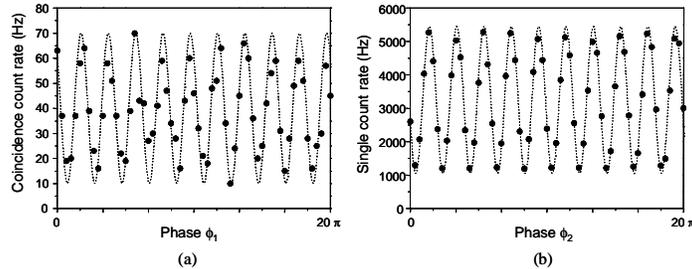


Fig. 3. Photon count rates as functions of the phases in path qubit projections: (a) coincidence count rate vs ϕ_1 , where the polarizers for photons 1 and 2 are H- and V-polarized, respectively, and shutters 1, 2, 3 are open; (b) single count rate of photon counter 2 vs ϕ_2 , where the polarizer for photon 2 is V-polarized and shutters 3, 4 are open.

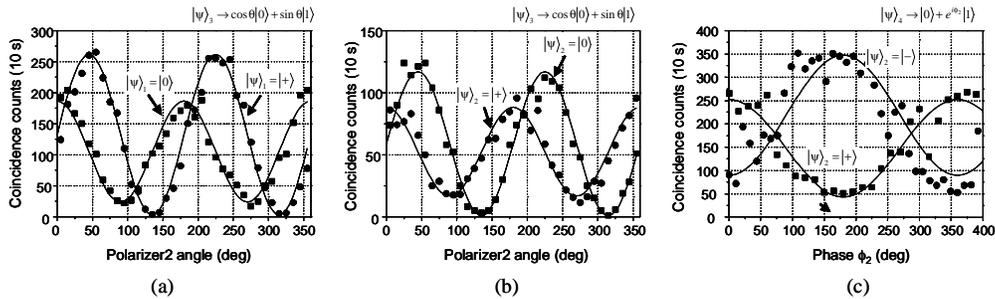


Fig. 4. Correlations between two entangled qubits selected from the cluster state $|\Psi\rangle$: (a) qubits 1 (polarization) and 3 (polarization) entangled as $1/2^{1/2}(|0\rangle_1|0\rangle_3+|1\rangle_1|1\rangle_3)$; (b) qubits 2 (path) and 3 (polarization) entangled as $1/2^{1/2}(|0\rangle_2|+\rangle_3+|1\rangle_2|-\rangle_3)$; (c) qubits 2 (path) and 4 (path) entangled as $1/2^{1/2}(|0\rangle_2|0\rangle_4+|1\rangle_2|1\rangle_4)$. The state for the i -th qubit is denoted as $|\psi\rangle_i$.

and path qubits has been shown to be addressable for measurements of quantum-mechanical correlations between the qubits. Genuine four-qubit entanglement has been verified by evaluating the entanglement witness for the generated state. The fidelity of our experimental cluster state can be further enhanced by incorporating entangled-photon sources with higher efficiency and improving spectral mode matching. An improved version of our cluster state generator is expected to become useful for implementing small-scale quantum algorithms.

Acknowledgments

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