Stabilization of optical vortices in noninstantaneous self-focusing medium by small rotating intensity modulation

Shiuan-Yeh Chen, Tzu-Chun Lo, and Ming-Feng Shih

Department of Physics, National Taiwan University, Taipei, 106, Taiwan
mfshih@phys.ntu.edu.tw

Abstract: We observe the stabilization of a single-(double-) charge optical vortex propagating in a self-focusing medium. The optical vortex, which carries a phase singularity at its center, usually breaks up in a self-focusing medium due to the so-called azimuthal instability. However, by adding a small rotating azimuthally-periodic intensity modulation on the vortex light beam, which propagates in a noninstantaneous self-focusing medium, we successfully suppress the azimuthal instability. This observation is confirmed by both numerical simulation and perturbational analysis.

OCIS codes: (190.3100) Instability and chaos, (190.4420) Transverse effect.

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1. Introduction

Vortices, which are well-known phenomena associated with phase dislocations, appear in many branches of physics, from fluid mechanics to Bose-Einstein condensates [1]. In optics, when a broad light beam carrying a vortex propagates in a self-defocusing medium, the vortex core can be self-trapped to form an optical vortex soliton [2]. Such vortex solitons have been observed in different types of defocusing media with either coherent or partially incoherent light [3-7]. In contrast, in a self-focusing medium, a vortex light beam is highly unstable and can decay into several spots due to the so-called azimuthal instability (AI) [8-10]. This has been observed in saturable nonlinearity by atomic absorption [10], biased photorefractive crystals [6], and quadratic nonlinear media [11]. Even recently, more experiments continue to demonstrate the breakup of the optical vortices in the self-focusing medium [12]. Similar effect has also been predicted to happen in attractive Bose-Einstein condensates [13]. Although there was theoretical prediction of the possible existence of stable vortex solitons in the competing cubic-quintic nonlinear medium [14-17] or with nonlocal nonlinearity [18], until now, the only observed stabilization of optical vortices in a self-focusing medium is when the light is partially incoherent enough [19]. The mechanism is similar to that of the suppression of the modulation instability by partially incoherent light [20].

Long before the study of the AI in the self-focusing medium, a very similar phenomenon, modulation instability (MI) [2] had been observed and studied for many years. The AI occurs along the azimuthal direction of a vortex light beam, while the pattern due to the MI can orient in any transverse direction of a broad light beam. The formation of MI can be intuitively understood as following: When a light beam carrying noise propagates in a self-focusing medium, it yields slightly higher refractive index in regions with slightly higher intensity. As the light propagates, the higher index region attracts more light nearby and yields even higher index that attracts more light. As a result, the light intensity will be localized in some regions, and the MI pattern emerges. In a previous study [21], it is shown that if the noise varies too fast for the noninstantaneous self-focusing medium to respond, the MI can be arrested completely. Therefore it seems natural that similar suppression will also happen to the AI. In this letter, we will demonstrate the observation of the suppression of AI by adding a small rotating azimuthally-periodic intensity modulation on a finite-sized single- or double-charge vortex light beam propagating in a noninstantaneous self-focusing medium. We will also show the results of the simulation and the perturbational analysis confirming the observation.

2. Experimental observation

One of the perfect candidates for the noninstantaneous self-focusing medium is the biased photorefractive material, whose nonlinear mechanism includes time-consuming re-distribution

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of the charge carriers [22]. We conduct the experiment (Fig. 1) with a biased photorefractive SBN crystal (a x b x c = 5 x 10 x 5mm^3). A collimated cw laser light beam (at 488 nm) of extraordinary polarization is passed through a computer-generated hologram to imprint a single-charge vortex phase. We then focus the vortex light beam at the input face of the SBN crystal and let it propagate along the crystalline a-axis. The power of the light beam is a few micro-watts. The crystal is illuminated by a background light from the top side to make the nonlinearity in the saturation region as that for generating photorefractive solitons. Under this illumination, the response time of this photorefractive material is a few seconds. Then, a lens is used to project the images at the input or output face of the crystal onto a CCD camera.

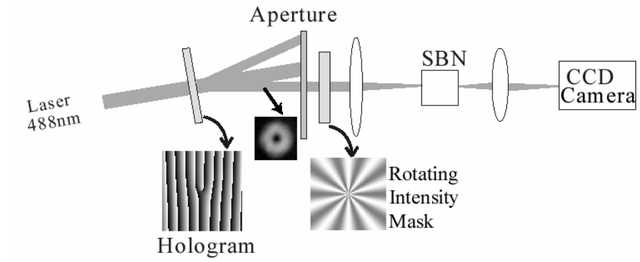

Fig. 1. The experimental setup.

We insert a intensity mask [Fig. 4(a)] in front of the focusing lens. The mask has a intensity transmission $0.95+0.05\cos(M\theta)$, where θ is the azimuthal angle. Several integers M larger than eight are used. Since the light beam propagates linearly in the free space before it enters the crystal, the modulation along the vortex ring is much weaker and cannot be seen at the input face of the crystal [Fig. 2(a)] due to the diffraction. We reproduce the experiment that the coherent vortex light beam cannot stably propagate in the self-focusing medium. With zero biasing voltage, Fig. 2(b) shows the diffraction of the vortex light beam at the output face of the crystal. While a 700-volt biasing voltage is applied on the photorefractive crystal creating a self-focusing index change about 2.4×10^{-4}, the vortex light beam breaks up into two pieces, shown in Fig. 2(c) at the output face of the crystal.

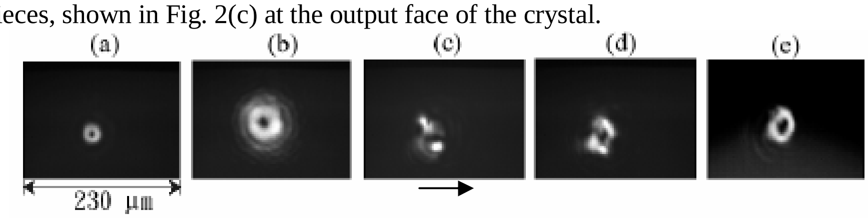

Fig. 2. Single-charge optical vortex: (a) at the input face; (b) at the output face without nonlinearity; at the output face with the nonlinearity turned on when the mask is (c) not rotating, (d) rotating at 0.05 turn per second, and (e) rotating at 5 turns per second. The arrow indicates the direction of the biasing field.

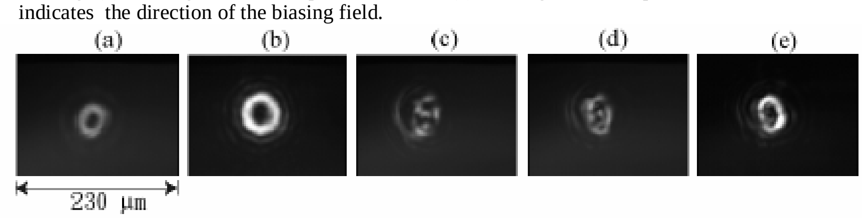

Fig. 3. Double-charge optical vortex: (a) at the input face; (b) at the output face without nonlinearity; at the output face with the nonlinearity when the modulation mask is (c) not rotating, (d) rotating at 0.05 turn per second, and (e) rotating at 5 turns per second.

We then let the mask rotate about its center. When the intensity mask is rotating at 0.05 turn per second, the azimuthal instability [Fig. 2(d)] is suppressed a bit. We then increase the

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rotation speed. When the rotation speed is 5 turns per second, the AI is suppressed and we observe the stabilized self-trapped vortex light beam in the self-focusing medium [Fig. 2(e)].

We continue our experiment for the double-charge vortex light beam. When the voltage is applied without the intensity modulation mask rotating, the double-charge vortex light beam breaks up into many pieces [Fig. 3(c)]. As we rotate the mask, the double-charge vortex light beam becomes more stable [Fig. 3(d)-(e)] at higher rotating speed of the mask.

3. Numerical simulation

To confirm and understand the stabilization mechanism, we approach via both the simulation based on beam propagation method (BPM) and perturbational analysis. We start from the wave equation $\nabla^2 E - \mu\partial^2 D/\partial t^2 = 0$ with $D = \varepsilon(|E|^2)E$ as the displacement. This equation is rewritten as $\nabla^2 E = \mu\cdot\partial^2(\varepsilon E)/\partial t^2 = \mu\varepsilon\cdot\partial^2 E/\partial t^2 + 2\mu(\partial\varepsilon/\partial t)(\partial E/\partial t) + \mu(\partial^2\varepsilon/\partial t^2)E$. Assuming $\varepsilon = \varepsilon_0(n_0+\delta n)^2$ and the nonlinearity relaxes with time constant τ, we have $\delta n(|E(t)|^2) = (1/\tau)\cdot\int_{-\infty}^{t} F[|E(t')|^2]\mathrm{e}^{-(t-t')/\tau}dt'$, where $F[I(t')] = n_2 I(t')/(1+\alpha I(t'))$ is the form of the nonlinearity and α defines the degree of saturation. For τ much larger than the optical period about 10^{-15} s [23], we neglect the last two terms of $\nabla^2 E$. Putting $E = A(\vec{r},t)\exp(i\omega t - ikz)$, and with the paraxial approximation, we obtain

$$\nabla_\perp^2 A - 2ik\partial A/\partial z + (2k^2/n_0)\delta n(|A|^2)A = 0. \quad (1)$$

We treat Eq. (1) in cylindrical coordinates and let $A = A_0 u_\ell(r)\exp(i\ell\theta - i\gamma z) = A_0 r^\ell U_\ell(r)\exp(i\ell\theta - i\gamma z)$. Here, $u_\ell(r)$ is the radial amplitude profile corresponding to the single-charge (ℓ=1) or double-charge (ℓ=2) vortex light beam. We then find the eigen-solution of Eq. (1) by numerical shooting method, which is to guess an eigen value γ, then to see if the profile satisfies the boundary conditions with $U_\ell'(0) = U_\ell'(\infty) = 0$, and $U_\ell(\infty) = 0$. Once the eigen-solution is found, we multiply $A(r,\theta,0)$, which is the field at the input face, by $0.97 + 0.03\cos(M\theta + 2\pi ft)$, and transform it back to the rectangular coordinates for the convenience in the BPM simulation. *f* is the rotating frequency of the intensity mask. Since the refractive index and the input wave are changing with time, we therefore need to iterate the BPM to trace the wave and the index change in the medium with respective to time. We follow Ref. 24 to handle the index change $\delta n(t) = (1/\tau)\cdot\int_{-\infty}^{t} F[I(t')]e^{-(t-t')/\tau}dt' \approx e^{-t/\tau}\cdot[e^{\Delta t/\tau}-1]\cdot\sum_{n=0}^{t/\Delta t-1}\{F[I(n\Delta t)]e^{n\Delta t/\tau}\}$, where the continuous time t' is replaced by the finite time step $n\Delta t$, with $\Delta t << \tau$. We can change the summation into the recursive form such that $\delta n(N) = F[I(N)](1-e^{-\Delta t/\tau}) + e^{-\Delta t/\tau}\delta n(N-1)$ for *N-th* iteration of the BPM and with initial value $\delta n(0) = 0$ by assuming the light beam is turned on at *t*=0. In the simulation, which has 256×256 grid points and propagation length 10 mm, we use a time step Δt=0.01τ. In every time step, the wave from the input face to the output face is obtained by the BPM with the concurrently updated index change δn. The BPM is iterated for at least 6000 time steps to insure that the profile of the light beam at the output face can reach steady state.

We first set *f=0* and ℓ=1. The vortex light beam [at the input face, shown in Fig. 4(b)] breaks up into two spots flying apart at 10 mm propagation[Fig. 4(c)] due to the AI, very similar to that observed in the experiment. When we increase *f* to 0.015/τ, the breakup is reduced greatly, shown in Fig. 4(d). When *f* is larger than 0.15/τ, the stabilized self-trapping of the vortex light beam is observed [Fig. 4(e)]. The phase of the beam is from 0 to 2π [Fig. 4(f)] along the ring, saying the vorticity of the light beam is perfectly reserved. Also as

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observed in the experiment [Figs. 2(c)-2(d)], with the increasing frequency *f*, the alignment of the flying-away spots [Figs. 4(c)-4(d)] rotates to a different direction. This simulation confirms our experiment that small rotating intensity modulation can stabilize the vortex light beam propagating in a noninstantaneous self-focusing medium. Similar simulation results are also obtained for the double-charge vortex light beam.

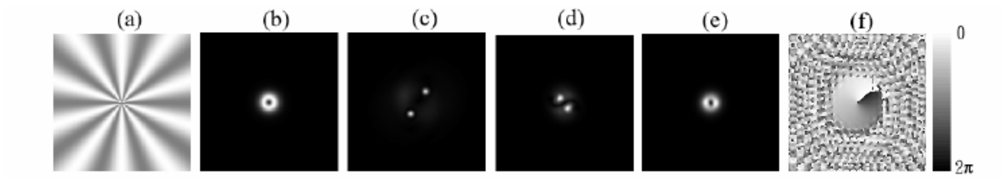

Fig. 4. (a). The intensity mask. Intensity of the single-charge optical vortex in the computer simulation: (b) at the input face; at the output face when (c) $f=0$, (d) $f=0.015/\tau$, and (e) $f=0.15/\tau$, at which frequency the phase at the output face is shown in (f).

4. Perturbational analysis

To further understand this stabilization by small rotating modulation, we follow Ref. 9 for the perturbational analysis. The analysis is rather lengthy; we therefore only describe how the analysis is done and its key results. We add the radial profile $u_\ell(r)$ of the eigen-solution for Eq. (1) by a perturbation $\varepsilon(z,r,\theta,t)=\xi_J^+(r)e^{i(\lambda z+J\theta+\Omega t)}+\xi_J^-(r)e^{-i(\lambda^* z+J\theta+\Omega t)}$, where $\xi_J^+(r)$ and $\xi_J^-(r)$ are complex valued, J is the azimuthal spatial frequency or the vorticity of the perturbation, Ω the temporal frequency of the rotating perturbation, and the imaginary part of λ is the most important quantity associated with the growth rate of the perturbation. By putting the perturbed field $A_0[u_\ell(r)+\varepsilon(z,r,\theta,t)]e^{i(\ell\theta-\gamma z)}$ into Eq. (1), we obtain

$$0=\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\varepsilon e^{i(\ell\theta-\gamma z)}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}(\varepsilon e^{i(\ell\theta-\gamma z)})+f(|u|^2)\varepsilon e^{i(\ell\theta-\gamma z)}-2ik\left[\frac{\partial\varepsilon}{\partial z}-i\gamma\varepsilon\right]e^{i(\ell\theta-\gamma z)} . \quad (2)$$

$f(|u|^2)\approx\frac{n_2|u_\ell|^2}{1+\alpha|u_\ell|^2}\left[1+\frac{1}{u_\ell(1+\alpha|u_\ell|^2)}\left(\frac{\xi_J^++\xi_J^{-*}}{1+i\Omega\tau}e^{i(\lambda z+J\theta+\Omega t)}+\frac{\xi_J^{+*}+\xi_J^-}{1-i\Omega\tau}e^{-i(\lambda^* z+J\theta+\Omega t)}\right)\right]$ is the unfolded nonlinearity in which the integration contained in δn has been carried out. Since Eq. (2) holds for all values of θ, z, and t, it renders $-2k\lambda\begin{bmatrix}\xi_J^+\\ \xi_J^{-*}\end{bmatrix}=\begin{bmatrix}C_+ & D\\ -D & -C_-\end{bmatrix}\begin{bmatrix}\xi_J^+\\ \xi_J^{-*}\end{bmatrix}$, with $C_\pm=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{(\ell\pm J)^2}{r^2}+\frac{n_2|u_\ell|^2}{(1+\alpha|u_\ell|^2)^2}\left[\frac{1}{1+i\Omega\tau}+(1+\alpha|u_\ell|^2)\right]-\gamma$, and $D=\frac{n_2|u_\ell|^2}{(1+\alpha|u_\ell|^2)^2}\frac{1}{1+i\Omega\tau}$. This can be solved numerically by replacing the differential operator by difference operator, and the radial coordinate *r* by *jR*, where *j* is an integer and *R* is the sampling distance. We first put $\alpha=0$, and $\Omega\tau=0$ for the instantaneous Kerr nonlinearity in both single-($\ell=1$) and double-($\ell=2$) charge cases. The obtained growth rates, $\mathrm{Im}(\lambda)$, are exactly the same as those from Ref. 9. We then put different values for $\Omega\tau$. The results are shown in Fig. 5. When $\Omega\tau=0$, meaning in the instantaneous Kerr self-focusing nonlinearity, the growth rate [Fig. 5(a)] is the largest for the perturbation with $J=2$ vorticity for the single-charge ($\ell=1$) vortex. This is exactly what we observe in the experiment and in the computer simulation that a single-charge vortex light beam will break up into two. As we increase the rotating frequency Ω of the perturbation and assume a constant relaxation time

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τ , we can see that when the value of $\Omega\tau$ is large enough, the growth rates are greatly reduced to be near zero for all vorticities *J*, i.e., this fast rotation of the perturbation can suppress the AI. Similar result is also shown for double charged (ℓ =2) vortex in Fig. 5(b). When $\Omega\tau$ =0, the vortex should break up into many spots since *J*=4 and *J*=5 have the largest and close growth rates. As $\Omega\tau$ increases to a large value, the growth rates for all vorticities *J* again fall to near zero, meaning the azimuthal instability is suppressed with the rotation. We also try for several different saturation degrees of α and obtain similar results. More details about the perturbational analysis and its comparison with the simulation will be published elsewhere.

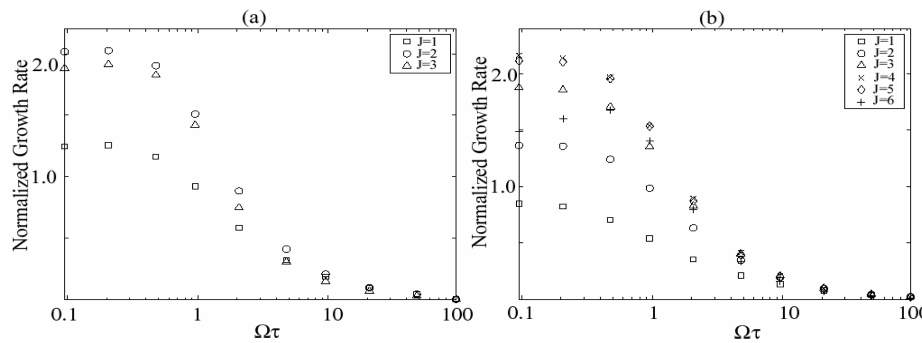

Fig. 5. The growth rates of the perturbation with different vorticity *J* for (a) the single-charge, and (b) the double-charge optical vortex in the noninstantaneous Kerr self-focusing medium.

5. Discussion and conclusion

First, though the photorefractive nonlinearity is used in the experiment, many other mechanisms also provide noninstantaneous nonlinearity [23]. It is therefore possible to observe the stabilization of the optical vortex in these self-focusing media. Second, the simulation shows that the optical vortex can keep its size as it propagates in the medium. However, this is not observed in the experiment. The possible reason is that the Laguerre-Gaussian light beam rather than the eigen-solution is launched at the input. Third, this rotation can suppress the AI but cannot suppress the instability of a two-dimensional light beam propagating in a Kerr-type self-focusing medium [25]. In the simulation, when added 5% more total energy, the vortex light beam first collapses, similar to the catastrophic self-focusing, and then breaks up into many pieces. If the total energy is 5% less, it simply does not maintain its size and keeps diffracting. Fourth, in the single-charge vortex experiment, the two split spots are always from the top and bottom parts of the original vortex ring. This is different from the simulation result, in which the split spots can come from any pair of opposite sides of the vortex ring. The main reason is that in the simulation the nonlinearity is local that it preserves the azimuthal symmetry. However, in the experiment the screening photorefractive nonlinearity is nonlocal and this generates, at the sides of the light beam along the direction of the applied bias, two antiguiding lobes [26]. The antiguiding lobes then prevent the split spots to be initiated there. Fifth, a new vortex entity called the azimuthon [27] is predicted to exist stably in the self-focusing medium. It contains a stair-type vortex phase core and several near-steady intensity peaks and valleys along the ring. This is different from our case here where the intensity modulation along the vortex ring is a rotating perturbation.

To conclude, we observe experimentally the stabilization of single-(double-) charged vortex light beam. This is done by adding a small rotating azimuthally-periodic intensity modulation on the optical vortex. We also confirm this by a computer simulation. With the help of the perturbational analysis, we understand this stabilization is due to the reduction to the growth rates of the perturbation of all vorticities by the rotation. This research is supported by National Science Council of Taiwan.

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