

Correlations of polarization in random electromagnetic fields

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Abstract: Random electromagnetic fields have a number of distinctive statistical properties that may depend on their origin. We show here that when two mutually coherent fields are overlapped, the individual characteristics are not completely lost. In particular, we demonstrate that if assumptions can be made regarding the coherence properties of one of the fields, both the relative average strength and the field correlation length of the second one can be retrieved using higher-order polarization properties of the combined field.

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1. Introduction

Random electromagnetic fields (REF) occur in a number of situations, usually manifesting themselves as the familiar intensity fluctuations known as speckles [1–5]. Their properties can be described as the coherent superposition of a large number of random phasors with uniformly distributed phases [1,6]. As long as their amplitudes are Rayleigh distributed, the resulting distribution of intensity follows a negative exponential and the optical contrast is unity. This is the so-called fully developed speckle. The spatial correlation length of the intensity variations in such a REF, i.e the average size of a speckle, depends on the size of the source and wavelength.

In practice, one sometimes encounters situations where two or more independent REFs overlap. When their superposition is incoherent, the REFs add in intensity and the outcome is the suppression of intensity fluctuations and an overall reduction in optical contrast [1]. Independent REFs can be generated as a result of illumination at different angles, with different wavelengths, or even with orthogonal polarization components [7–9]. The addition of multiple independent REFs is a common method to reduce or eliminate unwanted speckles in a number of imaging applications [10].

A more interesting situation is that of coherent superposition of REFs which have different characteristics [11–13]. In general, when two REFs add in amplitude, there is no reduction of intensity contrast, the speckles remain fully developed and the probability density of intensity distribution continues to follow a negative exponential. In this situation, when the overlapping fields are completely unknown, the information about an individual REF cannot be recovered. However, we will show here that if prior knowledge about one of the fields is available, some information about the other one can still be retrieved. In particular, we will demonstrate that if assumptions can be made regarding the field correlation length of one of the overlapping REFs, the relative average strength and the field correlation length of the second REF can be retrieved using the polarization properties of the combined field.

REFs can have different physical origins. For instance, depending on the type of interaction, there are different kinds of REFs that result from the coherent light scattering by randomly inhomogeneous media. Such fields are fully polarized locally but their global properties can vary from being fully polarized (same polarization state across the entire field) to being completely unpolarized (a uniform distribution of all possible polarization states). For instance, a strong multiply scattering medium will generate scattering fields that are globally unpolarized fields while uniformly polarized random fields can be the result of single scattering, surface scattering, ballistic propagation through the medium or combinations of those. Of course, in addition to different polarization properties, these fields can also differ in their spatial coherence properties, and this diversity may be used to trace their actual physical origin. The present paper provides means to determine the correlation length of such single scattering components in the presence of multiple scattering.

2. Coherent superposition of random electromagnetic fields

In the following we will describe the superposition of two locally polarized fields: one field being globally unpolarized $\mathbf{U}(\mathbf{r}) = E^U(r)\hat{\epsilon}(r)$ and the other one characterized by a uniform polarization state, $\mathcal{P}(\mathbf{r}) = E^P(r)\hat{\epsilon}_0$. If the fields are quasi-monochromatic and also mutually coherent, their addition leads to a new REF that is $E^R(r) = E^U(r)\hat{\epsilon}(r) + E^P(r)\hat{\epsilon}_0$. As mentioned before, a globally unpolarized field can be regarded as the superposition of two uncorrelated components that are orthogonally polarized, one along x and the other one along y , for instance. This is an example of adding two independent fields in intensity, which reduces the optical contrast. The addition of more independent fields further reduces this contrast. However, when a fully polarized and coherent REF is added to the field \mathbf{U} , the optical contrast of $E^R(r)$ actually increases because the uniformly polarized component increases the magnitude of the field amplitude along a certain direction, thus biasing the

overall polarization of $E^R(r)$. Not only does the addition of this coherent field increase the contrast, but it also increases the overall degree of polarization creating a partially polarized REF. This is similar to combining a completely unpolarized beam with a fully polarized one to create any partially polarized beam [14, 15].

The globally unpolarized field (\mathcal{U}) can be modeled as a REF where the complex amplitude components E_x and E_y are both circular Gaussian random functions. Each of these components can be represented as a sum of plane waves

$$E_\mu^U(\mathbf{r}) = \sum_j a_j \exp(i(k_j \cdot \mathbf{r} + \phi_{\mu,j})), \mu = x, y, \quad (1)$$

where a_j is an amplitude, k_j are transverse wavenumbers, \mathbf{r} is a position vector, and $\phi_{\mu,j}$ are uniformly random phases. When the field in Eq. (1) is added to a field uniformly polarized along x ,

$$E_x^P(\mathbf{r}) = \sum_j b_j \exp(i(k_j \cdot \mathbf{r} + \phi_{x,j})), \quad (2)$$

where b_j is a different set of amplitudes, there are several ways to characterize the properties of the resultant REF. One simple global measure is to compare the average intensity of the field \mathcal{P} with the average intensity in the total field $E^R(r)$. In our practical example, this ratio $\beta = \langle |E^P(r)|^2 \rangle / \langle |E^R(r)|^2 \rangle$ would indicate the strength of the scattering regime. For instance, for $\beta > 0.5$, the REF would favor the linearly polarized component. We note that this ratio of intensities relates to the global degree of polarization, \bar{P} , of the final REF, which is an ensemble quantity defined as [16]

$$\bar{P}(r) = \sqrt{\langle S_1(r) \rangle^2 + \langle S_2(r) \rangle^2 + \langle S_3(r) \rangle^2} / \langle I(r) \rangle, \quad (3)$$

where

$$\begin{aligned} S_1(r) &= E_x^{R*}(r)E_x^R(r) - E_y^{R*}(r)E_y^R(r), \\ S_2(r) &= E_x^{R*}(r)E_y^R(r) - E_y^{R*}(r)E_x^R(r), \\ S_3(r) &= i(E_x^{R*}(r)E_y^R(r) - E_y^{R*}(r)E_x^R(r)). \end{aligned} \quad (4)$$

As can be seen, the strength of the linearly polarized component determines the partial polarization of the resulting field. As β continues to increase, \bar{P} increases as well until unity saturation is reached.

Another characteristic of the resultant field is the extent of its field-field correlations. In addition to having different overall magnitudes and polarization characteristics, random fields may also have different field correlation lengths. In other words, the speckle sizes of the fields \mathcal{U} and \mathcal{P} can be different. The short-range correlation length for \mathcal{U} can be defined as

$$\langle E^U(r)E^U(r+\delta) \rangle_r = f(\delta^U), \quad (5)$$

and it has the same value for both x and y field components. The unpolarized field can be caused by any number of strongly scattering media but, for the purpose of this paper, it is assumed that the unpolarized field is examined near its source and, therefore, the field correlation length is of the order of a wavelength [17,18]. The field correlation for the linearly polarized component,

$$\langle E_x^P(r)E_x^P(r+\delta) \rangle_r = f(\delta^P), \quad (6)$$

is of course only along x . This can result from the scattering from a rough surface, ballistic scattering, and other types of scattering that conserve the state of polarization [1,19]. Along with the value of β , these two correlation lengths directly influence the length scales of the resulting REF.

As mentioned before, β , \bar{P} , and the f factors in Eqs. (5) and (6) are all global properties, evaluated as ensemble averages. While \bar{P} is indicative of the overlap between the fields \mathcal{U} and \mathcal{P} , its value does not take into account the field correlation in the resulting REF. The correlation length on the other hand is a structural characteristic evaluated using a two-point property. As the resulting REF has different levels of partial polarization depending on the strength of \mathcal{P} , the polarization structure of the final REF is important to consider.

In the past, we demonstrated that a two-point polarization similarity measure such as the complex degree of mutual polarization (CDMP) can conveniently describe the spatial structure of polarization in a REF without requiring an ensemble average [20]. In general, the CDMP factor measures the similarity between two polarization states [21] and it ranges from zero when the two states are orthogonal to unity when the polarization states are identical. For the purpose of the present analysis, the CDMP is defined such that it measures the correspondence between the state of polarization at position r and a chosen reference polarization state:

$$|V^2(r)| = \frac{(E_x^{R*}(r)E_x^P + E_y^{R*}(r)E_y^P)^2}{(|E_x^R(r)|^2 + |E_y^R(r)|^2)(|E_x^P|^2 + |E_y^P|^2)}. \quad (7)$$

In Eq. (7), the reference is the polarization state of the field \mathcal{P} . Using this definition, one can generate a two-dimensional CDMP map corresponding to this specific state of reference polarization as can be seen in Fig. 1. This two-dimensional graphical representation of polarization “speckle” is characterized by spatial features with different sizes and spatial frequencies. Similarly to conventional intensity speckles, it is expected that these features will depend on the properties of \mathcal{U} and \mathcal{P} . Of course, since the field \mathcal{P} is in the same polarization state of the reference, its CDMP map would have a uniform value of one; the CDMP map of \mathcal{U} on the other hand should be correlated over distances on the order of δ^U .

As a means to assess the spatial frequencies in these polarization maps, one can examine the power spectral density (PSD) defined as

$$P(\omega) = F\{|V^2(r)| * |V^2(r)|\}, \quad (8)$$

where $|V^2(r)| * |V^2(r)|$ represents the autocorrelation of a CDMP map. Because the analyzed REF is the superposition of two other fields that are mutually coherent, Eq. (8) can be further written as [12]

$$P(\omega) = \bar{I}_1^2 p_1(\omega) + \bar{I}_2^2 p_2(\omega) + \bar{I}_1 \bar{I}_2 p_{12}(\omega), \quad (9)$$

where

$$p_j(\omega) = F\{a_j^* a_j * a_j^* a_j\}, \quad j=1, 2$$

$$p_{12}(\omega) = F\{2a_1^* a_1 * a_2^* a_2 + (a_1^* a_2 + a_2^* a_1) * (a_1^* a_2 + a_2^* a_1)\}, \quad (10)$$

represent the power spectral densities of the individual components and the mixed (interference) term, respectively. In Eq. (10), a_1 and a_2 denote the individual, normalized x

field components $E^P(r)/S_0(r)$ and $E^U(r)/S_0(r)$, respectively. The properties of the spatial distribution of polarization states across a REF relate to the power spectrum of the CDMP map in Eq. (9), which, in our case, depends on the specific values of β and δ^P . Of course, the information content of this power spectrum in Eq. (9) is richer than that provided by the value of \bar{P} , which is only a global average of point-like properties.

In general, any REF can be decomposed into a globally unpolarized and a uniformly polarized component. These two components have a relative strength β and are also characterized by their, possibly different, coherence lengths δ^U and δ^P . These characteristics influence the global properties of REF in different ways. For instance, the global degree of polarization \bar{P} of the final REF depends only on the ratio β but is not influenced at all by δ^U or δ^P . The spatial properties of polarization on the other hand are determined by all these factors as can be seen in the power spectrum of the CDMP map in Eqs. (9) and (10). Because (i) the global degree of polarization \bar{P} can be determined independently and (ii) the coherence length δ^U is known to be of the order of the wavelength, one can use the power spectrum of the CDMP map to determine the unknown correlation length δ^P of the polarized field component. In the following we will illustrate this procedure using systematic numerical simulations.

3. Numerical simulations of overlapping REF

To illustrate some of the field properties resulting from the superposition of coherent REFs, a simple numerical simulation was performed. Using the plane wave decomposition in Eq. (1), plane waves originating from a circular array of source points with random phases were mapped onto an observation plane of 250 by 250 pixels. This creates a Gaussian random field originating from a beam with radius r , with a coherence length $\delta^{coh} = 3.83/(\kappa r)$ [1]. When $\kappa = 1$ and $r = 0.3$, the coherence length δ^U of the globally unpolarized field was set to be equal about 12 pixels in the observation plane. The uniformly polarized field was created in a similar manner using Eq. (2) in only one linear state of polarization. In addition, the spatial correlation length of this polarized field (δ^P) was controlled by adjusting the parameter r to produce different values that are larger than δ^U . These two random fields are then superposed coherently and the resulting intensity patterns are shown in Fig. 1 for the case where the coherence length of the polarized field is four times larger than the unpolarized component, i.e. $\delta^P = 4\delta^U$. In this example, the intensity patterns in Figs. 1(a) and (b) are characterized by a ratio $\beta = 0.15$, which corresponds to a global degree of polarization $\bar{P} = 0.11$.

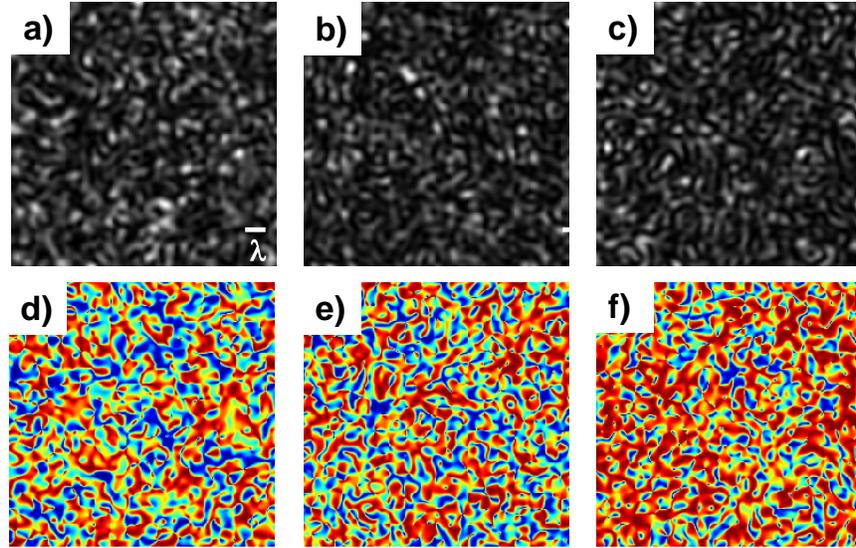


Fig. 1. Intensity speckle images of the superposition between an unpolarized field of coherence length δ^U and a polarized field characterized by: a) $\beta = 0.15$, $\delta^P = 4\delta^U$ b) $\beta = 0.15$, $\delta^P = \delta^U$ c) $\beta = 0.45$, $\delta^P = \delta^U$ and the corresponding CDMP maps for: d) $\beta = 0.15$, $\delta^P = 4\delta^U$ e) $\beta = 0.15$, $\delta^P = \delta^U$ and f) $\beta = 0.45$, $\delta^P = \delta^U$. Areas of blue and red correspond to CDMP values of 0 and 1, respectively. The values of $\beta = 0.15$ and $\beta = 0.45$ correspond to global degrees of polarization $\bar{P} = 0.11$ and $\bar{P} = 0.31$, respectively.

At such a low intensity ratio, adding an additional linearly polarized field has little impact and the resulting REF is almost globally unpolarized. As can be seen, even when the correlation length δ^P is four times larger than δ^U , there is practically very little change in the size of the final intensity speckles. However, when observing the CDMP maps, one can easily notice changes in the statistical nature of their structure. Even though the two REFs in Fig. 1(a) and 1(b) have the same global degree of polarization, there is a clear difference in the spatial frequency content of the corresponding CDMP maps as seen by the larger groupings of high CDMP values in Fig. 1(d).

The third speckle pattern in Fig. 1(c) corresponds to the situation where δ^P is equal to δ^U but the field \mathcal{P} now has a greater amplitude, i.e. the ratio $\beta = 0.45$ and, correspondingly, $\bar{P} = 0.31$. As can be seen, the spatial frequency content in the CDMP map of Fig. 1(f) is similar to the one in Fig. 1(e) but now with a higher prominence of locations where $|V^2(r)| = 1$.

To get a quantitative description on how the correlation length of the field \mathcal{P} affects the spatial distribution of polarization in the resulting REF, we have calculated the power spectral density of the CDMP maps resulting from the numerical procedure. An example is illustrated in Fig. 2 for three cases corresponding to fields \mathcal{P} having different correlation lengths and the same $\beta = 0.45$.

It is clearly seen that the characteristic shape of the curves in Fig. 2 appears to be composed of three different contributions. This is also described by Eq. (9) where the power spectral density contains three main terms that can be approximated by zero-mean Gaussians

with different widths. The widths of these Gaussians are representative of the correlation lengths of the fields \mathcal{P} and \mathcal{U} while their magnitudes depend on the relative strengths of the fields (β). We have also fitted the power spectral densities to the formulation in Eqs. (9) and (10) using the magnitudes I_1 and I_2 and the three Gaussian widths as fitting parameters. The results are included with continuous lines in Fig. 2. The first two terms correspond to the power spectral densities of the individual fields \mathcal{P} and \mathcal{U} . Since the CDMP maps for the individual fields do not change with β and δ^P , the widths of the first two Gaussians also remain unchanged.

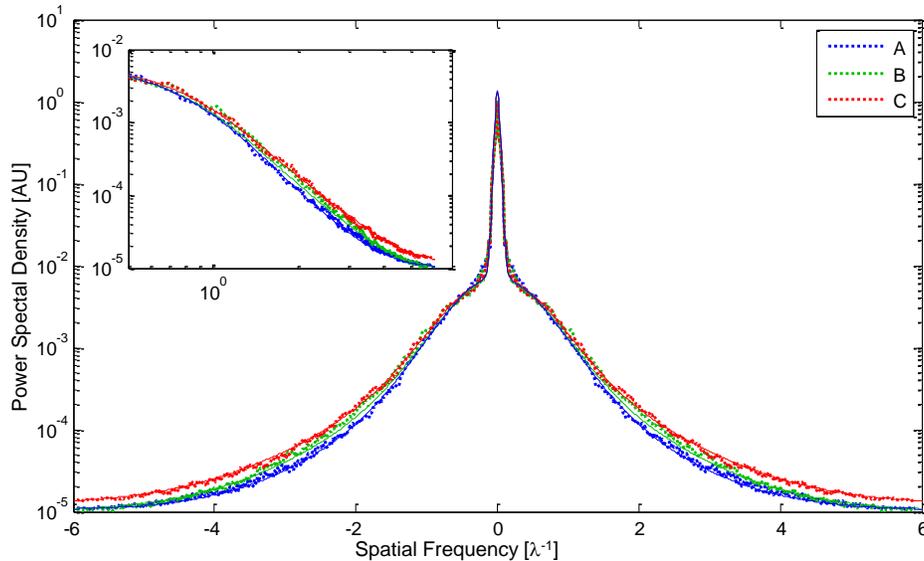


Fig. 2. The power spectral density of CDMP maps calculated for $\beta = 0.45$ and correlation lengths δ^P equal to A) $2\delta^U$, B) $4/3\delta^U$, and C) δ^U . Also shown with solid lines are the best fits with power spectrum dependence given in Eq. (9). The inset shows a log-log plot of the high spatial frequencies region.

In the specific case analyzed here, the first term, which is basically the PSD of the CDMP map with uniform unity value, has a small Gaussian width of 0.06 in our normalized units (the narrow central peak in Fig. 2). The second term represents the PSD corresponding to the unpolarized component and has a constant width of 0.9 due to the fixed correlation length δ^U . The third term in Eq. (9) describes the interference between the fields \mathcal{P} and \mathcal{U} with most of its contributions occurring in the high spatial frequency range. In the example presented in Fig. 2, only the width of this interference term and the magnitudes of the Gaussians depend on the characteristics of the interfering fields. However, because β is constant, all the magnitudes remain unchanged and only the width of the third component changes as δ^P varies. The contribution of this third term lies mostly in the high frequencies and can be fitted well by a Gaussian function with widths of 3.6, 4, and 5.3 for the PSD labeled A, B, and C, respectively. As can be seen, as the correlation length of \mathcal{P} decreases, the PSD width increases indicating that smaller spatial polarization features appear due to the interference between \mathcal{P} and \mathcal{U} .

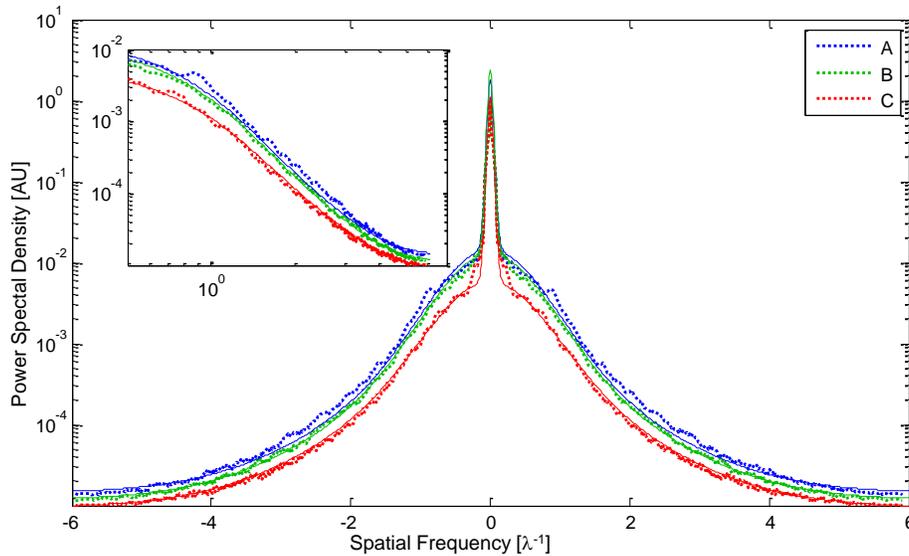


Fig. 3. The power spectral density of CDMP maps calculated for $\delta^P = 4\delta^U$ and field ratios β equal to A) 0.04, B) 0.19, and C) 0.45. Also shown with solid lines are the best fits with power spectrum dependence given in Eq. (9). The inset shows a log-log plot of the high spatial frequencies region.

A different example is illustrated in Fig. 3. Here the correlation length of \mathcal{P} is kept fixed and is four times larger than the field correlation length of \mathcal{U} but the relative strength β is varied. This corresponds to a gradual progression of different polarization regimes. Again, the most interesting features lie in the high spatial frequencies. When fitting the results of the simulation, only the magnitudes of the Gaussians are altered since now the underlying field correlations of the different components are unchanged. As a result, the curves are almost parallel to each other in the high spatial frequency range, as can be clearly seen in the inset. One can also note that, at low β , the influence of the correlation length of field \mathcal{P} is minimal. This is because, when the average strength of the uniformly polarized component increases, the overall content of high spatial frequencies decreases due to a decrease in the magnitude of the second term in Eq. (9). The values of this magnitude are 3.1, 3.0, and 2.6 for the PSDs labeled A, B, and C, respectively. The behavior seen in Fig. 3 demonstrates that, if the correlations of the underlying fields do not vary during the transition from polarized to globally unpolarized regimes, the shape of the PSD remains relatively unchanged.

4. Conclusion

There are practical situations when the emerging random electromagnetic fields can be thought as a combination of two interfering mutually coherent fields. When one of these two underlying fields is globally unpolarized and the other one is polarized uniformly, the spatial correlation of the polarization states contains information about both the relative strength and the extent of the field correlations in the two components.

We have shown that the complex degree of polarization (CDMP), a two-point vectorial descriptor quantifying the polarization similarities in the resulting random field, can be used to recover this information. In particular, we have demonstrated that the power spectral density of the spatial variation in the CDMP maps depends on the specific two-point properties of the underlying fields. We have also outlined a simple procedure to recover them when some prior information is available.

The physical situation discussed here is typical for random fields emerging from the interaction between coherent optical fields and randomly inhomogeneous media where a definite polarized component can be associated with ballistic, surface, or single scattering. Our results should be of interest for a variety of applications operating in these scattering regimes.

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