

Dynamics of high repetition rate regenerative amplifiers

M. Grishin^{1,2*}, V. Gulbinas¹ and A. Michailovas²

¹*Institute of Physics, Savanoriu ave 231, LT-02300, Vilnius, Lithuania*

²*EKSPLA uab, Savanoriu ave 231, LT-02300, Vilnius, Lithuania*

[*m.grishin@ekspla.com](mailto:m.grishin@ekspla.com)

Abstract: Dynamics features of high repetition rate continuously pumped solid-state regenerative amplifiers were studied numerically. A space independent rate equations and discrete-time dynamical system approach were used for system state evolution analysis. Regular single-energy operation, quasi-periodic pulsing and chaotic behavior regions are distinguished in space of control parameters. Diagrams of dynamical regimes comprehensively exhibiting operation features of the system are presented. Seed energy is shown to be an important parameter determining the stability space. Conditions of stable operation are described quantitatively.

©2007 Optical Society of America

OCIS codes: (140.3430) Laser theory; (140.3280) Laser amplifiers; (140.3580) Lasers, solid-state; (140.7090) Ultrafast lasers; (140.1540) Chaos.

References and links

1. W. Koechner, *Solid-State Laser Engineering* (Springer, 1996), Chap. 9.4.2.
2. S. Forget, F. Balembos, P. Georges, and P. Devilder, "A new 3D multipass amplifier based on Nd:YAG or Nd:YVO₄ crystals," *Appl. Phys. B* **75**, 481 (2002).
3. V. Kolev, M. Lederer, B. Luther-Davies, and A. Rode, "Passive mode locking of a Nd:YVO₄ laser with an extra-long optical resonator," *Opt. Lett.* **28**, 1275 (2003).
4. D. Nickel, C. Stolzenburg, A. Bevertt, A. Geisen, J. Häussermann, F. Butze, and M. Leitner, "200 kHz electro-optic switch for ultrafast laser systems," *Rev. Sci. Instrum.* **76**, 033111 (2005).
5. G. Raciukaitis, M. Grishin, R. Danielius, J. Pocius, L. Giniūnas, "High repetition rate ps- and fs- DPSS lasers for micromachining", *ICALEO 2006 Proceedings on CD-ROM*, (Laser Institute of America, 2006), **99**. <http://www.laserinstitute.org/store/CONF/599>
6. S. Valling, T. Fordell, and A. M. Lindberg, "Experimental and numerical intensity time series of an optically injected solid state laser," *Opt. Commun.* **254**, 282 (2005).
7. J. Döring, A. Killi, U. Morgner, A. Lang, M. Lederer, and D. Kopf, "Period doubling and deterministic chaos in continuously pumped regenerative amplifiers," *Opt. Express* **12**, 1759 (2004). <http://www.opticsinfobase.org/abstract.cfm?URI=oe-12-8-1759>
8. Orazio Svelto, *Principles of Lasers* (Plenum Press, 1998), Chap. 7.2.
9. Kathleen T. Alligood, Tim D. Sauer, and James A. Yorke, *Chaos. An Introduction to Dynamical Systems* (Springer, 1996), Chap. 1.

1. Introduction

Regenerative amplifiers (RA) are extensively used for ultra short laser pulse amplification [1]. This is a powerful technique providing several orders of magnitude gain and eliminating amplified spontaneous emission problem, well known for multi-pass amplifiers [2]. Originally solid state lasers with RA have been operated at repetition rates of up to several kHz. Nowadays a rising demand for efficient laser micro-processing has led to developments in a field of high repetition rate ultra-short pulse solid state lasers [3]. On the other hand, a new generation of fast electro-optical switches became available (such as Pockels cells based on β -barium borate along with improved high-voltage electronics reported in [4]). As a result, picosecond and femtosecond lasers with the repetition rate of the order of 100 kHz came up for last few years [5].

The required level of output energy (typically around 0.1-1mJ) for diode pumped systems implies that materials with a long upper laser level lifetime have to be used. Therefore Nd- and Yb-doped laser media are a common choice for RA-based systems intended for industrial applications. It has been recently found that dynamics of such systems operating at a high repetition rate is not trivial. Similar to other lasers demonstrating complex dynamics [6] they undergo bifurcations as control parameters (CP) are varied. Depending on the system parameters, continuously pumped RA based on Yb-doped glass tends to produce either regular or multi-energy or even chaotic sequences of output pulses [7]. However, a complete picture of RA behavior features was not elaborated in this paper. In particular, the influence of the seed energy on the system stability has not been thoroughly examined. More detailed exploration of control parameter influence on operation stability is of interest of practical system designing.

In this article we present comprehensive illustration of stability features of continuously pumped high repetition rate RA based on laser media with long relaxation time. The regions exhibiting different system behavior are mapped in the space of non-dimensional control parameters: repetition rate, round trip number and seed energy. The paper is organized in the following way. First we consider one cycle of RA operation and derive gain and pulse energy evolution rules employing rate equations and some additional assumptions. Then we describe evolution of the successive cycles in terms of the discrete-time dynamical system. The results of numerical study of the system dynamics are presented and discussed in the last section.

2. Analysis of a single cycle. Rate equations and basic assumptions

A schematic of a conventional solid-state diode-pumped regenerative amplifier is depicted in Fig. 1. It represents a resonator containing gain medium and electro-optical switch composed of the Pockels cell, retardation plate and the polarizer. The RA operation cycle consists of two successive stages called pump phase and amplification phase [1]. No voltage is applied to the Pockels cell during the pump phase. The retardation plate along with the polarizer provides high cavity loss, preventing lasing. Under continuous pumping conditions and in the absence of lasing the population inversion grows up. The amplification phase begins as soon as the seed pulse is injected into the cavity and high voltage is applied to the Pockels cell. Intra-cavity loss becomes minimal and the seed amplification takes place during several round trips, consuming certain part of the stored energy. As soon as the pulse energy reaches the required level, the cavity dumping occurs: the Pockels cell voltage is switched off again and the amplified pulse is ejected out of the cavity as the output pulse. The system returns into the initial state. In case of repetitive operation, such cycles reiterate continuously.

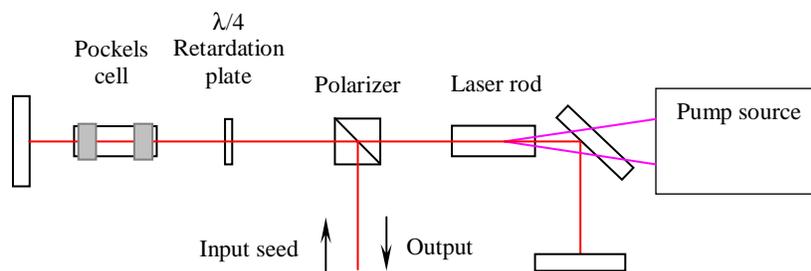


Fig. 1. Optical layout of the solid-state diode pumped regenerative amplifier.

The system evolution can be described by coupled differential equations for the population inversion density N and photon number ϕ . We employ the space independent rate equations that have been formulated for idealized four-level laser medium with a homogeneously broadened line [8]:

$$\frac{dN}{dt} = R_p - \frac{\sigma c}{V} \phi N - \frac{N}{T_1} \quad (1)$$

$$\frac{d\phi}{dt} = \left(\frac{\sigma c L_a A_a}{V} N - \frac{1}{T_c} \right) \phi, \quad (2)$$

where R_p is the pumping rate; c is the speed of light; σ is the stimulated emission cross-section; V is the mode volume within the laser cavity; L_a is the active medium length; A_a is the beam cross section in the laser medium; T_1 is the upper laser level lifetime; T_c is the photon lifetime in the laser cavity.

Concerning general validity of the rate equation model, we refer to [8] for details. Here we just mention that this is a conventional approach of laser dynamics study and it gives adequate results in particular for RA behavior analysis as it has been shown in [7].

It is advisable to re-arrange the equations to a form containing macroscopic non-dimensional terms. This is a common way to reduce the number of control parameters, that itself is advantageous for generalized description of the system. We introduce the normalization coefficient proportional to the pump level, $G_0 = R_p T_1 \sigma L_a$. It has simple physical meaning: $\exp(G_0)$ is the steady state small signal gain per pass. Thus we can proceed to equations for the normalized gain $g = N \sigma L_a / G_0$ and normalized energy $\varepsilon = \phi \sigma / (A_a G_0)$:

$$\frac{dg(\tau)}{d\tau} = -\varepsilon(\tau)g(\tau) + \frac{1-g(\tau)}{\tau_1} \quad (3)$$

$$\frac{d\varepsilon(\tau)}{d\tau} = \varepsilon(\tau)g(\tau) - \frac{\varepsilon(\tau)}{\tau_c}. \quad (4)$$

Additionally, we modify the time scale by introducing of an effective round trip number $\tau = t G_0 \beta / T_0$. Here T_0 is the cavity round trip time; β is the number of passes through the gain media for one round trip ($\beta=1$ for ring cavity; $\beta=2$ for linear cavity). The term τ is a product of the cavity round trip number t/T_0 and small signal gain per round trip $G_0 \beta$. As we will see later, τ assigns a natural time scale to the amplification phase and thus it conveniently defines the RA control parameter governing the amplification phase duration. The other terms of Eqs. (3) and (4) are assigned as follows: $\tau_1 = T_1 G_0 \beta / T_0$ is the normalized upper laser level lifetime; $\tau_c = T_c G_0 \beta / T_0$ is the normalized photon lifetime. Hereinafter, we often miss attributes "effective" and "normalized" just for shortening.

Below in this chapter we define relations between basic system parameters deriving the rate equations for separate phases of RA operation. At first let us consider the pump phase. There is no lasing during this stage (i.e. $\varepsilon=0$), therefore a set of Eqs. (3) and (4) transforms into a single equation:

$$\frac{dg(\tau)}{d\tau} = \frac{1-g(\tau)}{\tau_1}. \quad (5)$$

Initial condition specifies the gain at the beginning of the pump phase: $g(0)=g_1$. Taking into account that $\tau/\tau_1=t/T_1$, we can find relation between the initial gain and the final gain (g_p versus g_1) for a certain pump phase duration T :

$$g_p = 1 - (1 - g_1) \exp\left(-\frac{T}{T_1}\right). \quad (6)$$

Then we consider the amplification phase. The equations for this phase can be simplified as well. We remind here that we explore practically important situation for diode-pumped systems of laser media having long relaxation time. Since the buildup time of the pulse is

usually short in respect of the pump phase duration and the upper laser level lifetime, the population inversion change due to pumping and relaxation processes is much smaller than the inversion depletion caused by amplification. Hence, the terms containing spontaneous decay and optical pumping can be neglected in Eq. (3). The second assumption presumes low intra-cavity loss, $1/\tau_c \ll g$. Thus Eqs. (3) and (4), when describing amplification phase dynamics, reduce to the following:

$$\frac{dg(\tau)}{d\tau} = -\varepsilon(\tau)g(\tau) \quad (7)$$

$$\frac{d\varepsilon(\tau)}{d\tau} = \varepsilon(\tau)g(\tau). \quad (8)$$

Initial conditions specify the system state at the beginning of the amplification phase: $g(0)=g_0$ is the initial gain; $\varepsilon(0)=\varepsilon_s$ is the normalized energy of the seed pulse. With those assumptions the solution of the set of Eqs. (7) and (8) can be presented in analytic form:

$$g_a = \frac{g_0 + \varepsilon_s}{1 + \frac{\varepsilon_s}{g_0} \exp(g_0 \tau)} \quad (9)$$

$$\varepsilon = \frac{g_0 + \varepsilon_s}{1 + \frac{g_0}{\varepsilon_s} \exp(-g_0 \tau)}. \quad (10)$$

Equation (9) describes temporal evolution of the gain. In case of fixed amplification phase duration, expressed in terms of the effective round trip number τ , this equation shows how the final gain g_a depends on the initial one. Similarly, Eq. (10) defines the RA output energy determined by the normalized energy ε , reached on the amplification phase completion. The terms τ and ε_s are system control parameters, whereas the initial gain g_0 is a variable coupling the equations. The initial gain evaluation is a trivial task for the low dumping rate situation, i.e. when the pump phase lasts much longer than the inversion relaxation time, $T \gg T_1$. The gain in this case reaches saturation before the amplification phase starts, therefore $g_0=1$. In general case, the initial conditions for the current phase depends on the previous system state. Hence, the initial gain for each cycle depends not only on operation parameters but on the system pre-history as well. In order to determine evolution of the g_0 value we shall relate final and initial states for successive operation cycles.

3. Coupling of successive cycles. Discrete-time dynamical system approach

We consider gain and pulse energy at the time boundaries between neighboring phases of operation as variables defining the system state. We introduce the term $g_0(1)$ to be the initial gain of the amplification phase for the first cycle of operation. This stage finishes with the final gain denoted by $g_1(1)$. Obviously, the following pump phase begins with the same value, $g_1(1)$. Similarly, the gain evolution continuity should be taken into account for coupling of all consequent cycles. The legend of the system evolution in a discrete time scale is presented in Tab. 1. The corresponding time points can be described as follows: $t_0(k)=(k-1)(\tau T_0/\beta G_0+T)$ and $t_1(k)=k\tau T_0/\beta G_0+(k-1)T$. Here k is the operation cycle number. Note that in an assumption of short amplification phase, the T value approximately equals to the dumping period.

Unlike the gain, the intra-cavity energy evolution $\varepsilon_s \rightarrow \varepsilon(k)$ interrupts at the end of the amplification phase of each cycle at the moment of the pulse ejection and then it begins again with the fixed value of ε_s . Hence the term $\varepsilon(k)$ determining the RA output energy does not depend on the system pre-history. It does depend on the gain and can be found from Eq. (10) for any cycle as soon as the gain is known. Therefore, the gain becomes the only independent variable that needs to be analyzed.

Table 1. Legend of System State Evolution in Discrete Time Scale

Cycle number 1		Cycle number 2		...		Cycle number k	
Amplification phase		Pump phase		Amplification phase		Pump phase	
Start	End	Start	End	Start	End	Start	End
$g_0(1)$	$g_1(1)$	$g_1(1)$	$g_0(2)$	$g_0(2)$	$g_1(2)$	$g_1(2)$	$g_0(3)$
ε_s	$\varepsilon(1)$	0	0	ε_s	$\varepsilon(2)$	0	0
$t_0(1)$	$t_1(1)$	$t_1(1)$	$t_0(2)$	$t_0(2)$	$t_1(2)$	$t_1(2)$	$t_0(3)$

Now we consider the expressions obtained earlier [Eq. (6) for the pump phase and Eq. (9) for the amplification phase] as the rules of the system state updating. So we have a system defined by the evolution of a single variable on a discrete time scale and the rules that take the current state as input and update the situation by producing a new state. This new output state serves as the input for the next phase. Thus, the RA behavior is described in terms of the one-dimensional discrete-time dynamical system [9]. The sequence of the system states $g_0(1)$, $g_0(2)$, ..., $g_0(k)$ is called an orbit in terms of this technique. Taking into account that $g_1(k)=g_a[g_0(k)]$ and $g_0(k+1)=g_p[g_1(k)]$ we obtain the recurrent formula determining the present state of the system in terms of previous state: $g_0(k+1)=g_p\{g_a[g_0(k)]\}=g_\Sigma[g_0(k)]$. Here g_Σ is the composition of functions g_p and g_a exhibiting a gain updating rule for the complete cycle:

$$g_\Sigma(g_0) = 1 - \left[1 - \frac{g_0 + \varepsilon_s}{1 + \frac{\varepsilon_s}{g_0} \exp(g_0 \tau)} \right] \exp\left(-\frac{T}{T_1}\right). \quad (12)$$

It is obvious that in a regular single-energy regime the gain depletion during the amplification phase should be compensated by restoring the population inversion during the pump phase. In other words, the initial gain of the amplification stage eventually should reiterate, i.e. $g_0(k+1)=g_0(k)$. Consequently the system eigenstate satisfying the condition $g_0=g_\Sigma(g_0)$ should exist. The solution of this equation designated as g_f is known as a fixed point in the discrete-time dynamical system theory [9]. Being exactly in the fixed point the system state reproduces itself after each cycle that leads to operating in a regular manner. However, requirement of technical feasibility of such a regime establishes a more strict condition to be fulfilled. The system should return to the fixed point after some perturbation has occurred, i.e. the fixed point should be attracting. Thus, the RA dynamics study is reduced to the analysis of the fixed point existence conditions and its stability features.

We start the analysis with the graphical illustration of the system state evolution. For more intuitive presentation, the fixed point existence condition [$g_0=g_\Sigma(g_0)$ with the explicit form of g_Σ given by Eq. (12)] is rearranged into the following form:

$$1 - (1 - g_0) \exp\left(\frac{T}{T_1}\right) = g_1 = \frac{g_0 + \varepsilon_s}{1 + \frac{\varepsilon_s}{g_0} \exp(g_0 \tau)}. \quad (13)$$

The left-hand part can be interpreted as an inverse rule of the gain restoring during the pump phase [transformed Eq. (6)]. It expresses the gain value g_1 from which this phase should begin in order to be terminated with g_0 . The right hand part expresses the gain at the end of the amplification phase (g_1) as a function of the gain at its beginning [see Eq. (9)]. We can present

both g_1 versus g_0 dependences on a single plot (Fig. 2). The intersection of those curves gives solution of Eq. (13), i.e. the fixed point of the system. Simple analysis of the properties of two constituent parts of Eq. (13) shows that the intersection always exists and it is always single for any set of control parameters. This fact is important. One of two necessary requirements for existence of a stable single-energy regime, namely the fixed point uniqueness, is fulfilled. Hence, the main concern is the fixed point stability study. Figures 2(a) and 2(b) represent system state evolution diagrams for two different situations. Figure 2(a) presents a typical case of the orbit converging into an attracting fixed point. Such an orbit behavior means that RA eventually starts producing regular output energy. The fixed point becomes attracting if the derivative of g_Σ at $g_0=g_f$ satisfies the requirement $|g'_\Sigma(g_0)|<1$ [9]. The condition $|g'_\Sigma(g_f)|=1$ represents the boundary between stable and unstable operation regimes or in other terms defines the bifurcation point. In case of $|g'_\Sigma(g_f)|>1$ the fixed point is repelling, and consequently the stable operation becomes unrealizable.

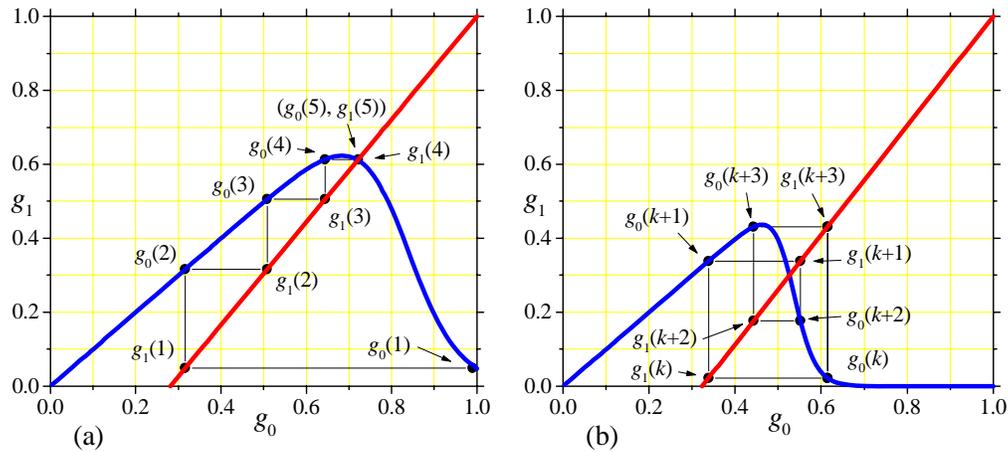


Fig. 2. Graphical presentation of the orbits in state space. Blue and red curves are right and left hand parts of Eq. (13), respectively. Transition to the stable (attracting) fixed point (a) at $\varepsilon=3\times 10^{-7}$; $\tau=18.0$; $T_1/T=3.0$. Period-4T orbit (b) at $\varepsilon=10^{-10}$; $\tau=42.0$; $T_1/T=2.56$.

Obviously in latter case the system state is unable to reproduce itself after one cycle of operation. However, such a reiteration may occur after two or several cycles. Corresponding sets of system states are called periodic orbits. An example of periodic orbit is depicted in Fig. 2(b). The condition for existence of the orbit with the period of $2T$ can be written by introducing composition of the function g_Σ . Define $g_\Sigma^2=g_\Sigma(g_\Sigma)$ to be the result of applying the function g_Σ to the system state two times. The system state g_{2f} such that $g_{2f}=g_\Sigma^2(g_{2f})$ is the fixed point analogue but suited for two successive operation cycles. Generally the orbit with the period of PT exists if there is a system eigenstate g_{pf} , satisfying the equation: $g_{pf}=g_\Sigma^P(g_{pf})$. Here P is an integer number exhibiting a factor of output pulse repeatability for the corresponding multi-energy regime. If such a regime is realized, the system produces quasi-periodic sequence of the output pulses. The pulses of identical magnitude in this sequence appear each time in a multiplied period equal to PT . The same as in the fixed point case the existence of periodic orbit does not imply realizability of the corresponding regime. Additionally, analysis of the orbit stability is required. A conventional procedure for doing that is calculation of the product of the derivatives at points along the orbit. The orbit is stable if $|(g_\Sigma^P)'[g(k)]|=|g'_\Sigma[g(1)]g'_\Sigma[g(2)]\cdots g'_\Sigma[g(k)]|<1$ for $k\rightarrow\infty$. Otherwise, being eventually non-

periodic it implies chaotic behavior. In this case the absolute value of such a product of derivatives is larger than 1. This is the Lyapunov number criterion of deterministic chaos [9].

4. Numerical consideration

Simplifying assumptions and non-dimensional effective parameters, introduced for the basic rate equation model, reduce the number of independent control parameters (CP) of the system to the set of three. These are the normalized pump phase duration T/T_1 , amplification phase duration expressed in terms of the effective round trip number τ and normalized seed energy ε_s . In an assumption of a short amplification phase, the term $(T/T_1)^{-1}$ corresponds to the normalized repetition rate of RA.

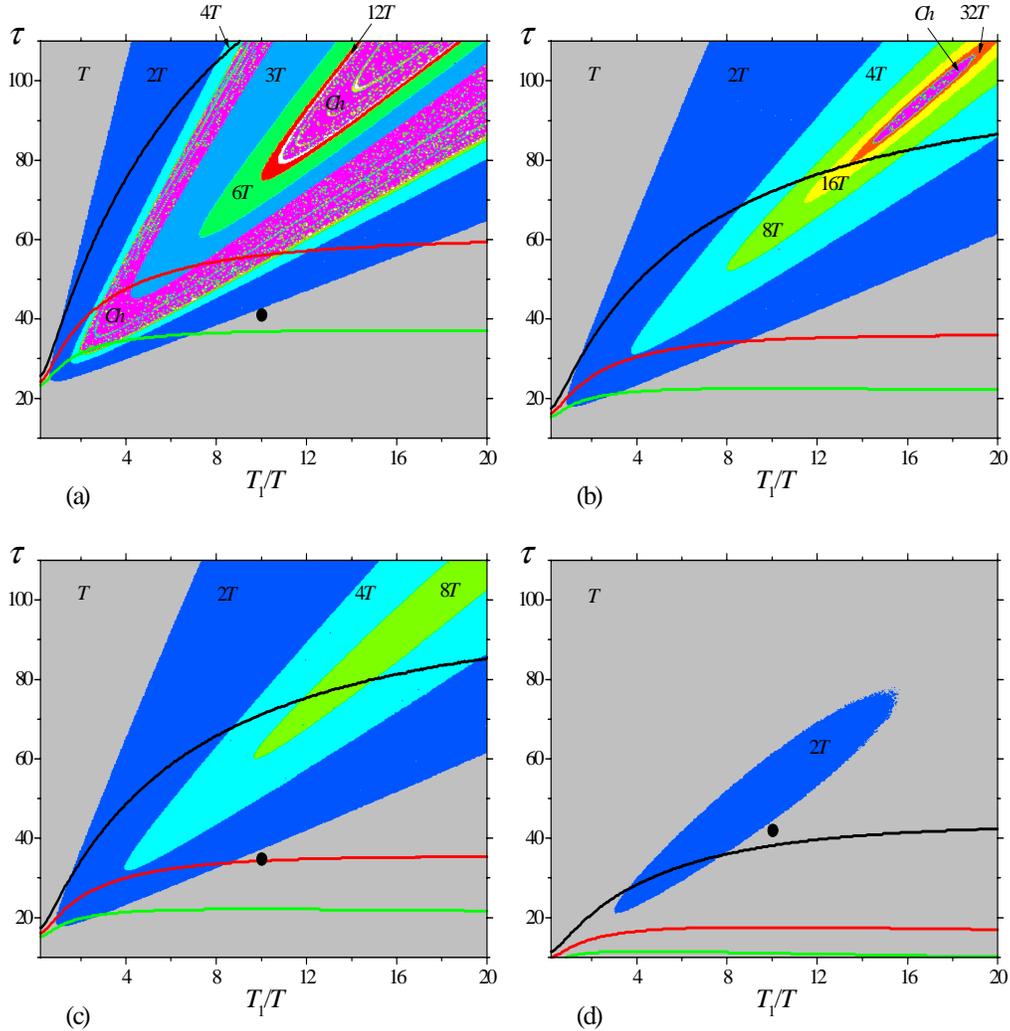


Fig. 3. Diagrams of RA dynamical regimes in parameter space for different seed energy: $\varepsilon_s=10^{-10}$ (a); $\varepsilon_s=2.5 \times 10^{-7}$ (b); $\varepsilon_s=3 \times 10^{-7}$ (c); $\varepsilon_s=1.3 \times 10^{-4}$ (d). Green, red, and black lines correspond to stored energy extraction efficiency of 50 %, 70 %, and 90 %, respectively.

The orbits have been calculated by iterating of Eq. (12) in the range of control parameters (CP) wide enough to comprehend all the relevant dynamics features: $0.02 < (T_1/T) < 20$; $10 < \tau < 120$; $10^{-11} < \varepsilon_s < 10^{-3}$. We employed as much as 3000 iterations, the sufficient number to be confident that the results are independent of the system state at the beginning of iterating.

The orbits have been analyzed in two stages. At first, the minimal number of cycles between repeating system states was revealed for each orbit in CP space. It has been performed by direct comparison of the system state sequences with themselves but shifted within a certain cycle number, $g_0(k)$ versus $g_0(k+P)$. In that way the periodic orbits up to $P=32$, including regular ones ($P=1$), have been identified. Then the Lyapunov number criterion was applied to the residual unidentified orbits. They were separated into two fundamentally different bunches: chaotic and eventually periodic.

Thus, the following relevant dynamics regions have been distinguished in three-dimensional space of control parameters: (1) The orbits evolving into stable fixed points ($P=1$) corresponding to the regular system behavior (single pulse energy output, i.e. $1T$ -regime). (2) Periodic orbits corresponding to multi-energy regimes with repeatability coefficients in the range of $2 \leq P \leq 32$. (3) Eventually periodic orbits having larger repeatability factor ($P > 32$), for which the P -number itself is not identified. (4) Regime of deterministic chaos in accordance with the Lyapunov number criterion.

The regions mapping different dynamics are depicted in coordinate plane of the repetition rate - round trip number for several seed energies (Fig. 3). It is found that the major part of the CP space is occupied by the regions corresponding to the following regimes: single-energy ($1T$); quasi-periodic with fundamental period of two ($2T$, $4T$, $8T$, $16T$, and $32T$); quasi-periodic with fundamental period of three ($3T$, $6T$, and $12T$); and chaotic behaviour. These domains are marked with different colours, whereas the rest of the space containing the remaining zones of eventually periodic orbits is left white. The boundaries between adjacent colors (i.e. between different regimes) represent system bifurcation manifolds in parameter space.

As it is seen, the dynamics turned out multifarious. Chaotic regime ordinarily comes out from the chain of successive period doubling bifurcations: $T-2T-4T-8T-16T-32T\dots$. The chaotic zone itself has fine structure. Quasi-periodic "windows" with various periods are disseminated in it. The situation strongly depends on the seed value. RA behavior for low seed level ($\epsilon_s < 10^{-9}$) is complex, the parameter space contains more than one clearly distinguishable chaotic regions [Fig. 3(a)]. The higher the seed energy, the simpler the diagram becomes. Chaotic domain shrinks to ellipse [Fig. 3(b)] and disappears from the parameter space. Furthermore, period doubling bifurcations with fundamental period of two only remain for $\epsilon_s > 2.52 \times 10^{-7}$. Then the number of bifurcations decreases [Figs. 3(c) and 3(d)] and finally the system becomes stable in the whole CP range for $\epsilon_s > 1.9 \times 10^{-4}$.

The sets of regimes realizable within a certain seed value range are extracted to Tab. 2. To give an idea about the scale of normalized seed value, we present corresponding absolute seed energies E_{seed} for RA with the following parameters: laser medium is the Nd:YVO₄ crystal ($\lambda=1064$ nm, $\sigma=1.35 \times 10^{-18}$ cm²); the pump provides steady-state small signal gain $G_0=1.0$; the beam diameter is 0.1 cm.

Table 2. Dynamic Regimes Realizable for Corresponding Seed Energy Range

Existing regimes	Seed value range	
	ϵ_s	E_{seed} (nJ)
Chaos and "all" periods	$\epsilon_s < 2.52 \times 10^{-7}$	$E_{seed} < 0.271$
$T, 2T, 4T, 8T, 16T, 32T\dots$	$2.52 \times 10^{-7} < \epsilon_s < 2.56 \times 10^{-7}$	$0.271 < E_{seed} < 0.275$
$T, 2T, 4T, 8T, 16T$	$2.56 \times 10^{-7} < \epsilon_s < 2.72 \times 10^{-7}$	$0.275 < E_{seed} < 0.293$
$T, 2T, 4T, 8T$	$2.72 \times 10^{-7} < \epsilon_s < 3.56 \times 10^{-7}$	$0.293 < E_{seed} < 0.383$
$T, 2T, 4T$	$3.56 \times 10^{-7} < \epsilon_s < 1.39 \times 10^{-6}$	$0.383 < E_{seed} < 1.49$
$T, 2T$	$1.39 \times 10^{-6} < \epsilon_s < 1.90 \times 10^{-4}$	$1.49 < E_{seed} < 204$
T (stable)	$\epsilon_s > 1.90 \times 10^{-4}$	$E_{seed} > 204$

The relation of the output energy to system parameters is presented in Fig. 3 as well. For this purpose numerically obtained curves of equal efficiency are plotted. Each curve represents a manifold of operating points in parameter space providing constant efficiency of the stored energy extraction. As it is seen the efficiency of single energy operation is limited by the dynamics peculiarity in a wide range of repetition rates. This phenomenon is the most significant when the seed energy is low [Fig. 3(a)]. Let us consider the above mentioned Nd:YVO₄ based RA operating at 100 kHz ($T_1/T=10.0$) as an example. So we can show what absolute energy can be extracted from the RA just before the system gets into unstable mode. The calculated output energies are equal to 56 μ J, 71 μ J, and 92 μ J as the seed energies are 0.11 pJ ($\epsilon_s=10^{-10}$), 0.32 nJ ($\epsilon_s=3\times 10^{-7}$), and 140 nJ ($\epsilon_s=1.3\times 10^{-7}$), respectively. The corresponding operating points are marked in Figs. 3(a), 3(c), and 3(d) with dots.

A stable single-energy operation is the only suitable regime for routine use of laser systems. RA stability diagrams are shown in Fig 4. These diagrams represent a set of the first order period doubling bifurcations ($1T-2T$ boundary). Each curve corresponding to a certain seed value separates stable and unstable operation zones. The unstable zones are situated in the right upper part of the plot. For higher seed values they shrink to ellipses. Note that the operation is always stable for low repetition rate ($T_1/T < 0.3$) and small round trip number ($\tau < 15$). The first condition just gives quantitative illustration of the well known fact that the low repetition rate RA has no dynamical features. The second condition shows that the system is stable if its total (through all the roundtrips) small signal gain is limited from above, i.e. $\exp(\tau) < 3.3\times 10^6$.

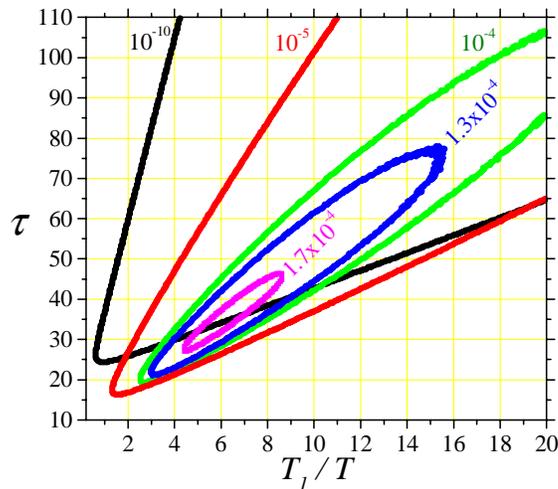


Fig. 4. Stability diagrams in system parameters space (normalized repetition rate - effective round trip number) for different seed energies.

Although the calculations have been done with 4-level rate equations the results are valid for quasi-3-level systems with a homogeneously broadened line as well. Simply, in the latter case the explicit expression of the normalization coefficient G_0 for the effective round trip number τ looks more sophisticated. However the physical meanings of both terms G_0 and τ remain identical to those defined for 4-level system.

5. Conclusion remarks

A complicated dynamics is an inherent property of continuously pumped high repetition rate RA based on long relaxation time laser media. The behavior of such systems is determined by equilibrium between population inversion depletion caused by amplification and its restoring

caused by pumping. Analytically this balance condition can be described in terms of fixed points within a one-dimensional discrete-time dynamical system approach. Our analysis shows that the fixed point exists and it is always single for any set of system governing parameters. Therefore all the feasible regimes are eventually independent of the system initial state, i.e. they are related to the control parameters unambiguously. Furthermore, the fixed point uniqueness leads to a quite certain structure of RA dynamics mapped in parameter space. The period doubling bifurcations result in the system transition from stable operation through the multi-energy pulsing to chaotic regime. The corresponding diagrams exhibiting relevant relations of RA control parameters to the dynamical regimes are presented in this paper.

The strong influence of the seed value on RA performance is the fundamental feature of the high repetition rate operation. For low repetition rate systems the seed value should just exceed the level of spontaneous emission in the amplification channel. In case of the high repetition rate in order not to get into unstable mode much higher seed energy is required. Another way of reaching stability is the round trip number (i.e. system total gain) decrease that again requires a higher seed value for obtaining the maximum output energy.

One-dimensional discrete-time dynamical system approach is quite a versatile tool for the RA behavior study. No matter in what form and by what means the necessary rules, determining the present system state in terms of previous state, are obtained. For this purpose we used the analytical expressions, derived within certain model assumptions, presented in this paper. The analytic form just simplifies fixed points and orbits evaluation. In general, the gain updating rules can be found by deriving more comprehensive rate equations numerically. In that way one can take into consideration the contributions not counted in the present study. It is possible to comprehend the intra-cavity loss and the pulse duration influence by elaborating the terminal level lifetime. These are important factors to be considered for more precise prediction of the instability-limited system efficiency. Thus practical guidelines can be formulated for designing and optimization of regenerative amplifiers. It is certainly a worthy goal for further research.

Acknowledgments

This work was partially financed by the European Union Structural Funds and Republic of Lithuania under the project No.S-BPD04-ERPF-3.1.7-03-04/0004.