Control of spontaneous emission from a microwave driven atomic system

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Abstract: We demonstrate coherent control of spontaneous emission from an indirectly coupled transition in a microwave driven four-level atomic system. The transition of concern is not directly coupled by any laser fields, while the ground state is coupled to another ground state by a microwave field. We show that the coupling of the microwave field produces interesting features such as double narrow lines in the emission spectrum. The heights, widths and positions of the emission peaks can be controlled by modifying the Rabi frequency and detuning of the microwave field. We discuss the spectra in the dressed states basis.

OCIS codes: (270.1670) Coherent optical effects; (270.4180) Multiphoton processes.

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1. Introduction

Spontaneous emission is a fundamental phenomenon originating from the interaction between radiation and matter. It has been one of the most attractive topics in the field of quantum optics for the last decades. We can draw a lot of information of quantum systems by focusing on spontaneous emission. For example, Das and Agarwal showed that the photon-photon correlations of the radiation can be viewed as a probe of vacuum-induced coherence effects in a four-level system [1]. We can also study many basic conceptions of quantum optics through the observation of spontaneous emission. In Ref [2], the authors investigated the emission from a $J = 1/2$ to $J = 1/2$ transition that is driven by a monochromatic laser field. They showed that the spectrum and intensity of the fluorescence reflects the quantum interference enforced by the principle of time-energy complementarity. In Ref [3], Temnov and Woggon found giant photon bunching in the cooperative spontaneous emission. In spontaneous emission from a continuously driven atomic ensemble, Norris et al. observed ground-state quantum beats [4].

Apart from the interest of fundamentals, modification and control of spontaneous emission can also find applications in many fields such as lasing without inversion [5–8], transparent high index materials [9], high-precision spectroscopy and magnetometry [10,11], spatial localization of atoms [12,13], quantum information and computing [14–16], and so on. Based on the narrowing of spontaneous emission, the determination of atomic multipole moments by means of the detection of the fluorescence spectrum is anticipated to increase in accuracy by several orders of magnitude [10]. In the study of spatial atom localization, Wan et al. and Ding et al. presented 2D atom localization schemes via controlled spontaneous emission from coherently driven four-level tripod [12] and five-level M-type [13] atomic systems respectively. Entangled photons can also be generated through spontaneous emission. Grünwald and Vogel showed that resonance fluorescence from regular atomic systems may serve as a continuous source of non-Gaussian entangled radiation propagating in two different directions [16]. Unwanted spontaneous emission prevents lasers from working at high frequencies, induces decoherence in a quantum system, set the ultimate precision of quantum measurements, therefore a great deal of work has been devoted to the elimination and suppression of this process [17–21].

We can alter the emission of atoms by placing them in different environments such as cavities [22,23], photonic crystals [24], and nanostructures [25]. For an atom in the free space, quantum coherence plays an important role in controlling spontaneous emission. In the presence of Spontaneously Generated Coherence (SGC), Zhu and Scully showed spectral line elimination and spontaneous emission cancellation in a four level system [17]. In a similar system, Paspalakis and Knight showed the possibility of controlling the emission spectrum with the relative phase of the driving fields [26]. Interesting features of spontaneous emission, such as narrowing [27], squeezing [28] and splitting [29], have been extensively studied in a variety of systems where SGC exists. We can also use quantum coherence generated by coherent fields to effectively modify the properties of spontaneous emission. Laser fields [30–33] and microwave fields [34–36], for example, have been used to couple different energy levels of atoms. It has been predicted [30] and experimental demonstrated [32] that laser fields may induce multi-peaks and sub-natural width peak in resonance fluorescence of three-level systems. In four-level systems, Wu et al. [33] showed that coherence generated by a laser field could produce quenching and narrowing of the emission spectra. In a five-level system, Li et al. [35] demonstrated similar features and showed the possibility of simulating SGC by coupling two ground states with a microwave field. In a four-level Y system where the upper two levels are coupled by a microwave field, the coupling of the microwave field is responsible for the phase dependent spectra [36].

Coupling close-lying levels with microwave fields has been shown to be an efficient way of controlling spontaneous emission [34–36]. It may produce enriched phenomena and provide flexibility for the control of spontaneous emission. However, most of the present researches on spontaneous emission deal with emission to such ground levels that is either unperturbed [17, 26, 33–36] or directly coupled to the excited levels by laser fields [27–32]. Spontaneous emission from an indirectly driven transition, which means it is not directly driven by any laser fields while the ground state is coupled to another ground level by a microwave field, has been seldom mentioned. Inspired by this situation, we investigate the spontaneous emission from a laser coupled excited state to a microwave coupled ground state in a four-level atomic system. We investigate the steady-state spectrum of the spontaneous emission, demonstrate phenomena such as double narrow lines of the spectrum, and show the possibility of controlling the spectrum with the Rabi frequency and detuning of the microwave field.

The paper is organized as follows. In Sec. 2, we introduce the model and the basic equations. In Sec. 3, we describe the numerical results and explain the corresponding features. Sec. 4 contains a summary of the results.

2. Schemes and equations

We consider a four-level system depicted in Fig. 1(a). This set contains an excited state $|3\rangle$, and three ground states $|1\rangle, |2\rangle$, and $|4\rangle$. The transitions from $|3\rangle$ to $|1\rangle, |2\rangle$, and $|4\rangle$ are optical dipoles with decay rates γ_{31}, γ_{32}, and γ_{34}, respectively. There are also nonradiative

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decaying rates (might owing to collision) between the three ground levels, denoted by γ_{12}, γ_{24}, and γ_{14}. The transitions $|3\rangle \to |1\rangle$ and $|3\rangle \to |2\rangle$ are resonantly driven by two laser fields ω_1 and ω_2. The ground states $|2\rangle$ and $|4\rangle$ are coupled by a microwave field ω_3. The detuning of the microwave field from the magnetic transition $|2\rangle \to |4\rangle$ is defined by $\Delta = \omega_{42} - \omega_3$. The Rabi frequencies of the two lasers and the microwave field are Ω_1, Ω_2, and Ω_3, respectively.

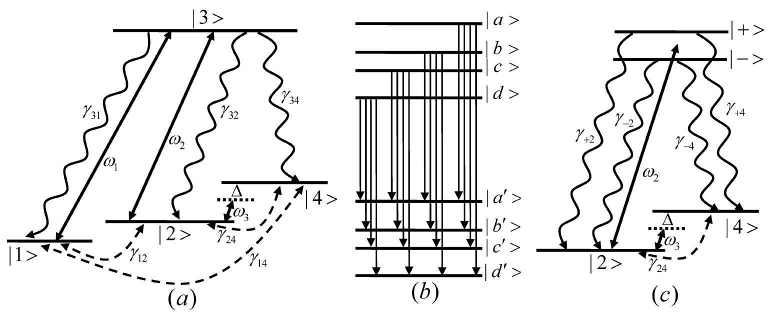

Fig. 1. The energy scheme under consideration. (a) In the bare-state basis. (b) In the dressed-state basis of the two laser fields and the microwave field. (c) In the dressed- state basis of the laser field ω_1.

In the interaction picture, the coherent part of the Hamiltonian in the rotating wave approximation and electric dipole approximation reads

$$H_I = \hbar\Delta|4\rangle\langle 4| - \hbar[\Omega_1|1\rangle\langle 3| + \Omega_2|2\rangle\langle 3| + \Omega_3|2\rangle\langle 4| + H.c.]. \tag{1}$$

The master equation of motion for the density operator in an arbitrary multilevel atomic system is given by

$$\frac{\partial\rho}{\partial t} = \frac{1}{i\hbar}[H, \rho] + L\rho, \tag{2}$$

where $L\rho$ represents the decay part in the system. By expanding Eq. (2), we can arrive at the following density matrix equations of motion.

$$\begin{aligned}
\dot{\rho}_{22} &= i\Omega_2(\rho_{32} - \rho_{23}) + i\Omega_3(\rho_{42} - \rho_{24}) + \gamma_{32}\rho_{33} + \gamma_{12}\rho_{11} + \gamma_{24}\rho_{44} - (\gamma_{12} + \gamma_{24})\rho_{22} \\
\dot{\rho}_{33} &= i\Omega_1(\rho_{13} - \rho_{31}) + i\Omega_2(\rho_{23} - \rho_{32}) - (\gamma_{31} + \gamma_{32} + \gamma_{34})\rho_{33} \\
\dot{\rho}_{44} &= i\Omega_3(\rho_{24} - \rho_{42}) + \gamma_{34}\rho_{33} + \gamma_{14}\rho_{11} + \gamma_{24}\rho_{22} - (\gamma_{12} + \gamma_{14})\rho_{44} \\
\dot{\rho}_{12} &= [-\gamma_{12} - (\gamma_{14} + \gamma_{24})/2]\rho_{12} + i\Omega_1\rho_{32} - i\Omega_2\rho_{13} - i\Omega_3\rho_{14} \\
\dot{\rho}_{13} &= [-(\gamma_{12} + \gamma_{14} + \gamma_{31} + \gamma_{32} + \gamma_{34})/2]\rho_{13} + i\Omega_1(\rho_{33} - \rho_{11}) - i\Omega_2\rho_{12} \\
\dot{\rho}_{14} &= [-\gamma_{14} - (\gamma_{12} + \gamma_{24}) + i\Delta]\rho_{14} - i\Omega_1\rho_{34} - i\Omega_3\rho_{12} \\
\dot{\rho}_{23} &= [-(\gamma_{12} + \gamma_{24} + \gamma_{31} + \gamma_{32} + \gamma_{34})/2]\rho_{23} + i\Omega_2(\rho_{33} - \rho_{22}) + i\Omega_3\rho_{43} \\
\dot{\rho}_{24} &= [-\gamma_{24} - (\gamma_{12} + \gamma_{14})/2 + i\Delta]\rho_{24} + i\Omega_3(\rho_{44} - \rho_{22}) + i\Omega_2\rho_{34} \\
\dot{\rho}_{34} &= [-(\gamma_{31} + \gamma_{32} + \gamma_{34} + \gamma_{14} + \gamma_{24})/2 + i\Delta]\rho_{34} + i\Omega_1\rho_{14} + i\Omega_2\rho_{24} - i\Omega_3\rho_{32}.
\end{aligned} \tag{3}$$

Closure of this atomic system requires $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$, and $\rho_{ij} = \rho_{ji}^*$.

We proceed to calculate the steady-state emission spectra with the common method [27, 29–31]. As is well known, the fluorescence emission spectrum is proportional to the Fourier transform of the steady-state correlation function $\lim_{t\to\infty}\langle E^{(-)}(r, \tau + t) \cdot E^{(+)}(r, t)\rangle$ [37], where $E^{(\pm)}(r,t)$ are the positive and negative frequency parts of the radiation field in the far zone, which consists of a free-field operator, and a source-field operator that is proportional to the atomic polarization operator. Therefore the incoherent fluorescence spectrum can be expressed in terms of the atomic correlation function,

$$S(\omega) = \mathrm{Re}\int_0^\infty \lim_{t\to\infty} < \Delta D^+(\tau+t)\cdot\Delta D^-(t) > e^{-i\omega t}d\tau. \tag{4}$$

Re denotes the real part and $\Delta D^\pm(t) = D^\pm(t) - < D^\pm(\infty) >$ represents the deviation of the dipole polarization operator $D^\pm(t)$ from its mean steady-state value, and

$$D^+(t) = \mu_{34}a_4^\dagger a_3,\, D^-(t) = [D^+(t)]^\dagger, \tag{5}$$

where μ_{34} is the dipole moment of the atomic transition from $|3\rangle$ to $|4\rangle$.

In order to calculate the emission spectrum, we write the equations of motion [see Eq. (3)] in the form

$$\frac{d}{dt}\Psi = L\Psi + I \tag{6}$$

where $\Psi = (\rho_{12},\rho_{13},\rho_{14},\rho_{21},\rho_{22},\rho_{23},\rho_{24},\rho_{31},\rho_{32},\rho_{33},\rho_{34},\rho_{41},\rho_{42},\rho_{43},\rho_{44})^T$, and L is a (15×15) matrix. The elements of L and I can be found explicitly from Eq. (3).

Following the common procedure [27, 29–31], we can obtain the steady spectrum of the spontaneous emission from the excited state $|3\rangle$ to the ground state $|4\rangle$ by means of the quantum regression theorem [38,39]. The result is

$$S(\delta_k) = \mathrm{Re}\left\{M_{14,12}\bar{\rho}_{31} + M_{14,13}\bar{\rho}_{32} + M_{14,14}\bar{\rho}_{33} + M_{14,15}\bar{\rho}_{34} + \sum_l N_{14,l}I_l\bar{\rho}_{34}\right\}, \tag{7}$$

where

$$M_{i,j} = \left[(z-L)^{-1}\Big|_{z=i\delta_k}\right]_{i,j}, N_{i,j} = \left[L^{-1}(z-L)^{-1}\Big|_{z=i\delta_k}\right]_{i,j}. \tag{8}$$

$\bar{\rho}_{ij}$ is the steady-state population ($i=j$) and atomic coherence ($i\neq j$), which can be obtained by setting $\dot{\rho}_{ij}=0$ and solving numerically Eq. (6). δ_k is the detuing between the fluorescence and the transition $|3\rangle\to|4\rangle$.

We assume that the decay rates from $|3\rangle$ to $|1\rangle,|2\rangle$, and $|4\rangle$ are $\gamma_{12}=\gamma_{14}=\gamma_{24}=\gamma.$ The Rabi frequencies of the two laser fields are fixed at $\Omega_1=\Omega_2=\gamma.$ We tune the Rabi frequency Ω_3 and the detuning Δ of the microwave field, and calculate the corresponding emission spectrum.

3. Results and discussions

First of all, we assume that the microwave field is resonant with the corresponding transition and concentrate on controlling the emission properties with the Rabi frequency of the microwave field. The numerical results at different values of Ω_3 are shown in Fig. 2. When $\Omega_3=0,$ which means no microwave field is applied, the fluorescence spectrum is doubly peaked with normal width restricted by the spontaneous decay rate γ [see Fig. 2(a)]. Then we apply a weak microwave field to the system by setting $\Omega_3=0.05\gamma.$ Two narrow lines can be observed in the middle of the spontaneous emission spectrum [see Fig. 2(b)]. The heights of

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theses narrow lines exceed the two broad peaks when we increase the Rabi frequency of the microwave field to $\Omega_3 = 0.1\gamma$ [see Fig. 2(c)]. It is clear that the narrow lines are more pronounced when we increase the value of the Rabi frequency to $\Omega_3 = 0.2\gamma$ [see Fig. 3(d)]. We can also see the distance between the two narrow lines grows larger with an increase of Ω_3.

If the increase of Ω_3 continues, narrow lines become broadened [see Fig. 2(e)]. Meanwhile, an emission peak emerges at the frequency of $\omega = \omega_{24}$ [see Fig. 2(e)]. Moreover, the central peak can be greatly enhanced with a larger Ω_3 [see Fig. 2(f)]. We can also observe more peaks at both sides of the central peak.

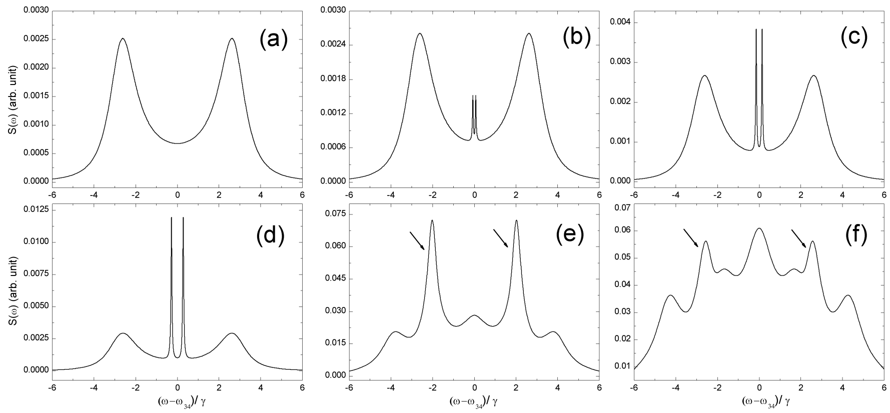

Fig. 2. Numerical results of the emission spectrum with different Rabi frequency of the microwave field. The parameters used are $(a)\,\Omega_3 = 0, (b)\,\Omega_3 = 0.05\gamma, (c)\,\Omega_3 = 0.1\gamma, (d)\,\Omega_3 = 0.2\gamma, (e)\,\Omega_3 = 1.5\gamma, (f)\,\Omega_3 = 2\gamma.$

The observed spectrum can be analyzed in the dressed state basis. Under the resonant coupling of the two laser fields and the microwave field, we obtain the dressed levels by finding the eigenvectors of the coherent part of the interaction Hamiltonian [see Eq. (1)]. The eigen energies are

$$
\begin{aligned}
\lambda_a &= \sqrt{\frac{(\Omega_1^2+\Omega_2^2+\Omega_3^2)+\sqrt{(\Omega_1^2+\Omega_2^2+\Omega_3^2)^2-4\Omega_1^2\Omega_3^2}}{2}}\\
\lambda_b &= \sqrt{\frac{(\Omega_1^2+\Omega_2^2+\Omega_3^2)-\sqrt{(\Omega_1^2+\Omega_2^2+\Omega_3^2)^2-4\Omega_1^2\Omega_3^2}}{2}}\\
\lambda_c &= -\sqrt{\frac{(\Omega_1^2+\Omega_2^2+\Omega_3^2)-\sqrt{(\Omega_1^2+\Omega_2^2+\Omega_3^2)^2-4\Omega_1^2\Omega_3^2}}{2}}\\
\lambda_d &= -\sqrt{\frac{(\Omega_1^2+\Omega_2^2+\Omega_3^2)+\sqrt{(\Omega_1^2+\Omega_2^2+\Omega_3^2)^2-4\Omega_1^2\Omega_3^2}}{2}}.
\end{aligned}
\tag{9}
$$

The corresponding dressed levels can be expressed as

$$
\begin{aligned}
|a\rangle &= x_1|1\rangle + x_2|2\rangle + x_3|3\rangle + x_4|4\rangle\\
|b\rangle &= -x_1|1\rangle - x_2|2\rangle + x_3|3\rangle + x_4|4\rangle\\
|c\rangle &= -x_1|1\rangle - x_2|2\rangle + x_3|3\rangle + x_4|4\rangle\\
|d\rangle &= x_1|1\rangle + x_2|2\rangle + x_3|3\rangle + x_4|4\rangle
\end{aligned}
\tag{10}
$$

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where

$$
\begin{aligned}
x_1 &= \frac{\dfrac{\Omega_1(\lambda_i^2-\Omega_3^2)}{\Omega_2\Omega_3\lambda_i}}{\sqrt{1+\left(\dfrac{\lambda_i}{\Omega_3}\right)^2+\left(\dfrac{\lambda_i^2-\Omega_3^2}{\Omega_2\Omega_3}\right)^2+\left(\dfrac{\Omega_1(\lambda_i^2-\Omega_3^2)}{\Omega_2\Omega_3\lambda_i}\right)^2}} \\
x_2 &= \frac{\dfrac{\lambda_i}{\Omega_3}}{\sqrt{1+\left(\dfrac{\lambda_i}{\Omega_3}\right)^2+\left(\dfrac{\lambda_i^2-\Omega_3^2}{\Omega_2\Omega_3}\right)^2+\left(\dfrac{\Omega_1(\lambda_i^2-\Omega_3^2)}{\Omega_2\Omega_3\lambda_i}\right)^2}} \\
x_3 &= \frac{\dfrac{\lambda_i^2-\Omega_3^2}{\Omega_2\Omega_3}}{\sqrt{1+\left(\dfrac{\lambda_i}{\Omega_3}\right)^2+\left(\dfrac{\lambda_i^2-\Omega_3^2}{\Omega_2\Omega_3}\right)^2+\left(\dfrac{\Omega_1(\lambda_i^2-\Omega_3^2)}{\Omega_2\Omega_3\lambda_i}\right)^2}} \\
x_4 &= \frac{1}{\sqrt{1+\left(\dfrac{\lambda_i}{\Omega_3}\right)^2+\left(\dfrac{\lambda_i^2-\Omega_3^2}{\Omega_2\Omega_3}\right)^2+\left(\dfrac{\Omega_1(\lambda_i^2-\Omega_3^2)}{\Omega_3\Omega_2\lambda_i}\right)^2}},
\end{aligned}
\tag{11}
$$

with $i = a, b, c, d$.

For simplicity, we write Eq. (10) in the following form:

$$
\left|i\right\rangle = \sum C_{ik}\left|k\right\rangle, \quad (i = a, b, c, d; k = 1, 2, 3, 4),
\tag{12}
$$

where C_{ik} is a function of Ω_3.

We can see that both the excited state $\left|3\right\rangle$ and the ground state $\left|4\right\rangle$ [see Fig. 1(a)] are split into four dressed levels [see Fig. 1(b)], which are $\left|a\right\rangle, \left|b\right\rangle, \left|c\right\rangle$, and $\left|d\right\rangle$ for the bare-state level $\left|3\right\rangle$ while $\left|a'\right\rangle, \left|b'\right\rangle, \left|c'\right\rangle$, and $\left|d'\right\rangle$ for the bare-state level $\left|4\right\rangle$. Therefore the spontaneous emission from the excited state $\left|3\right\rangle$ to the ground state $\left|4\right\rangle$ has 16 dipole transitions in the dressed-state representation. This result is similar to that in Ref [40], where a two-level system has 4 dipole transitions in the dressed-state representation. Note that the dressed levels $\left|i\right\rangle$ has the same expressions as the dressed levels $\left|i'\right\rangle$ with $i = a, b, c, d$, but $\left|i\right\rangle$ and $\left|i'\right\rangle$ are different in energy by a constant value, i. e. by the energy difference between level $\left|3\right\rangle$ and $\left|4\right\rangle$. It is well known that positions, widths, and heights of emission peaks are determined by the energies, electronic dipole moments, and steady-state populations of dressed states. In order to interpret the numerical results, we investigate the properties of the dressed levels. We plot the eigen energies $(\lambda_i, i = a, b, c, d)$ and the populations of the dressed levels $(\rho_{aa}, \rho_{bb}, \rho_{cc}, \rho_{dd})$ as functions of the Rabi frequency Ω_3 [see Fig. 3]. We can see that the energies of the dressed levels $\left|a\right\rangle$ and $\left|d\right\rangle$ depend weakly on Ω_3, and the splitting between the dressed levels $\left|b\right\rangle$ and $\left|c\right\rangle$ increases with a larger value of Ω_3 [see Fig. 3(a)]. When Ω_3 is relatively small (i. e. $\Omega_3 < 2\gamma$), all the four dressed levels are well populated [see Fig. 3(b)]. We can also see the influence of Ω_3 on the populations of the dressed levels.

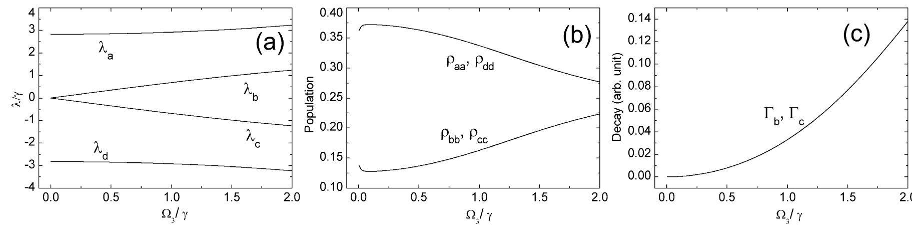

Fig. 3. Properties as a function of the Rabi frequency Ω_3, (a) The eigen energies given by Eq. (2), (b) Steady-state population, (c) decay rates of $|b\rangle$ and $|c\rangle$.

The decay rate of the transition between the dressed levels $|i\rangle$ to $|j'\rangle$ is proportional to the squared dipole moments $R_{ij} = |\langle i|P|j\rangle|^2$, where $P = \mu_{43}|4\rangle\langle 3|$ is the transition dipole moment operator between $|3\rangle$ and $|4\rangle$ in the bare state basis [see Fig. 1(a)]. R_{ij} can be calculated with the expression:

$$R_{ij} = |\langle j|P|i\rangle|^2 = |C_{j4}|^2 \mu_{43}^2 |C_{i3}|^2, (i, j = a, b, c, d), \tag{13}$$

where C_{ij} is the coefficient in Eq. (12).

The two broad side bands [see Fig. 2(b)–2(d)] are associated to the decay of the four transitions $|a\rangle \to |b'\rangle, |a\rangle \to |c'\rangle, |d\rangle \to |b'\rangle$, and $|d\rangle \to |c'\rangle$. The splitting of the dressed levels $|b'\rangle$ and $|c'\rangle$ is smaller than the linewidths of the emission peaks, so we can see two broad bands instead of four. With a weak coupling of the microwave field (e. g. $\Omega_3 < 0.2\gamma$), the decay rates of the dressed levels $|b\rangle$ and $|c\rangle$ takes very low values (approximately 1/400 of the decay rates of $|a\rangle$ and $|d\rangle$ when $\Omega_3 = 0.2\gamma$). The slow decay of the transition $|b\rangle \to |c'\rangle$ and $|c\rangle \to |b'\rangle$ leads to the double narrow lines in the middle of the emission spectrum. The position of the emission peaks are determined by the eigen energies of the dressed levels [see Fig. 3(a)]. The dependence of λ_b and λ_c on Ω_3 are responsible for the variation of the splitting between the two narrow lines. The positions of the two broad side bands are almost unchanged owing to the fact that λ_a and λ_d depend weakly on Ω_3. The decay rate of the dressed levels $|b\rangle$ and $|c\rangle$ are sensitive to Ω_3 [see Fig. 3(c)], so the heights of the two narrow lines are greatly enhanced with an increase of this Rabi frequency [see Fig. 2(b)–2(d)]. Under the weak coupling of the microwave field, the decay of other transitions, such as $|a\rangle \to |d'\rangle$ and $|d\rangle \to |a'\rangle$, are too weak to be observed.

When Ω_3 is comparable to the Rabi frequencies of the laser fields Ω_1 and Ω_2, the decay of the dressed levels $|b\rangle$ and $|c\rangle$ becomes much faster [see Fig. 3(c)], and results in the broadening of the corresponding emission [see the peaks marked with arrows in Fig. 2(e), 2(f)]. When we tune the value of Ω_3, the populations and the decay rate of the dressed levels change accordingly. Then all the sixteen transitions between the dressed levels contribute to the emission spectrum, and results in more peaks in the spectrum [see Fig. 2(e), 2(f)]. Owing to the degeneracy of the transitions, we can obtain as many as nine peaks with sufficient Rabi frequencies of the driving fields (not shown here).

The spontaneous emission is also sensitive to the detuning of the microwave field relative to the magnetic transition $|4\rangle \to |2\rangle$. We illustrate the effects of the detuning Δ on the

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emission spectrum in Fig. 4. The Rabi frequency of the microwave field is fixed at $\Omega_3 = 0.2\gamma$.

By varying the detuning Δ, we get the results in Fig. 4(a)–4(c). When the microwave field is slightly detuned, the spectrum becomes asymmetric [see Fig. 4(a)]. We can also observe a narrow line at the frequency of $\omega = \omega_{34}$. When the detuning is increased to $\Delta = 0.2\gamma$, the central peak turns to be comparable with the other two narrow lines in height [see Fig. 4(b)]. The narrow peak in the center continues to grow higher with a greater detuning [see Fig. 4(c), where $\Delta = 0.4\gamma$]. There are also larger splitings between the three narrow peaks with an increase of the detuning.

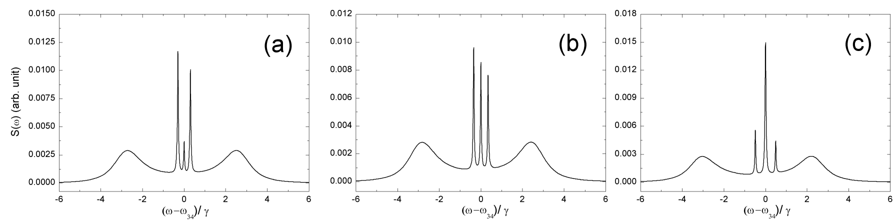

Fig. 4. Numerical results of the emission spectrum with different detuning of the microwave field. The parameters used are (a) $\Delta = 0.1$, (b) $\Delta = 0.2\gamma$, (c) $\Delta = 0.4\gamma$.

With the detuning of the microwave field Δ , it is difficult to obtain the analytical expression of the dressed levels as Eqs. (9)–(11). We give qualitative explanations for the results in Fig. 4. Under the coupling of the two laser fields and the microwave field, we get dressed levels similar to those in Fig. 1(b). Compared with the case of resonant coupling, the properties of the dressed levels, such as the positions, populations and dipole moments between them, are modified by the detuing of the microwave field Δ. The modified populations and decay rates of the dressed levels lead to the asymmetry and additional narrow peak in the emission spectra shown in Fig. 4. The larger is the detuing Δ, the greater is the modification. As a result, we see the variation of the spectrum from Fig. 4(a) to 4(c). Furthermore, the detuing Δ modifies the positions of the dressed levels and induces splitings between the degenerated transitions in Fig. 3(b). Therefore the spectrum is further affected. As a matter of fact, we can get as much as 13 peaks in the emission spectrum (no shown here) with proper chosen Rabi frequencies and detunings of the coupling fields.

The observed results can also be viewed as the combined effects of the microwave coupling and SGC between dressed levels. In the dressed-state representation of the laser field ω_1, the system turns to be a four-level scheme with two close-lying excited levels [see Fig. 1(c)], where $|+\rangle = (|1\rangle - |3\rangle)/\sqrt{2}$ and $|-\rangle = (|1\rangle + |3\rangle)/\sqrt{2}$. The coupling of the microwave field together with SGC between $|+\rangle$ and $|-\rangle$ affect the decay from the two excited levels to the ground state $|4\rangle$. The combined effects of SGC and the coupling of the microwave field result in the interesting emission spectrum such as narrow peaks. We can certainly realize the modification and control of spontaneous emission by tuning the parameters of the microwave field.

As is well known, it is very difficult to realize SGC in real atoms owing to the rigorous requirements: there are at least two near degenerate levels subject to the condition that the dipole moments from them to another level are not orthogonal. The system presented here is more realistic as compared with the systems with SGC [17, 26–29] because no stringent condition is required.

Our system is a modified version of a tripod four-level atomic system [33], where fluorescence quenching and spectral-line narrowing can be obtained by tuning the detunings of the laser fields. With the coupling of the microwave field, we observe enriched phenomena in this work, such as more narrow peaks in the spectrum. It has been shown that emission

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spectrum can be modified by detunings and phases of laser fields [33, 35,36], while we demonstrate an alternative way for the coherent control of spontaneous emission with the Rabi frequency and detuning of the microwave field. Our work can be extended to other systems such as N-type and M-type atomic systems. We can anticipate more interesting phenomena and flexible control of spontaneous emission by including the coupling of microwave fields.

Finally, we would like to mention the limitation of the current scheme. We obtain the emission spectrum with numerical calculations, and resort to the dressed-state basis for the physics behind the phenomena. In more sophisticated systems, it might be difficult to give explicit expressions of dressed levels and we might need to solve the problem numerically.

4. Conclusion

We have investigated the steady-state spectrum of spontaneous emission from an indirectly driven transition in a four-level atomic system driven by a microwave field and two laser fields. We have obtained a few interesting features such as double narrow lines between two broad bands. We can control the heights, linewidths and splittings of the narrow peaks by tuning the Rabi frequency and detuning of the microwave field. In the mean time, the number and the relative heights of emission peaks can also be modified. The numerical results have been analyzed with the decay rates of the dressed levels.

The phenomena predicted in this paper can be experimentally observed in cold atoms ^{87}Rb. The magnetic sublevels of $5S_{1/2}, F=2$ can serve as the ground states $|1\rangle$ and $|2\rangle$. The excited state $|3\rangle$ and ground state $|4\rangle$ can be provided by $5P_{1/2}, F=2$ and $5S_{1/2}, F=1$. We note that the populations in the ground state $|4\rangle$ can be pumped by the microwave field in our system, so that we do not need processes such as incoherent pumping to maintain the intensity of the fluorescence. Moreover, the spontaneous emission of interest can be spectrally separated from the scattering of the lasers. These features may facilitate the experimental observation.

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