On the delayed self-heterodyne interferometric technique for determining the linewidth of fiber lasers

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Abstract: The delayed self-heterodyne interferometric technique, first proposed in the context of semiconductor lasers, has been commonly used for over 20 years in the determination of the optical linewidth of lasers. We examine this technique in the light of recent work on fiber lasers, and point out further constraints in the applicability of these measurements. An approximate but simple and intuitive expression is provided for assessing the self-heterodyne technique when applied to fiber lasers.

OCIS codes: (140.3510) Lasers, fiber; (300.3700) Linewidth.

References and links

1. T. Okoshi, K. Kikuchi and A. Nakayama, "Novel method for high resolution measurement of laser output spectrum," Electron. Lett. **16**, 630-631 (1980).
2. W. H. Loh, B. N. Samson, L. Dong, G. J. Cowle and K. Hsu, "High performance single frequency fiber grating-based Erbium:Ytterbium-codoped fiber lasers," J. Lightwave Technol., **16**, 114-118 (1998).
3. M. Sejka, P. Varming, J. Hubner and M. Kristensen, "Distributed feedback Er^{3+}-doped fibre laser," Electron. Lett. **31**, 1445-1446 (1995).
4. J. J. Pan and Y. Shi, "166-mW single-frequency output power interactive fiber lasers with low noise," IEEE Photon. Technol. Lett. **11**, 36-38 (1999).
5. C. Spiegelberg, J. Geng, Y. Hu, Y. Kaneda, S. Jiang and N. Peyghambarian, "Low-noise narrow-linewidth fiber laser at 1550nm," J. Lightwave Technol. **22**, 57-62 (2004).
6. L. E. Richter, H. I. Mandelburg, M. S. Kruger and P. A. McGrath, "Linewidth determination from self-heterodyne measurements with subcoherence delay times," IEEE J. Quantum. Electron. **QE-22**, 2070-2074 (1986).
7. C. H. Henry, "Theory of the linewidth of semiconductor lasers," IEEE J. Quantum. Electron. **QE-18**, 259-264 (1982).
8. G. A. Ball, C. G. Hull-Allen and J. Livas, "Frequency noise of a Bragg grating fibre laser," Electron. Lett. **30**, 1229-1230 (1994).
9. K. Kikuchi, "Effect of 1/f-type FM noise on semiconductor laser linewidth residual in high-power limit," IEEE J. Quantum. Electron. **QE-25**, 684-688 (1989).
10. L. B. Mercer, "1/f frequency noise effects on self-heterodyne linewidth measurements," J. Lightwave Technol. **9**, 485-493 (1991).
11. P. Horak, N. Y. Voo, M. Ibsen and W. H. Loh, "Pump-noise induced linewidth contributions in distributed feedback fiber lasers," IEEE Photon. Technol. Lett. (to be published).
12. J. W. Dawson, N. Park and K. J. Vahala, "An improved delayed self-heterodyne interferometer for linewidth measurements," IEEE Photon. Technol. Lett. **4**, 1063-1066 (1992).
13. N. Park, J. W. Dawson and K. J. Vahala, "Linewidth and frequency jitter measurement of an erbium-doped fiber ring laser by using loss-compensated, delayed self-heterodyne interferometer," Opt. Lett. **17**, 1274-1276 (1992).

1. Background

The delayed self-heterodyne interferometer (DSHI) has been widely used for the measurement of laser linewidth since it was introduced in 1980 [1]. Originally used in the context of diode lasers, it has been very effective in characterizing the linewidth behavior of these lasers. Its experimental simplicity has led to its adoption for the measurement of linewidth of other laser

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types, in particular to fiber lasers [2-5], leading to the reporting in some instances of very narrow linewidth fiber lasers. However, in spite of its apparent simplicity, the data that the DSHI yields can be difficult to interpret correctly. It is the purpose of this work to conduct a closer examination on the applicability of this measurement technique, and point out new constraints on the delay parameter when applied to fiber lasers.

2. Analysis

The basic operation behind the delayed self-heterodyne interferometric technique is simple: the laser light is split into two paths, one of which is delayed and the light frequency-shifted with respect to the other before both beams are recombined, and the resulting beat response is measured. The laser linewidth is usually inferred from the width of this beat spectrum. At first glance, it seems reasonable to assume that the measurement will be accurate so long as the delay path is greater than the coherence length of the laser light, as in that case the two combining beams would be effectively uncorrelated. The validity and the resolution $\Delta\nu_{res}$ of the DSHI is thus often cited as

$$\tau_{coherence} << \tau_d \tag{1}$$

$$\Delta\upsilon_{res} \cong c / nL_d = 1/\tau_d \tag{2}$$

where $\tau_{coherence}$ is the laser coherence time and L_d (τ_d) is the fiber delay length (time). The validity of the above relations has been variously analyzed, e.g. see [6]; however, the analyses are generally based on a white frequency noise spectrum as the dominant contribution to the laser linewidth. For diode lasers, this is a valid assumption, as the optical linewidth stems primarily from spontaneous emission-induced refractive index changes in the semiconductor [7]. However, in the case of fiber lasers, the spontaneous emission contribution is extremely small (in the region of Hz [8]), and the linewidth behavior is instead overwhelmingly dominated by other (colored) noise sources.

To understand the consequences of the latter noise behavior on the measurement, we start with the beat spectrum $S(f)$ measured by the DSHI [9]

$$S(f) = \Im[\exp\{-8I(\tau)\}] \tag{3}$$

where

$$I(\tau) = \int_0^\infty S_F(\nu)\frac{\sin^2 \pi\nu\tau}{\nu^2}\sin^2(\pi\nu\tau_d)d\nu \tag{4}$$

$\Im$ denotes the Fourier transform and $S_F(f)$ the frequency (FM) noise spectrum. We first note that, in the case where $S_F(f)$ represents white (frequency-independent) noise, i.e. $S_F(f) = S_{F0}$, the DSHI beat spectrum can be simplified to [10]

$$S(f) \propto \frac{2S_{F0}}{(S_{F0})^2 + 2\pi(f - f_0)^2} \times$$
$$(1 - e^{-S_{F0}\tau_d}(\cos[2\pi\tau_d(f - f_0)] + \frac{S_{F0}}{2\pi(f - f_0)}\sin[2\pi\tau_d(f - f_0)])) \tag{5}$$

where f_o is the laser center frequency. It is easy to see from the exponent in Eq. (5) that the Lorentzian laser lineshape (with the linewidth given simply by $S_{F0} = 1/\tau_{coherence}$) is faithfully reproduced for $\tau_d >> 1/S_{F0}$, providing a sound basis for relation (1).

In the case of fiber lasers, however, there is considerable work indicating that the dominant contribution to the frequency noise and linewidth is not white in nature, but has a low frequency (kHz) spectrum [8, 11], arising in no small part from pump-noise induced temperature fluctuations. Equation (5) is thus not applicable, and it is necessary to examine more closely the adequacy of (1) and (2) in the current context.

For a noise spectrum with a low frequency cut-off f_c, we can to a reasonable approximation restrict the integration in Eq. (4) to the frequency interval $[0, f_c]$. Expanding the term $\sin^2(\pi\nu\tau)/\nu^2$ in powers of τ, we have

$$I(\tau) = \tau^2 \int_0^{f_c} S_F(\nu)\pi^2 \sin^2(\pi\nu\tau_d)d\nu - \tau^4 \int_0^{f_c} S_F(\nu)\frac{\pi^4}{3}\nu^2 \sin^2(\pi\nu\tau_d)d\nu + O(\tau^6) \tag{6}$$

Next, we define a time τ_g by

$$1/\tau_g^2 = \int_0^{f_c} S_F(\nu)\pi^2 \sin^2(\pi\nu\tau_d)d\nu \tag{7}$$

and write Eq. (6) as

$$I(\tau) = \tau^2/\tau_g^2 + O(\tau^4) \tag{8}$$

From Eq. (6) it can be seen that the higher order terms are much smaller than one for $\tau<\tau_g$ if $f_c^2\tau_g^2 << 1$, i.e., if

$$f_c/\bar{S}_F << 1 \tag{9}$$

where

$$\bar{S}_F = \frac{1}{f_c}\int_0^{f_c} S_F(\nu)\sin^2(\pi\nu\tau_d)d\nu \tag{10}$$

can be viewed as an average FM noise level in the relevant frequency range.

On the other hand, for very large values of τ, the term $\sin^2(\pi\nu\tau)$ in Eq. (4) is rapidly fluctuating and averages to ½. We thus find

$$I(\infty) = \frac{1}{2}\int_0^{f_c} S_F(\nu)\sin^2(\pi\nu\tau_d)\frac{1}{\nu^2}d\nu > \frac{1}{2\pi^2 f_c^2\tau_g^2} >> 1 \tag{11}$$

where the last inequality again holds if Eq. (9) is fulfilled.

Generally, the function $I(\tau)$ will increase from $I(\tau_g)=1$ to much larger values with increasing τ. However, the exact behavior will depend on the form of $S_F(f)$ and $I(\tau)$ may show some oscillations in the intermediate regime. An example of this behavior is found if the noise spectrum is modeled simply as a low pass function with a single cut-off frequency, i.e.

$$\begin{aligned} S_F(f) &= S_{F0} \qquad &\text{for } f < f_c \\ &= 0 \qquad &\text{for } f > f_c \end{aligned} \tag{12}$$

$I(\tau)$ is readily computed for arbitrary values of τ_d, as shown in Fig. 1. In all cases, $I(\tau)$ is parabolic for $\tau f_c<1/\pi$ and approaches a constant for large τ. The oscillations for intermediate times never exceed about 20% of the asymptotic value.

Summarizing the above discussion, we may therefore rewrite Eq. (3) to a good approximation as

$$\exp\{-8I(\tau)\} \cong \exp\{-8\tau^2/\tau_g^2\} \tag{13}$$

for *all* values of τ, with only exponentially small corrections. The laser spectrum is thus given by

$$S(f) \cong \Im[\exp\{-8\tau^2/\tau_g^2\}] \tag{14}$$

which corresponds to a Gaussian lineshape.

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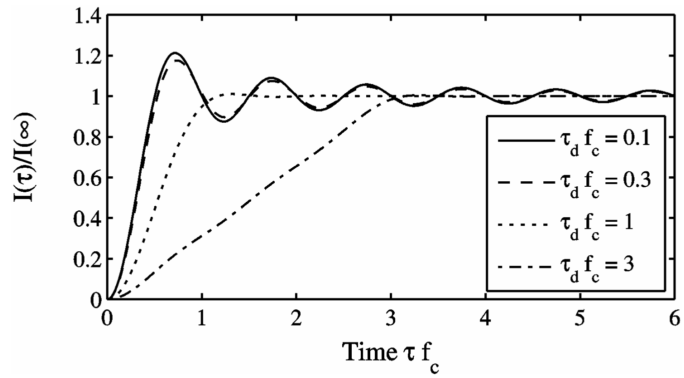

Fig. 1. Normalized integral $I(\tau)$ for a simple low-pass noise function $S_F(f)$.

Finally, from Eq. (14), we can obtain the FWHM of the Gaussian frequency spectrum

$$\Delta \upsilon_{1/2} \cong 4\{2\log(2)\int_0^{f_c} S_F(f)\sin^2(\pi f\tau_d)df\}^{1/2} \tag{15}$$

Relation (9) underlines the fact that the Gaussian lineshape is associated with high noise levels over a narrow frequency range (e.g. temperature fluctuations). The Gaussian approximation in turn fails for broadband, low-level noise (e.g. spontaneous emission noise), in agreement with a Lorentzian lineshape for the latter noise behavior.

3. Results and discussion

Equation (15) yields an approximate but intuitive expression that affords very useful insight towards clarifying the validity of the DSHI technique when used to measure the linewidth of fiber lasers. Effectively, we see that the DSHI technique 'samples' the fiber laser frequency noise spectrum via the area under the term $\sin^2(\pi f\tau_d)$. It should be evident, therefore, that for the measurement to be reliable, the delay time τ_d has to be long enough to generate a sufficient number of cycles of $\sin^2(\pi f\tau_d)$ within the frequency noise spectrum. This is illustrated more clearly in Fig. 2.

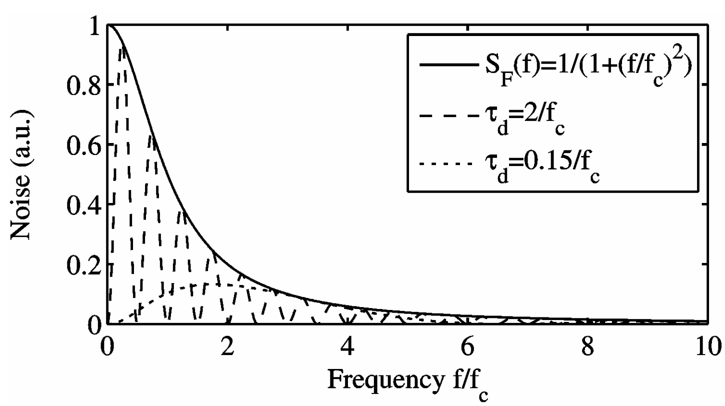

Fig. 2. Sampling of the frequency noise for two different delay lengths. The integral in Eq. (15) is given by the area under the dashed (dotted) curve. It is clear that with the fiber delay time τ_d insufficiently long, the laser frequency noise will be significantly under-estimated, and the resulting linewidth correspondingly under-reported.

As an instructive example, we consider again the case where the laser frequency noise spectrum is modeled as a low pass function with a single cut-off frequency, as in Eq. (12). Equation (15) is then readily integrated to yield the analytic linewidth result

$$\Delta \upsilon_{1/2} = 4\sqrt{\log(2)\, f_c\, S_{F0} (1 - \frac{\sin x}{x})} \tag{16}$$

where $x = 2\pi f_c \tau_d$. This linewidth behavior with delay time is quite distinctive: it initially increases linearly with x (τ_d), and for large $x > \pi$, the linewidth tends towards the final value $(4\log(2) f_c S_{F0})^{1/2}$. The corresponding requirement for τ_d is therefore simply $\tau_d > (2f_c)^{-1}$ which, for a low frequency cut-off of 1 kHz, yields a considerable delay time (fiber length) of 0.5 ms (100 km).

Fiber delay lines of 100km or more are not commonly used in DSHI due to propagation losses. However, a recirculating 11 km delay line incorporating an EDFA for loss compensation was experimentally demonstrated over a decade ago, and applied to characterize a fiber laser [12, 13]. To our knowledge, however, a quantitative explanation for the measured linewidths with loop distance has yet to be adequately provided. It is therefore interesting to compare our current analysis with the data reported there. Figure 3 shows the fit of Eq. (16) with the data in [12]; there is very good general agreement. We note in this case that it takes a fiber delay length of 200km for the linewidth to approach its final value; from Eq. (16), we can also infer that the fiber laser tested had a frequency noise spectrum with a low frequency cut-off f_c ~ 0.5kHz., and a laser FM noise level S_{F0} ~ 11.5 kHz.

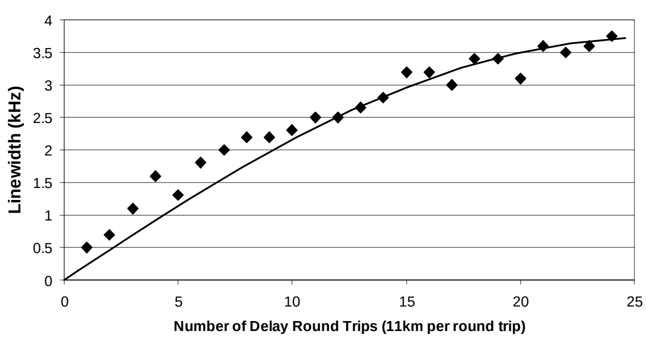

Fig. 3. Dependence of measured linewidth on fiber delay, and comparison to previously reported experimental data. Solid line: Eq. (16). Data points are from Ref. [12].

Our analysis clearly indicates that condition (1) is not sufficient for the DSHI technique to be accurate when applied to the linewidth determination of fiber lasers, as their noise behavior is not frequency-independent. Furthermore, our findings indicate the DSHI measured linewidth may not necessarily be valid even if the fiber laser linewidth is broad due to a high FM noise level S_{F0}, if the condition $\tau_d > (2f_c)^{-1}$ is not satisfied as, unlike a spontaneous emission (or white noise) induced linewidth which is governed by the parameter S_{F0}, the fiber laser linewidth is determined by the product $(f_c S_{F0})^{1/2}$. Therefore, the observation of a linewidth larger than the 'instrument resolution' commonly specified by relation (2) does not in itself validate the accuracy of the measurement, as the delay time may still not be long enough to have fully sampled the frequency noise spectrum, and the actual linewidth may be broader yet.

Finally, we note that in many applications, the noise spectra at very low frequencies is not always of concern, as very slow fluctuations can be tracked or compensated easily. Equation

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(15) is still useful, however, as one can simply replace the lower limit of the integral with a low (non-zero) frequency appropriate to the application, and arrive at the relevant linewidth.

4. Conclusions

We have analyzed the delayed self-heterodyne interferometric technique for measuring the linewidth of fiber lasers. Unlike diode lasers, fiber lasers are not dominated by white noise; instead, their noise characteristic more closely resembles a low pass filter function. The differences between the spectral behavior of the dominant noise sources for fiber lasers and diode lasers call into question the conventional criteria governing the resolution of this measurement technique. We have presented an approximate but simple and intuitive expression for the delayed self-heterodyne measured linewidth and its dependence on delay time, from which additional criteria is inferred for improving the usefulness and accuracy of this approach when used in assessing fiber lasers.

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