

Experimental study of influence of smooth surface reflectance and diffuse reflectance on estimation of root mean square roughness

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Abstract: To estimate the root mean square roughness (σ) of a surface from reflected power, it is necessary to know the diffuse reflectance (DR) and the reflectance (SSR) of a smooth surface made from the same material as the rough surface. In our study, σ is estimated from value of power reflected from one-dimensionally rough steel surfaces in the specular direction without considering SSR and DR. An expression describing dependence of an error of the estimation on SSR and DR is derived. Linear polarized light with $\lambda=660\text{nm}$ and the azimuth of polarization of 49° was used in the experiment. The angle of incidence was varied from 30° to 74° . It was found that absolute relative errors caused by influence of SSR and DR are smaller than 0.03 in the angular ranges of $46\text{--}54^\circ$ and $30\text{--}58^\circ$ for $\sigma=10.2\text{nm}$ and $\sigma = 49.8\text{nm}$, respectively. Out of these ranges, SSR is the main reason for the errors lying in the wide range of $\sim 0.05\text{--}2.5$.

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1. Introduction

Remote optical methods for estimating the root mean square (rms) roughness of a surface from reflected power have been studied in many scientific works. Part of these works (e.g., [1–11]) is based on the assumption that contribution of the diffuse reflectance $R_D(\theta)$ to reflectance $R(\theta)$ in the specular direction is negligible. In this case the general relation for $R(\theta)$ [12,13]:

$$R(\theta) = R_0(\theta)[R_S(\theta) + R_D(\theta)] \quad (1)$$

is reduced to the relation:

$$R(\theta) = R_0(\theta)R_S(\theta), \quad (2)$$

where θ is the angle of incidence defined as the angle between the direction of incident light and the normal to the surface; $R_0(\theta)$ is the reflectance of a smooth surface made from the same material as the rough surface; $R_S(\theta) = \exp\left[-(4\pi\sigma \cos\theta/\lambda)^2\right]$ is the specular reflectance of a perfectly conducting rough surface; σ is the rms roughness of the rough surface; and λ is the wavelength. In [1–11] values of θ and λ were known *a priori* and value of $R_0(\theta)$ was determined experimentally.

In [1–6] value of $R_0(\theta)$ was determined as the ratio between the total scattered power (specularly reflected power and total diffusely scattered power) and incident power. In [7–11] optical constants of the smooth surface were determined from measurements performed for a rough surface and value of $R_0(\theta)$ was calculated from these constants.

In the present paper, the rms roughness is estimated from value of power reflected from one-dimensionally rough steel surfaces in the specular direction without considering the reflectance of the smooth surface and the diffuse reflectance. Influence of the reflectance of the smooth surface and the diffuse reflectance on an error of the estimation of the rms roughness is studied. In Section 2, a relation describing dependence of the estimation error on the reflectance of the smooth surface and the diffuse reflectance is derived from the scattering theory [13] for a one-dimensionally rough surface. In Section 3, influence of the reflectance of the smooth surface and the diffuse reflectance on the estimation error is analyzed from experimental data. The surfaces with the rms roughness of 10.2nm and 49.8nm and linear polarized light with the wavelength of 660nm and the azimuth of polarization of 49° were used in the experiment. The results are summarized in Section 4.

2. Basic relations

As it follows from the scattering theory, the average power $P(\theta, \alpha)$ scattered in the specular direction from a one-dimensionally rough surface having stationary roughness illuminated by linear polarized light can be described by the relation:

$$\begin{aligned} P(\theta, \alpha) &= P_S(\theta, \alpha) + P_D(\theta, \alpha) \\ &= P_1 R_0(\theta, \alpha)[R_S(\theta) + R_D(\theta)], \end{aligned} \quad (3)$$

where α is the azimuth of polarization of the incident light defined as the angle between the polarization plane and the plane of incidence; $P_s(\theta, \alpha)$ is the power of the specularly reflected light; $P_D(\theta, \alpha)$ is the power diffusely scattered in the specular direction; P_I is the incident power; $R_0(\theta, \alpha) = R_{\parallel}(\theta)\cos^2\alpha + R_{\perp}(\theta)\sin^2\alpha$ [14]; the \parallel and \perp symbols correspond to $\alpha = 0$ and $\alpha = 90^\circ$, respectively; the functions $R_{\parallel}(\theta)$ and $R_{\perp}(\theta)$ depend on the angle of incidence and the optical constants.

Taking into account Eq. (3), a logarithmic index $\ln[P(\theta_1, \alpha)/P(\theta_2, \alpha)]$ which describes variation of the function $P(\theta, \alpha)$ in the θ range from θ_1 to θ_2 can be written as:

$$\begin{aligned} \ln\left[\frac{P(\theta_1, \alpha)}{P(\theta_2, \alpha)}\right] &= \ln\left[\frac{R_0(\theta_1, \alpha)}{R_0(\theta_2, \alpha)}\right] \\ &\quad + \left(\frac{4\pi\sigma}{\lambda}\right)^2 (\cos^2\theta_2 - \cos^2\theta_1) + \ln\left[\frac{1+R_D(\theta_1)/R_S(\theta_1)}{1+R_D(\theta_2)/R_S(\theta_2)}\right] \\ &= \left(\frac{4\pi\sigma}{\lambda}\right)^2 (\cos^2\theta_2 - \cos^2\theta_1) [1 + A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)], \end{aligned} \quad (4)$$

where

$$A_0(\theta_1, \theta_2, \alpha) = \frac{\ln[R_0(\theta_1, \alpha)/R_0(\theta_2, \alpha)]}{(4\pi\sigma/\lambda)^2 (\cos^2\theta_2 - \cos^2\theta_1)} \quad (5)$$

and

$$A_D(\theta_1, \theta_2) = \frac{\ln\left\{\left[1+R_D(\theta_1)/R_S(\theta_1)\right]/\left[1+R_D(\theta_2)/R_S(\theta_2)\right]\right\}}{(4\pi\sigma/\lambda)^2 (\cos^2\theta_2 - \cos^2\theta_1)}. \quad (6)$$

Here the function $A_0(\theta_1, \theta_2, \alpha)$ characterizes the influence of the reflectance of the smooth surface on the index $\ln[P(\theta_1, \alpha)/P(\theta_2, \alpha)]$ and the function $A_D(\theta_1, \theta_2)$ is the characteristics of influence of the diffuse reflectance on the index $\ln[P(\theta_1, \alpha)/P(\theta_2, \alpha)]$.

If to assume that the influence of the reflectance of the smooth surface and the diffuse reflectance on $\ln[P(\theta_1, \alpha)/P(\theta_2, \alpha)]$ is weak, that is:

$$|A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)| \ll 1, \quad (7)$$

then Eq. (4) can be written as:

$$\ln\frac{P(\theta_1, \alpha)}{P(\theta_2, \alpha)} \approx \left(\frac{4\pi\sigma}{\lambda}\right)^2 (\cos^2\theta_2 - \cos^2\theta_1). \quad (8)$$

From Eq. (8) we obtain the relation for estimation of the rms roughness:

$$\sigma^*(\theta_1, \theta_2, \alpha) = \frac{\lambda}{4\pi} \sqrt{\frac{\ln[P(\theta_1, \alpha)/P(\theta_2, \alpha)]}{\cos^2\theta_2 - \cos^2\theta_1}}. \quad (9)$$

Also the relation for σ , which follows from Eq. (4), can be written as:

$$\sigma = \frac{\lambda}{4\pi} \sqrt{\frac{\ln[P(\theta_1, \alpha) / P(\theta_2, \alpha)]}{(\cos^2 \theta_2 - \cos^2 \theta_1) [1 + A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)]}}. \quad (10)$$

Then the relative estimation error $\Delta\sigma^*(\theta_1, \theta_2, \alpha)$ characterizing influence of the reflectance of the smooth surface and the diffuse reflectance on the estimation of σ :

$$\Delta\sigma^*(\theta_1, \theta_2, \alpha) = [\sigma - \sigma^*(\theta_1, \theta_2, \alpha)] / \sigma \quad (11)$$

can be written in the final form:

$$\Delta\sigma^*(\theta_1, \theta_2, \alpha) = 1 - \sqrt{1 + A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)}. \quad (12)$$

3. Experimental study

The experimental study was performed for two plane steel samples having one-dimensionally rough surfaces [15]. Statistical characteristics of the surfaces were determined with a Talystep profiler and the TalyProfile 3.2.0 program according to ISO 4288:1996 [16]. The profiler had a shovel-shaped stylus with tip dimensions of $0.1\mu\text{m} \times 2.5\mu\text{m}$ with the $0.1\mu\text{m}$ dimension in the traverse direction perpendicular to the grooves.

For each sample, thirty profiles were measured. The sampling length of the profiles was 0.08mm for the sample #1 and 0.25mm for the sample #2. In the measurements, the Gaussian filter was used and the cut-off wavelength equaled to the sampling length. From profile data, the average rms roughness, the height probability density function, and the height autocorrelation function were estimated. Shape of the estimated functions was close to Gaussian. Value of σ and correlation length T were respectively 10.2nm and 328nm for the sample #1 and 49.8nm and 472nm for the sample #2.

A schematic diagram of a goniometric instrument used for the experimental study is shown in Fig. 1. A light beam ($\lambda=660\text{ nm}$) of a LasirisTM semiconductor laser passed through a circular aperture (2) and a Glan prism (3) and illuminated a steel surface (4). Azimuth of polarization of the laser beam incident on the surface was 49° . Light reflected from the surface passed through a neutral density filter (5) and a circular aperture (6) onto a Hamamatsu S2281-01 silicon diode (7). Output signal of the diode was digitized by a Hamamatsu C9329 photosensor (8) connected to a computer (9). The θ_1 angle was varied from 30° to 70° with the increment of $\Delta\theta = 4^\circ$. The value of θ_2 was $\theta_1 + \Delta\theta$.

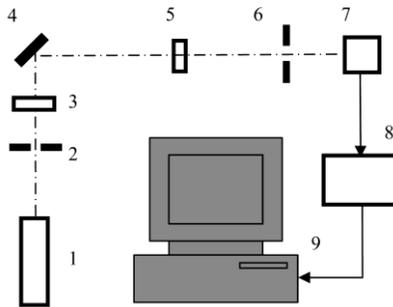


Fig. 1. Schematic diagram of the goniometric instrument: (1) laser, (2) and (6) diaphragms, (3) Glan prism, (4) sample, (5) neutral density filter, (7) silicon diode, (8) photosensor, and (9) computer.

In the experiment, it was measured the $P(\theta_1, \alpha)$ power and the $P(\theta_2, \alpha)$ one reflected in the specular direction and $P_D(\theta_1 - 4^\circ, \alpha)$ power and $P_D(\theta_2 - 4^\circ, \alpha)$ one diffusely scattered out of the specular direction at the $\theta_1 - 4^\circ$ angle and the $\theta_2 - 4^\circ$ one.

We assumed that the scattering theory is valid for the experiment. This assumption was based on the experimental results [2,3,17] obtained for conditions which were close to the conditions for our experiment ($\sigma/\lambda=0.015$ and $\sigma/T=0.031$ for the sample #1, $\sigma/\lambda=0.075$ and $\sigma/T=0.105$ for the sample #2, and the angles of incidence of $30-74^\circ$). The theory was valid for $\sigma/\lambda=0.126$, $\sigma/T=0.053$, and the angle of incidence of 54° in [2], for the σ/λ range of $0.011-0.047$ in the ranges of the angles of incidence of $2.6-75^\circ$ in [3], and for $\sigma/\lambda=0.21$, $\sigma/T=0.109$, and the angles of incidence from 20° to 70° in [17].

The measured values of $P(\theta_1, \alpha)$ and $P(\theta_2, \alpha)$ were used to determine the experimental value of $\sigma^*(\theta_1, \theta_2, \alpha)$ from Eq. (9). Values of $\Delta\sigma^*(\theta_1, \theta_2, \alpha)$ were calculated from Eq. (11) with use of the σ values obtained with the profiler and the values of $\sigma^*(\theta_1, \theta_2, \alpha)$ determined from Eq. (9). The values of $A_0(\theta_1, \theta_2, \alpha)$ and $A_D(\theta_1, \theta_2)$ were also determined experimentally. Assuming that $P_D(\theta_1, \alpha) = P_D(\theta_1 - 4^\circ, \alpha)$ and $P_D(\theta_2, \alpha) = P_D(\theta_2 - 4^\circ, \alpha)$, values of the terms of Eqs. (5) and (6) were determined from the relations:

$$\ln \frac{R_0(\theta_1, \alpha)}{R_0(\theta_2, \alpha)} = \ln \frac{P_S(\theta_1, \alpha)}{P_S(\theta_2, \alpha)} - \left(\frac{4\pi\sigma}{\lambda} \right)^2 (\cos^2 \theta_2 - \cos^2 \theta_1), \quad (13)$$

$$\frac{R_D(\theta_1)}{R_S(\theta_1)} = \frac{P_D(\theta_1, \alpha)}{P_S(\theta_1, \alpha)}, \quad (14)$$

$$\frac{R_D(\theta_2)}{R_S(\theta_2)} = \frac{P_D(\theta_2, \alpha)}{P_S(\theta_2, \alpha)}, \quad (15)$$

where

$$P_S(\theta_1, \alpha) = P(\theta_1, \alpha) - P_D(\theta_1 - 4^\circ, \alpha) \quad (16)$$

and

$$P_S(\theta_2, \alpha) = P(\theta_2, \alpha) - P_D(\theta_2 - 4^\circ, \alpha). \quad (17)$$

Figure 2 shows the experimental values of $\Delta\sigma^*(\theta_1, \theta_2, \alpha)$ corresponding to the experimental values of $A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)$ and the plot of the theoretical dependence of $\Delta\sigma^*(\theta_1, \theta_2, \alpha)$ on $A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)$ described by Eq. (12). As we can see from Fig. 2, values of the estimation errors calculated from Eq. (12) are in good agreement with that obtained experimentally.

In Fig. 3, the experimental values of the functions $\Delta\sigma^*(\theta_1, \theta_2, \alpha)$, $A_0(\theta_1, \theta_2, \alpha)$, $A_D(\theta_1, \theta_2)$, and $A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)$ are presented for the samples #1 and #2.

As we can see from Figs. 3(a) and 3(b), the inequality $|A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)| < 0.06$ is fulfilled in the θ_1 ranges of $46-54^\circ$ and $30-58^\circ$ for $\sigma=10.2\text{nm}$ and $\sigma=49.8\text{nm}$, respectively. For

these θ_1 ranges, value of $|\Delta\sigma^*(\theta_1, \theta_2, \alpha)|$ is smaller than 0.03. That shows that influence of the reflectance of the smooth surface and the diffuse reflectance on the estimation of the

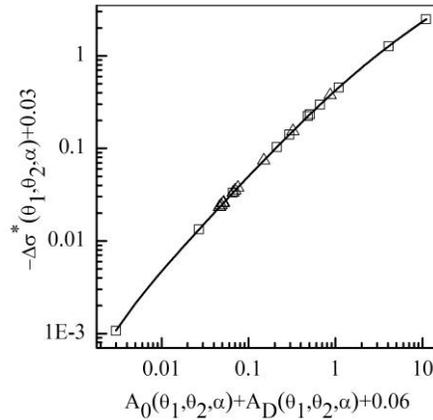


Fig. 2. Theoretical values of $\Delta\sigma^*(\theta_1, \theta_2, \alpha)$ obtained from Eq. (12) (solid line) and the experimental values of $\Delta\sigma^*(\theta_1, \theta_2, \alpha)$ corresponding to the experimental values of $A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)$. The square and triangle symbols correspond to the experimental values for $\sigma = 10.2\text{nm}$ and $\sigma = 49.8\text{nm}$, respectively.

rms roughness is weak in these θ_1 ranges. Out of these θ_1 ranges, the $|\Delta\sigma^*(\theta_1, \theta_2, \alpha)|$ values are in the wide range of ~ 0.05 - 2.5 . Since $|A_0(\theta_1, \theta_2, \alpha)| \gg |A_D(\theta_1, \theta_2)|$, we can consider that the main contribution to these estimation errors is connected with the reflectance of the

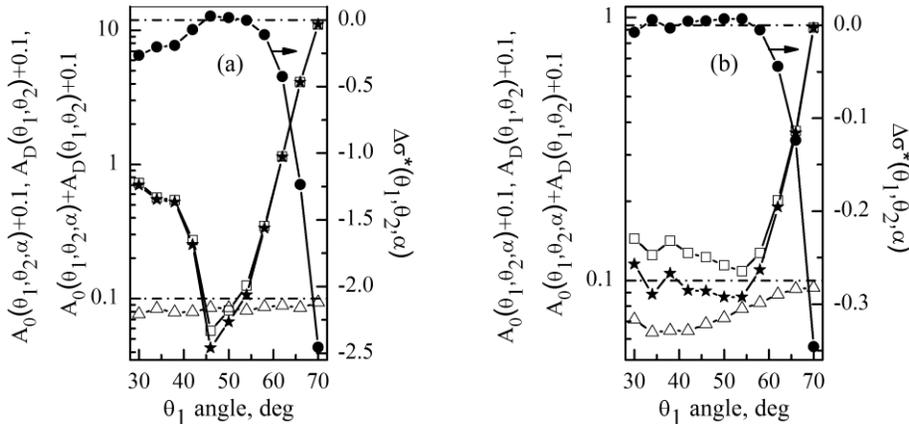


Fig. 3. Experimental values of $A_0(\theta_1, \theta_2, \alpha)$, $A_D(\theta_1, \theta_2)$, $A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)$, and $\Delta\sigma^*(\theta_1, \theta_2, \alpha)$ for $\sigma = 10.2\text{nm}$ (a) and $\sigma = 49.8\text{nm}$ (b). The , , , 8, and x symbols correspond to $\Delta\sigma^*(\theta_1, \theta_2, \alpha)$, $A_0(\theta_1, \theta_2, \alpha)$, $A_D(\theta_1, \theta_2)$, and $A_0(\theta_1, \theta_2, \alpha) + A_D(\theta_1, \theta_2)$, respectively.

smooth surface. Note that increase of σ leads to decrease of this contribution and increase of contribution of the diffuse reflectance to $|\Delta\sigma^*(\theta_1, \theta_2, \alpha)|$.

4. Conclusions

It was studied estimation of the rms roughness from value of power reflected from one-dimensionally rough steel surfaces in the specular direction without considering the reflectance of the smooth surface and the diffuse reflectance. An expression describing dependence of an error of the estimation on the reflectance of the smooth surface and the diffuse reflectance was derived from the scattering theory for a one-dimensionally rough surface. Influence of the reflectance of the smooth surface and the diffuse reflectance on the estimation error was studied experimentally for the one-dimensionally rough steel surfaces. Linear polarized light with the wavelength of 660nm and the azimuth of polarization of 49° was used. The angle of incidence was varied from 30° to 74°. The dependence of the estimation error on the reflectance of the smooth surface and the diffuse reflectance obtained experimentally was in good agreement with that obtained from the derived expression. It was found that absolute relative errors caused by influence of the reflectance of the smooth surface and the diffuse reflectance are smaller than 0.03 in the angular ranges of 46-54° and 30-58° for $\sigma = 10.2\text{nm}$ and $\sigma = 49.8\text{nm}$, respectively. Out of these ranges, the reflectance of the smooth surface is the main reason for the errors lying in the wide range of ~0.05-2.5. It was also found that the errors caused by the reflectance of the smooth surface decrease with increasing the rms roughness.

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