Constellation of phase singularities in a speckle-like pattern for optical vortex metrology applied to biological kinematic analysis

Wei Wang,[1,*] Yu Qiao,[2] Reika Ishijima,[3] Tomoaki Yokozeki,[3] Daigo Honda,[3] Akihiro Matsuda,[3] Steen G. Hanson,[4] and Mitsuo Takeda[3]

[1]*Department of Mechanical Engineering, School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh, EH14 4AS, United Kingdom*
[2]*Department of Frontier Informatics, Graduate School of Frontier Sciences, The University of Tokyo, 5-1-5, Kashiwanoha, Kashiwa-shi, Chiba, 277-8561, Japan*
[3]*Laboratory for Information Photonics and Wave Signal Processing, Department of Information and Communication Engineering, The University of Electro-Communications,1-5-1, Chofugaoka, Chofu, Tokyo, 182-8585, Japan*
[4]*DTU Fotonik, Department of Photonics Engineering, P. O. Box 49, DK-4000, Roskilde, Denmark*
[*]*Corresponding author: w.wang@hw.ac.uk*

Abstract: A novel technique for biological kinematic analysis is proposed that makes use of the pseudophase singularities in a complex signal generated from a speckle-like pattern. In addition to the information about the locations and the anisotropic core structures of the pseudophase singularities, we also detect the spatial structures of a cluster of phase singularities, which serves as a unique constellation characterizing the mutual position relation between the individual pseudophase singularities. Experimental results of *in vivo* measurements for a swimming fish along with its kinematic analysis are presented, which demonstrate the validity of the proposed technique.

OCIS codes: (120.4290) Nondestructive testing; (110.4153) Motion estimation and optical flow; (170.3010) Image reconstruction technique; (280.1415) Biological sensing and sensors.

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1. Introduction

In modern design of engineering systems, the transfer of technology from natural life forms to synthetic constructs is often advantageous because evolutionary pressure typically forces natural systems to become highly optimized and efficient [1]. To imitate mechanisms found in nature successfully, the in vivo measurement of dynamic structures plays a key role in both the field of mechanics and, to a lesser extent, in the field of biology. This objective has previously been pursued with partial success by establishing fix points on the object’s surface as markers to be followed later. Needless to say, the rather arbitrarily placing of these points will inevitably influence the measurement result. Furthermore, these methods will usually not possess the desired accuracy. Meanwhile, dynamic measurements by using speckle techniques have been explored extensively, and considerable progress has been achieved due to its theoretical interest and practical importance [2,3]. Although many attempts have been made to extend the applications of speckle metrology, the most difficult issue for its biological applications lies in the fact that the structures, themselves, are flexible, and thus are not easily followed by standard cross-correlation techniques. Let alone, that the entire structure might undergo rotation and maybe even scaling during movement. Finally, it is usually of importance to track the dynamical behavior of parts of the sample individually. Therefore, the need for establishing an autonomous method for tracking the dynamics of biological specimen with a high accuracy becomes obvious with an increase of interests in bionics.

The concept of phase singularities in optical fields has during the last two decades been a subject for intensive studies [4,5]. These points in a coherent field are positioned where the intensity vanishes, and thus the phase of the field is undetermined. Because the phase singularities hinder unique phase unwrapping of the measured phase map, many efforts have been concentrated on how to avoid such nuisance in interferometry [6,7]. Contrary to the general belief in optical metrology, we have proposed a new technique, referred to as Optical Vortex Metrology (OVM), which makes use of phase singularities. The technique was based on the fact that phase singularities are well-defined geometrical points with unique core structures, which serve as their fingerprints and endow the phase singularities with valuable information and capability as identifiable optimal markers [8-11]. Though we have demonstrated a high spatial resolution and flexibility with large dynamic range, the performance of OVM are often affected by the instability of the phase singularities due to the inherent speckle decorrelation.

In this paper, we will show the benefits of converting a sequence of real-valued images of a dynamic structure into a corresponding sequence of complex analytical fields, from which only the information of the phase singularities will be utilized for dynamic analysis. In

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addition to the information about the anisotropic core structure for an individual phase singularity as its unique fingerprint for reliable identification, we also detect a group of optical vortices (singularity clusters) with specific mutual spatial structure to further strengthen an unambiguous tracking over the entire field of view.

First, a series of images of a biological specimen, here a swimming fugu fish, can be transformed. Then, the constellation of phase singularities will be identified and tracked during the specimen's motion across the entire field of view of the camera. Not only can the position and the velocity be monitored, but it will be shown that also the rotation, the scaling and the biological kinematic analysis can be derived. Finally, the tracking of individual parts of the biological specimen will be depicted. This article will consist of a theoretical description of the detection of singularity clusters and a kinematic analysis, followed by the experimental demonstration on a living specimen, ending with a conclusion.

2. Principle

2.1 Complex signal representation of a speckle-like pattern

Usually, ordinary real-valued images do not *per se* have phase singularities, as they are not complex analytical functions. But based on the real-valued image one can construct a unique complex analytical field by deriving the imaginary part based on a Hilbert transform of the intensity, and subsequently merging the real- and imaginary parts. Unfortunately, this way of creating an analytical function is asymmetrical with respect to the coordinates, and thus less useful [8]. A rotationally symmetric transformation (the Riesz transform or Laguerre-Gauss transform) has been applied for a complex signal representation and will better serve our purpose [9-11]. Having converted the incoherent image into an analytical field facilitates the finding of the singularities, which uniquely depicts the structure under investigation. Before explaining the proposed improvement to optical vortex metrology for biological kinematic analysis, we first briefly review the two-dimensional isotropic complex signal representation of a speckle pattern along the line of our previous investigation.

Assume we are given a real-valued speckle pattern $g(x,y)$, then we can represent $g(x,y)$ by a complex signal $\tilde{g}(x,y)$ through an isotropic filter. Thus, our definition is

$$\tilde{g}(x,y) = \left|\tilde{g}(x,y)\right| \exp[j\theta(x,y)] = g(x,y) * \mathbb{LG}(x,y)\,, \tag{1}$$

where $*$ denotes the convolution operation, and $\mathbb{LG}(x,y)$ is a Laguerre-Gauss (L-G) function in the spatial signal domain,

$$\begin{aligned}\mathbb{LG}(x,y) &= (j\pi^2\omega^4)(x+jy)\exp[-\pi^2\omega^2(x^2+y^2)] \\ &= (j\pi^2\omega^4)[r\exp(-\pi^2 r^2\omega^2)\exp(j\alpha)]\,.\end{aligned} \tag{2}$$

with $r=\sqrt{x^2+y^2}$, $\alpha = \arctan(y/x)$ being the spatial polar coordinates. Alternatively, the complex signal $\tilde{g}(x,y)$ may also be calculated by means of the inverse Fourier transform of the spectrum $G(f_x,f_y)$ of $g(x,y)$ multiplied by a Laguerre-Gauss filter. After straightforward algebra, we have

$$\tilde{g}(x,y) = \mathbb{F}^{-1}\left\{LG(f_x,f_y)\cdot G(f_x,f_y)\right\}. \tag{3}$$

Here, $\mathbb{F}^{-1}$ is inverse Fourier transform, $LG(f_x,f_y)$ is a Laguerre-Gauss filter in the frequency domain defined as follows:

$$LG(f_x,f_y) = (f_x + jf_y)\exp\left[-(f_x^2+f_y^2)/\omega^2\right] = \rho\exp(-\rho^2/\omega^2)\exp(j\beta)\,, \tag{4}$$

where $\rho=\sqrt{f_x^2+f_y^2}$, $\beta=\arctan(f_y/f_x)$ are the polar coordinates in the spatial frequency domain. Because the spiral phase in L-G filter has its unique property of a signum function along any section through the origin [12,13], a spatial isotropy exists in the generated complex signal representation for a real-valued image. It is also readily seen from Eq. (1) and (4) that

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the generated complex signal $\tilde{g}(x, y)$ can be referred to as *Laguerre-Gauss signal* due to its origin from a Laguerre-Gauss filter.

The phase associated with $\tilde{g}(x, y)$ is referred to as the pseudophase to distinguish it from the true phase of the laser speckle field. The complex signal representation has the advantage that it permits one to use the associated pseudophase information for general random patterns other than laser speckles, such as a printed or projected random dot pattern or the random texture found on a natural object. Therefore, such transform of an incoherent image into a complex signal has the versatility that expands applications beyond those known for laser speckle metrology.

2.2 Constellation of pseudophase singularities for dynamic decomposition

Similar to a random speckle intensity pattern serving as marks on a coherently illuminated rough surface, many randomly distributed pseudophase singularities in the phase map of the complex signal representation also imprint unique marks on the object surface. When observed with focus on the object surface, the displacement of each phase singularity has a direct relation to the local displacement of an object surface. From the detected information of pseudophase singularities, we can trace and measure a complicated displacement of the object.

Usually, the phase change around pseudophase singularities are not uniform, and the typical core structure around the phase singularities is strongly anisotropic [14]. To identify the corresponding singularities before and after displacement, recently, we have adopted the ellipticity, zero crossing angle, and vorticity as singularity fingerprint [10]. However, these *ad hoc* parameters are not the most natural measures giving the best performance. More recently, we proposed to use an alternative set of parameters, known as the Pseudo-Stokes parameters, for characterizing the anisotropy of phase singularities [11]. These Pseudo-Stokes parameters and its associated Poincaré sphere representation provide a more natural and elegant description of the anisotropic phase singularity with a mathematical analogy to the Stokes parameters in optical polarization [15-17]. Though we have demonstrated the validity of Optical Vortex Metrology based on the anisotropic core structures of phase singularities as fingerprints for identification, the performance was affected or undermined due to the instability of phase singularities stemming from the decorrelation in the speckle-like pattern.

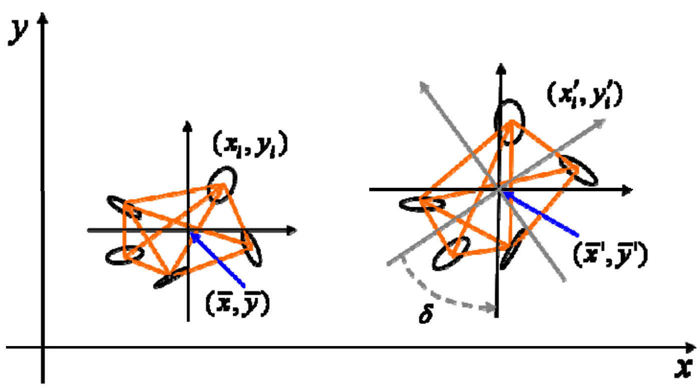

Fig. 1. Schematic diagram for the constellation of pseudophase singularities. (a) Before movement; (b) After movement.

To solve this problem, we propose to detect the spatial structures of a group of the pseudophase singularities in addition to the information about the location and the core structures of the individual phase singularities. As shown in Fig. 1, a cluster of pseudophase singularities are connected together to form a specific figure or picture. This spatial structure can serve as a constellation of pseudophase singularities with a fixed configuration that uniquely characterizes the mutual position between the individual phase singularities. Similar to what is the case for stars in a constellation in the sky with different size, brightness and other characteristics, each optical vortex in a constellation of pseudophase singularities has its anisotropic core structure. After identifying the corresponding phase singularities of the speckle pattern after displacement through their core structures, we can trace a cluster of pseudophase singularities, and therefore conduct the kinematic analysis based on the spatial structures in the constellation of singularities. Just as a stellar constellation will experience translation, rotation and scaling during different seasons of a year or as viewed from different places on the earth, the constellation of pseudophase singularities will have similar movements and maintain its particular configuration. It is this constancy and stability of the configuration in a constellation of phase singularities that has made possible the decorrelation-robust tracing with high reliability for a complicated movement of the specimen.

Let (x_i, y_i) , (x_i', y_i') be the coordinates of the i -th pseudophase singularity in one constellation before and after movement, respectively. Making use of the theory of elasticity [18], it is possible to decompose a complicated movement into a sum of translation, rotation and deformation (scaling). That is

$$\begin{pmatrix} x'_i - \bar{x}' \\ y'_i - \bar{y}' \end{pmatrix} = \beta \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} x_i - \bar{x} \\ y_i - \bar{y} \end{pmatrix}, \tag{5}$$

where β is a scale factor, δ is a rotation angle. Here, $(\bar{x}, \bar{y})$ and $(\bar{x}', \bar{y}')$ can be considered as the coordinates of the gravity centers before and after movement given by

$$\bar{x} = \sum\nolimits_i x_i \big/ N \text{, and } \bar{y} = \sum\nolimits_i y_i \big/ N \text{;} \tag{6}$$

$$\bar{x}' = \sum\nolimits_i x_i' \big/ N \text{, and } \bar{y}' = \sum\nolimits_i y_i' \big/ N \text{;} \tag{7}$$

with N being the total number of pseudophase singularities in this constellation. Therefore, the figure-of-merit for best matching of the phase singularities constellation has been chosen as

$$E = \sum\nolimits_i \left\{ \left[x'_i - \bar{x}' - C(x_i - \bar{x}) - S(y_i - \bar{y}) \right]^2 + \left[y'_i - \bar{y}' + S(x_i - \bar{x}) - C(y_i - \bar{y}) \right]^2 \right\}, \tag{8}$$

where $C = \beta\cos\delta$ and $S = \beta\sin\delta$. On the basis of the least square fitting routine, the condition for E to be a minimum is that $\partial E / \partial C = 0$ and $\partial E / \partial S = 0$. After straightforward algebra, we find that

$$C = \sum\nolimits_i \left(\mathbf{x}_i' \mathbf{x}_i + \mathbf{y}_i' \mathbf{y}_i \right) \Big/ \sum\nolimits_i \left(\mathbf{x}_i^2 + \mathbf{y}_i^2 \right), \tag{8}$$

$$S = \sum\nolimits_i \left(\mathbf{x}_i' \mathbf{y}_i \text{ - } \mathbf{x}_i \mathbf{y}_i' \right) \Big/ \sum\nolimits_i \left(\mathbf{x}_i^2 + \mathbf{y}_i^2 \right), \tag{9}$$

where $\mathbf{x}_i' = x_i' - \bar{x}'$, $\mathbf{x}_i = x_i - \bar{x}$, $\mathbf{y}_i' = y_i' - \bar{y}'$ and $\mathbf{y}_i = y_i - \bar{y}$. From the definition for S and C , we have

$$\beta = \sqrt{S^2 + C^2} \text{ ,} \tag{10}$$

$$\delta = \arctan(S/C)\text{,} \tag{11}$$

$$\Delta x = \bar{x}' - \bar{x} \text{ ,} \tag{12}$$

$$\Delta y = \bar{y}' - \bar{y} \text{ .} \tag{13}$$

Thus, we can conduct the kinematic analysis of an object under a complicated movement from the information of translation $(\Delta x, \Delta y)$, deformation β , and rotation angle δ by tracing the constellation of the pseudophase singularities.

3. Experiments

Experiments have been conducted to demonstrate the validity of the proposed principle for biological kinematic analysis. In the experiments, the biological sample is a swimming fugu fish with a speckle-like intensity pattern on its body surface. The fugu is imaged by a

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COSMICAR television lens ($f = 12.5$ mm) onto the sensor plane of a high-speed camera, FASTCAM-NET 500/1000/Max (PHOTRON) with a pixel size of $7.4\mu m \times 7.4\mu m$. From the nominal magnification of the imaging optics and pixel separation in the high-speed camera, a unit pixel shift in the image plane corresponds to an actual physical displacement of $42.4\mu m$. Under the illumination given by a halogen lamp, sequences of the swimming fugu have been taken at a rate of 30 frames per second. From the recorded images, we generated an isotropic complex signal by Laguerre-Gauss filtering, and retrieved the pseudophase information. Due to the typical doughnut shape in the amplitude of Laguerre-Gauss function serving as a band-pass filter, we can adjust the average speckle size by choosing a proper bandwidth of the L-G filter ω in Eq. (2). In the experiment, we have controlled the density of the pseudophase singularities carefully so that the phase singularities within the fugu body are clearly separated with a large average separation distance. After identifying the corresponding pseudophase singularities inside the fugu by using their core structures as the fingerprints, and determining their coordinates for every two consecutive pictures, we conducted the biological kinematic analysis by the proposed optical vortex metrology.

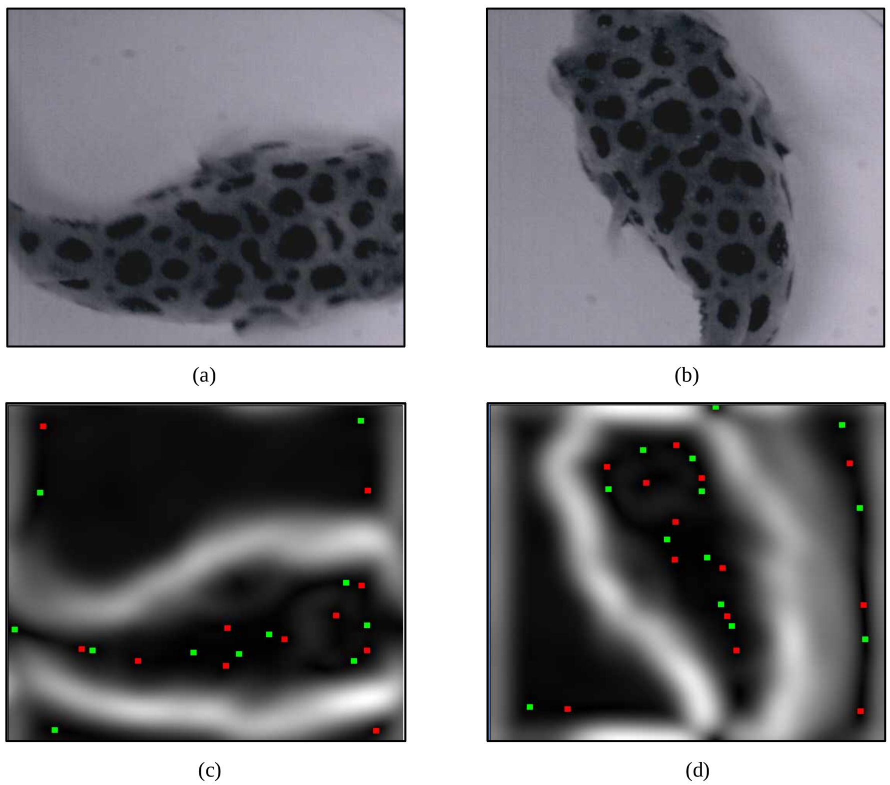

Fig. 2. (Media 1) Recorded images for the swimming fugu at different instants of time and the generated Laguerre-Gauss signals with pseudophase singularities inserted.

Two examples of the recorded temporally separated images for the swimming fugu are represented in Fig. 2(a) and (b). The image in Fig. 2(b) is delayed by 3.3s from that in Fig. 2(a). The corresponding amplitude distributions of the generated complex signals for the recorded images are shown in Fig. 2(c) and (d), where the locations for the positive and negative pseudophase singularities have been indicated by red and green points, respectively. As expected, a big constellation of pseudophase singularities can be observed on the surface of the fugu body through its complex signal representation, and the constellation boundary is readily drawn up due to the edge enhancement effect introduced by the Laguerre-Gauss filter

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[13]. Meanwhile, it is easily observed from Fig. 2 (c) and (d) that the constellation configuration of pseudophase singularities from the L-G signal is in all respects similar to the network of optical vortices and polarization singularities in scalar and vector speckle field [19-20], because the generated complex signal representation shares the same statistical properties as the laser speckle field [21].

In all of these figures, we can observe that the fugu has experienced a large rotation during the recording period, and the pseudophase singularities show structures similarly transformed with respect to each other by an amount that corresponds to the complicated movements of the fugu. Meanwhile, most of the pseudophase singularities can find their correct counterparts, and constitute a stable constellation with specific spatial configuration. The coordinate differences between the corresponding phase singularities inside the constellation gave a good estimation for the swimming fugu, and could be used for biological kinematic analysis as far as we use only the matched phase singularities.

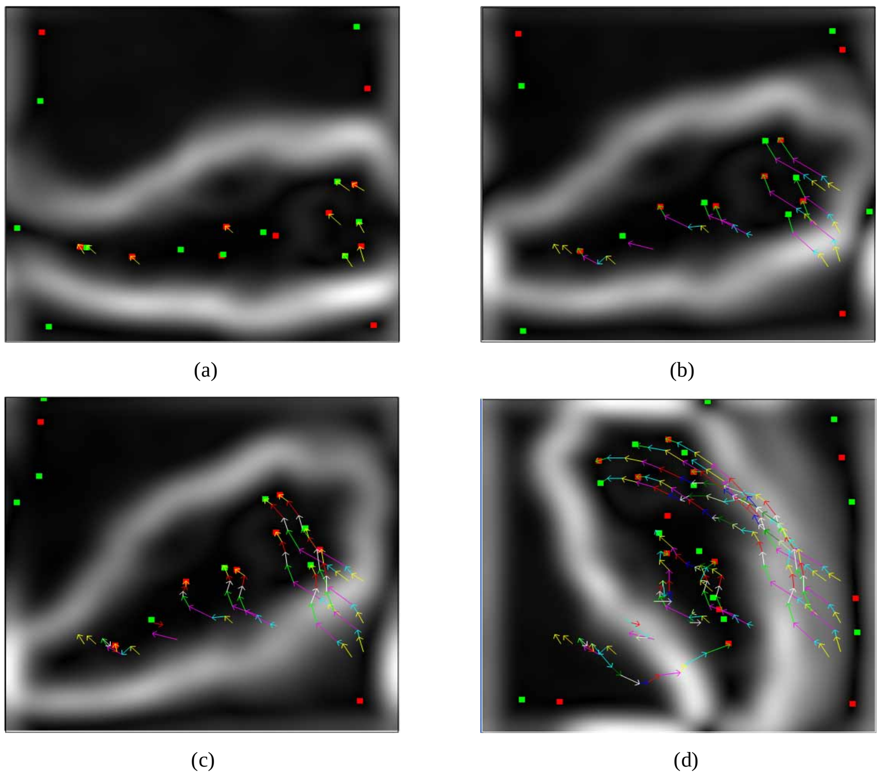

Fig. 3. (Media 2) The trajectory of the pseudophase singularities inside the fugu's body at different instants of time (a) $t = 0.067s$,(b) $t = 0.699s$, (c) $t = 1.665s$, (d) $t = 3.33s$.

However, we also found that some pseudophase singularities failed to find their correct counterparts. This phenomenon may be attributed to the following: Because of the change in the illumination condition and the body distortion during the fugu's swimming, the recorded images begin to change their shapes; in other words, speckle-like decorrelation increases when the time interval between two recorded images becomes large. This gives rise to the creation or annihilation of singularities in the phase map. Meanwhile, the local shape change in the recorded images also introduces distortions of the core structures of the phase singularities. These newly created or annihilated pseudophase singularities definitely have no counterparts in the phase map. Because the majority of pseudophase singularities remain stable within their own constellation, we can slightly adjust the constellation configuration within the fugu body by eliminating those phase singularities without their correct counterparts during the process of kinematic decomposition. Therefore, the constellations of pseudophase singularities are always adaptive with slight change of their configuration for any two consecutive images.

After identifying corresponding pseudophase singularities for each pair of consecutive images making use of their core structures as fingerprints, we can trace the movement of the swimming fugu through its trajectory as shown in Fig. 3, where the arrows indicate their movement directions at different instants of time. As expected for the rotation of fugu's swimming, the trajectory exhibits an arch shape, and body part far from the gravity center of fugu shows a larger arc length. From the coordinate information for each phase singularity in the constellation, we can conduct the *in vivo* measurement and obtain instant information for this swimming fugu about its translation, rotation, and scaling based on Eqs.(9)-(12). Meanwhile, we can also obtain the fugu's dynamic information about the linear and angular velocities from the calculated displacement in translation $(\Delta x, \Delta y)$ and rotation angle δ for every time interval of exposures with $\Delta t = 33.3$ ms.

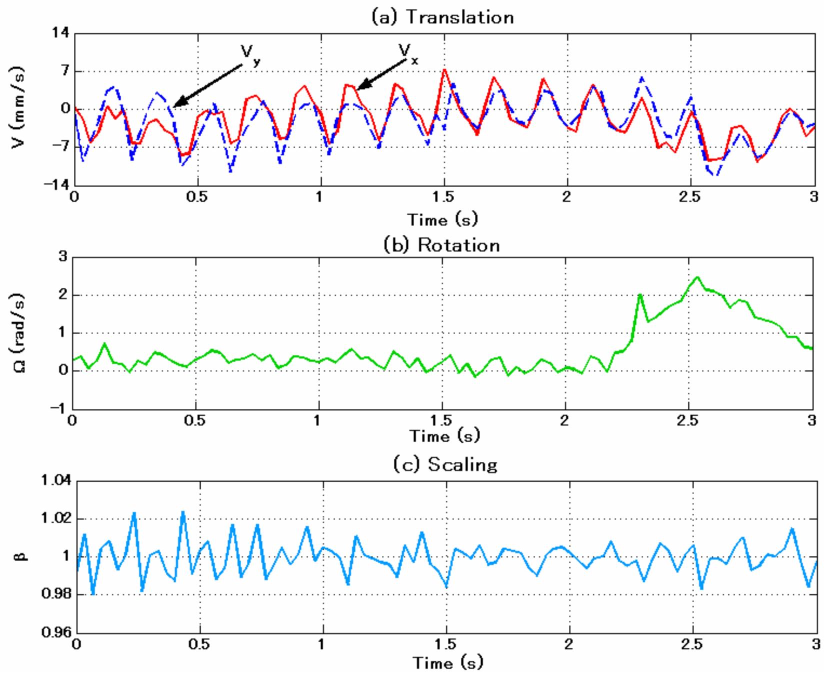

Fig. 4. Movement decomposition based on the pseudophase singularities constellation

Figure 4 shows an example of the movement decomposition for the swimming fugu based on the constellation of phase singularities. As shown in the top of Fig. 4, the linear velocity for translation has a typical periodical structure with an average frequency around 5 hertz, where the solid and dashed line stand for v_x and v_y ($v_x = \Delta x/\Delta t$, $v_y = \Delta y/\Delta t$), respectively. This rhythmic swimming behavior was produced by the cyclical rotation of fugu's fin to provide the internal force through striking water, which resulted in the forward-and-back movements. As expected, it is evident from Fig. 4(b) that this swimming fugu has a large angular velocity $(\Omega = \Delta\delta/\Delta t)$ at the time t equal to $2.6s$. Meanwhile, a slight depth change inside the water can also be observed for the fugu from Fig. 4(c). After calculating the finite difference for the horizontal, vertical velocity components, and the angular velocity, we obtained the variation of the linear and angular accelerations as shown in Fig. 5. Under the reasonable assumption that the mass and the moment of inertia for the fugu are constant

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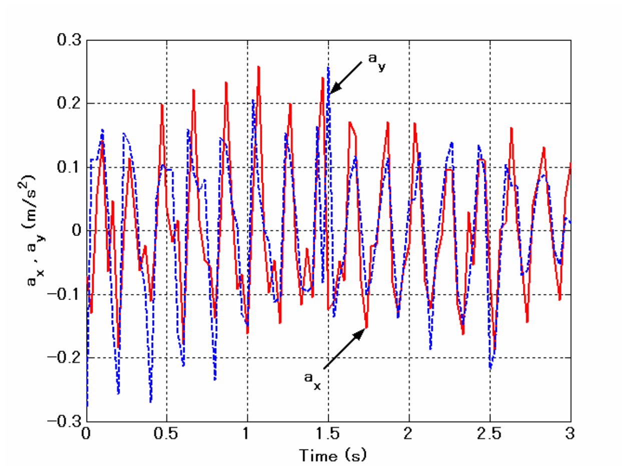
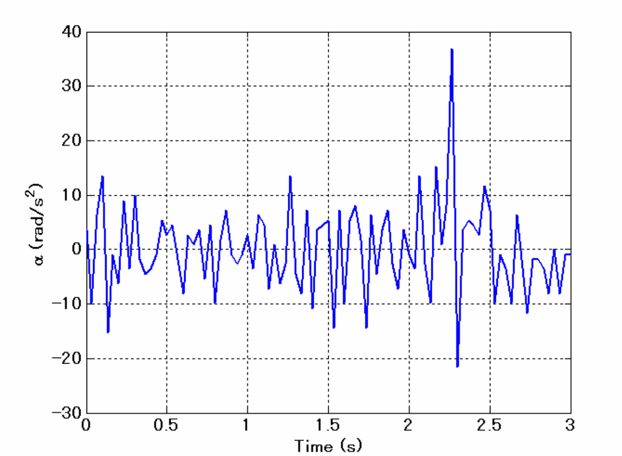
during the recording process, Fig. 5(a) and (b) provide the history for sum force and torque applied to the fugu at the different instants of time, because the linear and angular accelerations are proportional to the force F and torque τ, i.e. $F_x \propto a_x = \Delta v_x / \Delta t$, $F_y \propto a_y = \Delta v_y / \Delta t$, and $\tau \propto \alpha = \Delta\Omega / \Delta t$, respectively. As anticipated for the periodical linear velocity, the fugu, which can be regarded as a driven harmonic oscillator, had experienced a sinusoid-like force to produce an oscillation. From the torque diagram shown in Fig. 5(b), we can also observe the local torque peak corresponding to a quick change of angular velocity around $t = 2.3s$, which stems from the clockwise bending of fugu's tail to give a large rotation. Figures 4 and 5 serve as an experimental demonstration of the validity of the proposed technique for biological kinematic analysis based on the constellation of pseudophase singularities.



(a)



(b)

Fig. 5. History of the sum force (a) and torque (b) applied to the fugu.

4. Conclusion

We have proposed a technique for biological kinematic analysis based on the spatial structures of the constellation of pseudophase singularities in the complex signal generated by Laguerre-Gauss filtering. This singularity constellation uniquely characterizes the mutual position

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relation between the individual phase singularities, and can be used for the purpose of unique identification of a cluster of pseudophase singularities. Experimental results of velocity measurement and kinematic analysis for the swimming fugu are presented, which demonstrate the validity of the proposed technique. Furthermore, the proposed method for decomposition of the biological movement establishes a new relationship between singular optics and biological dynamics, which appears to have been regarded as being based on mutually independent disciplines, and thus introduces new opportunities to explore other potential application of the phase singularities in biological phenomena.

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