Multimode unraveling of master equation and decoherence problem

D. B. Horoshko and S. Ya. Kilin

Institute of Physics, National Academy of Sciences of Belarus
F.Skarina Ave. 70, Minsk 220072 Belarus

horoshko@ifanbel.bas-net.by

Abstract: An unraveling of master equation for a set of fields interfering with one another is developed and conditions are found under which decoherence can be avoided for conditional and unconditional evolution of one of this fields.

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It is well known that a system coupled to a noncoherent thermal reservoir comes after some characteristic time t_{eq} to the state of thermal equilibrium. In quantum physics a system coupled to a reservoir displays yet another fundamental phenomenon: in the timescale much shorter than t_{eq} the density matrix of quantum system becomes diagonal in some basis, defined by the system-reservoir interaction Hamiltonian, and then tends to thermal equilibrium preserving this diagonality [1-3]. This phenomenon, generally known as quantum decoherence, has been widely studied in recent years for such a simple system as single-mode optical or microwave field in an open cavity, for the up to date quantum optical experimental techniques provide a possibility to observe and modify

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this phenomenon with high precision, and from the other side decoherence causes additional difficulties in such applications of quantum optics as quantum cryptography[4] and development of quantum computers [5]. In this work we show that decoherence is not an intrinsic property of a decaying quantum system and study conditions under which it can be completely excluded.

As the first example we consider a single mode of the field in the cavity described by operators of photon annihilation and creation a and a^+ respectively. One of the cavity mirrors is semi-transparent, so that the field in the cavity interacts with the modes of external, initially vacuum field. The density operator of such a system obeys a well-known master equation of Lindblad form:

$$\dot{\rho} = -i[H_a, \rho] + \frac{\gamma_a}{2}\left(2a\rho a^+ - a^+ a\rho - \rho a^+ a\right), \tag{1}$$

where $H = \omega a^+ a$ is a Hamiltonian of internal evolution, ω is the mode frequency, γ_a is the decay constant of the cavity and $\hbar$ is set to unity. The solution of Eq. (1) can be represented as integral over so-called quantum trajectories [6-8]:

$$\rho(t) = \sum_{n=0}^{+\infty} \int_0^t dt_n ... \int_0^{t_2} dt_1 \rho(t|t_1, ..., t_n), \tag{2}$$

where

$$\rho(t|t_1, ..., t_n) = S(t - t_n) J_a S(t_n - t_{n-1}) J_a ... J_a S(t_1)\rho, \tag{3}$$

and the superoperators J_a and $S(t)$ are defined as follows:

$$J_a\rho = \gamma_a a\rho a^+, \tag{4}$$

$$S(t)\rho = e^{(-iH - \gamma_a a^+ a/2)t} \rho e^{(iH - \gamma_a a^+ a/2)t}. \tag{5}$$

The representation given by Eq.(2) is usually called an unraveling of the master equation, Eq. (1). In general any equation

$$\dot{\rho} = L\rho,$$

where L is any Liuvillian superoperator, can be unraveled in the following way:

$$\rho(t) = \sum_{n=0}^{+\infty} \int_0^t dt_n ... \int_0^{t_2} dt_1 e^{(L-J)(t-t_n)} J e^{(L-J)(t_n - t_{n-1})} J ... J e^{(L-J)t_1} \rho, \tag{6}$$

for any superoperator J, choice of which determines the physical sense of such an expansion.

The unraveling given by Eqs. (2-5) obtains its physical meaning if we place outside the cavity a photodetector with unity quantum efficiency collecting all outgoing radiation. In this case $\rho(t|t_1, ..., t_n)$ represents the conditional (non-normalized) density operator of the cavity field given exactly n photons were registered by the photodetector in the time interval $[0, t)$ at times $t_1, ..., t_n$, while its norm $p(t|t_1, ..., t_n) = Tr\{\rho(t|t_1, ..., t_n)\}$ gives the probability of the corresponding outcome. In this interpretation Eq. (2) can be considered just as an averaging over all possible detector records. The structure of Eq. (3) shows that the conditional state of the field under continuous measurement consists of jumps at times $t_1, ..., t_n$ described by superoperator J_a and continuous non-unitary evolution between the counts, described by superoperator $S(t)$. It is easy to see that if the initial state of the field is a pure state $\rho(0) = |\psi_a(0)\rangle \langle\psi_a(0)|$ it remains pure under conditional evolution:

$$\rho(t|t_1, ..., t_n) = \left|\psi_a^{cond}(t)\right\rangle \left\langle\psi_a^{cond}(t)\right|, \tag{7}$$

where

$$\left|\psi_a^{cond}\right\rangle = (\gamma_a)^{n/2} e^{(-iH-\gamma_a a^+ a/2)(t-t_n)} a...a e^{(-iH-\gamma_a a^+ a/2)t_1} |\psi_a(0)\rangle. \tag{8}$$

Because of term $e^{-\gamma_a t a^+ a/2}$ in Eq. (8) all conditional states tend to vacuum (equilibrium) state, however without any decoherence, which arises only as a result of averaging over all possible trajectories, given by Eq. (2). This provides the following interpretation of the decoherence process: due to coupling of the cavity mode to external modes on the semi-transparent mirror, photons leave cavity at some times $t_1, ..., t_n$ and the state of the system decays to vacuum. If we know exactly these times, gathering all photons on the surface of a photodetector, the conditional state of the field preserves its quantum coherence and only the loss of information connected with non-unitary detector efficiency or not perfect photon resolution brings about the decoherence of the system state. Thus, to avoid decoherence caused by interaction with a set of bosonic field modes, it is enough to collect all outgoing radiation on a photodetector and resolve all counts - the conditional state of the system with respect to observed photocount sequence will remain pure.

It seems that such a corpuscular interpretation gives a complete description of field decoherence. However the external field cannot be completely described by photon arrival times $t_1, ..., t_n$, it possesses a fundamental property of interference with other fields and itself. Moreover, in many experiments one is interested in phase properties of the cavity field, which are measured with the use of homodyne techniques, that is the external field undergoes interference with the field of local oscillator before being measured. So we should answer the following question: can we avoid in principle decoherence of the cavity state if the external field is subject to interference with other auxiliary field, so that the times $t_1, ..., t_n$ at which photons left the cavity cannot be more obtained directly by photodetection? The answer on this question is non-trivial and generally depends on the state of the auxiliary field.

Let us suppose that the auxiliary field is produced by another cavity, the single mode of which described by annihilation operator b is initially exited in the state $|\psi_b(0)\rangle$. The external fields of the two cavities are mixed on a symmetric beam-splitter and measured by two photodetectors. The master equation for the joint density operator of modes a and b is

$$\dot{\rho} = -i[H_a, \rho] - i[H_b, \rho] + \frac{\gamma_a}{2}\left(2a\rho a^+ - a^+ a\rho - \rho a^+ a\right) + \frac{\gamma_b}{2}\left(2b\rho b^+ - b^+ b\rho - \rho b^+ b\right), \tag{9}$$

where γ_b and H_b are the decay rate and Hamiltonian of the corresponding cavity. Eq. (9) can be unraveled in such a way that it will provide an expression for the conditional state of two modes for given photocount sequences at two detectors. We make this unraveling using the rule that the jump of the conditional wave function after detecting a photon at a photodetector is given by the operator of the field incident on this detector [7].

Disregarding the phase shifts introduced by beam-splitters we write the field operators on two detectors as

$$E_a = \sqrt{\frac{\gamma_a}{2}} a + \sqrt{\frac{\gamma_b}{2}} b, \tag{10}$$

$$E_b = \sqrt{\frac{\gamma_a}{2}} a - \sqrt{\frac{\gamma_b}{2}} b. \tag{11}$$

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Now a photocount at the first detector brings about reduction

$$\rho \to E_a \rho E_a^+ = J_a \rho, \tag{12}$$

while a photocount at the second causes reduction

$$\rho \to E_b \rho E_b^+ = J_b \rho. \tag{13}$$

Between counts evolution is described by superoperator $S(t)$, satisfying the equation

$$\dot{S}\rho = (L - J_a - J_b)\,\rho = -i[H_a + H_b, \rho] - \frac{\gamma_a}{2}\left(a^+ a\rho + \rho a^+ a\right) - \frac{\gamma_b}{2}\left(b^+ b\rho + \rho b^+ b\right), \tag{14}$$

that is

$$S(t)\rho = e^{-Kt}\rho e^{-K^+ t}, \tag{15}$$

where

$$K = iH_a + iH_b + \frac{\gamma_a}{2}a^+ a + \frac{\gamma_b}{2}b^+ b. \tag{16}$$

We see again that all superoperators J_a, J_b and $S(t)$ preserve the purity of the state. So the conditional state of two modes is a pure state at any time provided that it was pure at the beginning.

Using Eqs.(12), (13) and (15) we can construct a conditional state of two modes for any given sequences of photocounts at the detectors. For example, if one count was observed at the first detector at time t_1, and one at the second detector at time $t_2 > t_1$, the conditional (non-normalized) state at time $t > t_1, t_2$ is

$$\begin{aligned} \left|\psi_{ab}^{cond}(t|t_1;t_2)\right\rangle &= e^{-K(t-t_2)}\left(\sqrt{\frac{\gamma_a}{2}}a - \sqrt{\frac{\gamma_b}{2}}b\right)e^{-K(t_2-t_1)}\left(\sqrt{\frac{\gamma_a}{2}}a + \sqrt{\frac{\gamma_b}{2}}b\right) \\ &\quad \times e^{-Kt_1}\left|\psi_a(0)\right\rangle\left|\psi_b(0)\right\rangle. \end{aligned} \tag{17}$$

This wave function can be considered as a sum of four parts:

$$\begin{aligned} \left|\psi_{ab}^{cond}(t|t_1;t_2)\right\rangle &= \frac{\gamma_a}{2}e^{-K(t-t_2)}ae^{-K(t_2-t_1)}ae^{-Kt_1}\left|\psi_a(0)\right\rangle\left|\psi_b(0)\right\rangle \\ &\quad + \sqrt{\frac{\gamma_a\gamma_b}{4}}e^{-K(t-t_2)}ae^{-K(t_2-t_1)}be^{-Kt_1}\left|\psi_a(0)\right\rangle\left|\psi_b(0)\right\rangle \\ &\quad - \sqrt{\frac{\gamma_a\gamma_b}{4}}e^{-K(t-t_2)}be^{-K(t_2-t_1)}ae^{-Kt_1}\left|\psi_a(0)\right\rangle\left|\psi_b(0)\right\rangle \\ &\quad - \frac{\gamma_b}{2}e^{-K(t-t_2)}be^{-K(t_2-t_1)}be^{-Kt_1}\left|\psi_a(0)\right\rangle\left|\psi_b(0)\right\rangle, \end{aligned} \tag{18}$$

which describe the four possible ways of obtaining one photon at first and one at second detector: (i) both photons came from the cavity a (first term), (ii) first photon came from the cavity b, second came from the cavity a (second term), (iii) vice versa (third term), and (iv) both photons came from the cavity b (forth term). as we can in no way determine the paths of photons the resulting wavefunction is a superposition of four possible wavefunctions.

So the joint conditional state of two modes under continuous measurement does not decohere on its way to equilibrium (vacuum state). However we cannot say it about the mode a alone! The reason is that the states of two modes become entangled, that is the joint state does not factorize:

$$\left|\psi_{ab}^{cond}(t)\right\rangle \neq \left|\psi_a(t)\right\rangle\left|\psi_b(t)\right\rangle. \tag{19}$$

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It is most easily seen from Eq. (17), where each term factorizes, while the sum generally not. If we now average over mode b, the state of mode a becomes a mixture.

When such an entanglement is produced? Obviously not between the counts, as the superoperator $S(t)$ does not entangle states:

$$S(t)\rho_a\rho_b = e^{(-iH_a-\gamma_a a^+a/2)t}\rho_a e^{(iH_a-\gamma_a a^+a/2)t} e^{(-iH_b-\gamma_b b^+b/2)t}\rho_b e^{(iH_b-\gamma_b b^+b/2)t}. \quad (20)$$

It is produced by quantum jumps connected with photon detection, because generally

$$\left(\sqrt{\frac{\gamma_a}{2}}a+\sqrt{\frac{\gamma_b}{2}}b\right)|\psi_a\rangle\,|\psi_b\rangle \neq \left|\psi_a^{'}\right\rangle\left|\psi_b^{'}\right\rangle. \quad (21)$$

In the Appendix we show that the equality in Eq. (21) holds if and only if one of the states $|\psi_a\rangle$ or $|\psi_b\rangle$ is a coherent state. It is easy to see from the implicit form of the superoperators J_a, J_b and $S(t)$ that coherent state of mode a or b remains coherent under conditional evolution of the joint state. So we can state that the joint conditional state of two modes under continuous measurement performed after interference on a beam-splitter is disentangled if and only if one of these modes is in a coherent state initially.

This is just the situation met in a homodyne measurement. It means that if in homodyne measurement of cavity field all photons are gathered on photodetectors with unity quantum efficiencies and all photocounts are resolved, then the conditional state of the cavity field does not decohere, and the cause of decoherence, as in case of direct detection, is the loss of information.

The scheme with two interfering beams considered above can be generalized for the case of arbitrary number of modes. Let us consider n single mode fields of the same frequency ω described by operators a_i, $i=1,...,n$ in n cavities with decay rates γ_i, $i=1,...,n$. Such a system is described by master equation

$$\dot{\rho}=\sum_i\left(-i[H_i,\rho]+\frac{\gamma_i}{2}\left(2a_i\rho a_i^+ - a_i^+a_i\rho-\rho a_i^+a_i\right)\right), \quad (22)$$

where $H_i=\omega a_i^+a_i$ is the Hamiltonian of internal evolution of the ith mode. We suppose that the outgoing fields of the cavities are mixed with one another on a set of beam splitters and the resulting n beams are detected by n photodetectors. The field operators on the detectors E_i^{out} are given by relation

$$E_i^{out}=\sum_j U_{ij}E_j^{in}, \quad (23)$$

where $U_{ij}=U_{ji}^*$ is a unitary matrix, while

$$E_j^{in}=\sqrt{\gamma_i}a_i, \quad (24)$$

is the field operator just outside the cavity. Taking into account that

$$\begin{aligned}\sum_i \gamma_i a_i\rho a_i^+ &= \sum_i E_i^{in}\rho E_i^{in+}=\sum_{ijk}U_{ij}U_{ki}^*E_j^{out}\rho E_k^{out+}\\ &= \sum_{jk}\delta_{jk}E_j^{out}\rho E_k^{out+}=\sum_k E_k^{out}\rho E_k^{out+} \end{aligned} \quad (25)$$

we rewrite Eq. (20) in the following form

$$\dot{\rho}=-i[\sum_i H_i,\rho]+\sum_i E_i^{out}\rho E_i^{out+}-\sum_i\frac{\gamma_i}{2}a_i^+a_i\rho-\rho\sum_i\frac{\gamma_i}{2}a_i^+a_i. \quad (26)$$

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The last equation can be interpreted as follows: observing a photon at the ith detector brings about a reduction $\rho \to E_i^{out} \rho E_i^{out+}$ while between counts the state evolution is described by the superoperator $S(t)$:

$$S(t) = \exp\left\{ \left(-i \sum_i H_i - \sum_i \frac{\gamma_i}{2} a_i^+ a_i \right) t \right\} \rho \exp\left\{ \left(i \sum_i H_i - \sum_i \frac{\gamma_i}{2} a_i^+ a_i \right) t \right\}. \quad (27)$$

The conditional state of n-mode field is again a pure and generally entangled state.

So we have seen how it is possible to avoid decoherence in conditional state evolution. In some cases we can obtain also a non-conditional evolution without decoherence by means of electro-optical feedback between photodetectors and cavity field. Below we consider how this can be done in a special case of decaying superposition of coherent states in an open cavity.

We consider again a single mode cavity with photon annihilation operator a and decay rate γ_a. Initially the field inside the cavity is exited in a superposition of coherent states with amplitudes α and $-\alpha$ (a so-called Schrödinger cat state [10])

$$|\psi_a(0)\rangle = N\left(|\alpha\rangle + e^{i\varphi}|-\alpha\rangle\right), \quad (28)$$

where φ is arbitrary real phase and N is a normalization constant. The conditional evolution of the state vector of the field under continuous measurement is given by Eq. (8). Substituting Eq. (28) into Eq. (8) we obtain

$$\left|\psi_a^{cond}(t)\right\rangle = (\sqrt{\gamma_a}\alpha)^n N\left(\left|\alpha e^{-\gamma_a t/2 - i\omega t}\right\rangle + (-1)^n e^{i\varphi}\left|-\alpha e^{-\gamma_a t/2 - i\omega t}\right\rangle\right), \quad (29)$$

where n is number of counts in the time interval $[0, t)$. Eq. (29) shows that the conditional state of field remains a superposition of two coherent states with altering sign between them [11]. In contrast to this the unconditional evolution of field, i.e. the solution of Eq. (1) for initial state given by Eq. (29) gives fast decoherence of the field into statistical mixture of states $|\alpha\rangle\langle\alpha|$ and $|-\alpha\rangle\langle-\alpha|$. Decoherence again arises as a result of averaging over all possible detector records, however in this special case the conditional state does not depend on times $t_1, ..., t_n$ of photon arrivals, but only on total number of photons in the interval $[0, t)$, or more precisely there are only two possible wave functions $\left|\psi_a^{cond}(t)\right\rangle$: one for even and one for odd values of n, which differ only by sign between states $|\alpha\rangle$ and $|-\alpha\rangle$. When a photon leaves the cavity this sign is switched to the opposite value. This sign switching has a simple physical interpretation only for the case of $\varphi = \frac{\pi}{2}$, when it corresponds to shifting the phase of the intracavity field by π, and can be realized by the phase shift operator $\exp\{i\pi a^+ a\}$:

$$e^{i\pi a^+ a}\frac{1}{\sqrt{2}}\left(|\alpha\rangle + i|-\alpha\rangle\right) = \frac{1}{\sqrt{2}}\left(|-\alpha\rangle + i|\alpha\rangle\right) = \frac{i}{\sqrt{2}}\left(|\alpha\rangle - i|-\alpha\rangle\right). \quad (30)$$

The last relation shows how we can convert a conditional non-decohering evolution into unconditional non-decohering one [12]. It can be done by means of electro-optical modulator placed inside the cavity, controlled by photocurrent in such a way that each photocount is followed by a field phase shift by π, which must be performed during a time small compared to cavity photon lifetime γ_a^{-1} and average distance between photocounts $\gamma_a^{-1}|\alpha|^{-2}$. In such a scheme the conditional state of the field is given, instead of Eq. (8), by the following expression:

$$\left|\psi_a^{cond}(t)\right\rangle = (\gamma_a)^{n/2} e^{(-iH - \gamma_a a^+ a/2)(t - t_n)} e^{i\pi a^+ a} a ... e^{i\pi a^+ a} a e^{(-iH - \gamma_a a^+ a/2)t_1} |\psi_a(0)\rangle, \quad (31)$$

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which gives for initial condition Eq. (28)

$$\left|\psi_a^{cond}(t)\right\rangle = (\sqrt{\gamma_a}\alpha)^n \frac{1}{\sqrt{2}}\left(\left|\alpha e^{-\gamma_a t/2 - i\omega t}\right\rangle + i\left|-\alpha e^{-\gamma_a t/2 - i\omega t}\right\rangle\right). \tag{32}$$

Now conditional state does not depend on detector records at all. So to obtain an unconditional state we should only normalize it:

$$\left|\psi_a(t)\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\alpha e^{-\gamma_a t/2 - i\omega t}\right\rangle + i\left|-\alpha e^{-\gamma_a t/2 - i\omega t}\right\rangle\right) \tag{33}$$

i.e. the decaying state inside the cavity remains a superposition of coherent states with opposite amplitudes. Some of the difficulties which a practical realization of this method can meet, such as non-unity efficiency of photodetector, are discussed in Ref.[12].

In general case we may try to find such a feedback, which will preserve the state purity for any given Hamiltonian H and initial state $|\psi_a(0)\rangle$. We may describe the effect of instant feedback by some operator U and write a more general form of Eq. (31):

$$\left|\psi_a^{cond}(t)\right\rangle = (\gamma_a)^{n/2}\, e^{(-iH - \gamma_a a^+ a/2)(t-t_n)} U a ... U a e^{(-iH - \gamma_a a^+ a/2)t_1}\left|\psi_a(0)\right\rangle. \tag{34}$$

Now we need the field state (normalized) at time t to be independent of photon arrival times t_n and photocount number n, that is

$$\left|\psi_a^{cond}(t|n, t_1, ...t_n)\right\rangle = c(t|n, t_1, ...t_n)\left|\psi_a^{'}(t)\right\rangle, \tag{35}$$

where $c(t|n, t_1, ...t_n)$ is a c-number defining the probability of the corresponding count sequence and $\left|\psi_a^{'}(t)\right\rangle$ is some vector depending only on time. Letting $n = 0$ in Eq. (35) we find that $\left|\psi_a^{'}(t)\right\rangle$ can be chosen as

$$\left|\psi_a^{'}(t)\right\rangle = e^{(-iH - \gamma_a a^+ a/2)t}\left|\psi_a(0)\right\rangle. \tag{36}$$

It is readily seen from the structure of Eq. (34) that Eq. (35) is satisfied for any photocount sequence if and only if each of the states $\left|\psi_a^{'}(t)\right\rangle$, $t \in [0, +\infty)$ is an eigenstate of operator Ua:

$$Ua\left|\psi_a^{'}(t)\right\rangle = c(t)\left|\psi_a^{'}(t)\right\rangle, \tag{37}$$

where $c(t)$ is a c-number. So, if for given H and $|\psi_a(0)\rangle$ we find an operator U satisfying Eq. (37) and the corresponding physical process, which can be controlled by feedback loop, then the decoherence of intracavity field induced by decay can be completely removed.

The previous considerations are closely related to approach developed in Ref. [13], where authors consider the possibility to "invert" quantum jumps produced by operator a. For this purpose they propose to use feedback described by operator U, such that Ua is a c-number on a sub-space containing states $\left|\psi_a^{'}(t)\right\rangle$, $t \in [0, +\infty)$, which is much more strong a condition than that required by Eq.(37). The former condition is enough but not necessary for preserving the state purity, as it implies that the non-normalized state given by Eq. (34) is completely independent of photocount arrival times t_n, that is not only the state of the field is independent of these times, but also the probabilities of different count sequences are equal for given n. Contrary to this, Eq. (37) gives the necessary and enough condition, which must be satisfied by feedback operator preserving the intracavity field from decoherence.

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So we have shown using a single-mode field in an open cavity as an example that decoherence is not an intrinsic property of a system coupled to a reservoir and can be avoided if some information about the reservoir state is available. In some cases the necessary information can be much less than the complete information about the reservoir state, and sometimes it can be used to avoid decoherence in unconditional dynamics of the system.

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Appendix

We consider conditions under which equality in Eq. (21) holds for both signs, that is

$$\left(\sqrt{\frac{\gamma_a}{2}}a+\sqrt{\frac{\gamma_b}{2}}b\right)|\psi_a\rangle\,|\psi_b\rangle=\left|\psi_a^{'}\right\rangle\left|\psi_b^{'}\right\rangle, \tag{A1}$$

$$\left(\sqrt{\frac{\gamma_a}{2}}a-\sqrt{\frac{\gamma_b}{2}}b\right)|\psi_a\rangle\,|\psi_b\rangle=\left|\psi_a^{''}\right\rangle\left|\psi_b^{''}\right\rangle \tag{A2}$$

for some states $|\psi_a\rangle, |\psi_b\rangle, \left|\psi_a^{'}\right\rangle, \left|\psi_b^{'}\right\rangle, \left|\psi_a^{''}\right\rangle$, and $\left|\psi_b^{''}\right\rangle$. Multiplying both sides of Eq. (A1) by bra vector $\langle\varphi_a|$ such that $\langle\varphi_a|\psi_a\rangle=0$ we obtain

$$\sqrt{\frac{\gamma_a}{2}}\langle\varphi_a|\,a\,|\psi_a\rangle\,|\psi_b\rangle=\left\langle\varphi_a|\psi_a^{'}\right\rangle\left|\psi_b^{'}\right\rangle.$$

This condition can hold if and only if either (i)

$$\left|\psi_b^{'}\right\rangle=c\,|\psi_b\rangle\,,$$

where c is a non-zero c-number, or (ii)

$$\langle\varphi_a|\,a\,|\psi_a\rangle=\left\langle\varphi_a|\psi_a^{'}\right\rangle=0.$$

In the case (i) Eq. (A1) is rewritten as

$$\left(\sqrt{\frac{\gamma_a}{2}}a+\sqrt{\frac{\gamma_b}{2}}b\right)|\psi_a\rangle\,|\psi_b\rangle=c\left|\psi_a^{'}\right\rangle|\psi_b\rangle\,,$$

which after multiplying both sides by $\langle\psi_a|$ gives

$$\sqrt{\frac{\gamma_b}{2}}b\,|\psi_b\rangle=\left(c\left\langle\psi_a|\psi_a^{'}\right\rangle-\sqrt{\frac{\gamma_a}{2}}\langle\psi_a|\,a\,|\psi_a\rangle\right)|\psi_b\rangle\,,$$

i.e. $|\psi_b\rangle$ is an eigenstate of operator b(coherent state).

In the case (ii) we have: $a\,|\psi_a\rangle\perp|\varphi_a\rangle$ for any $|\varphi_a\rangle\perp|\psi_a\rangle$, therefore $a\,|\psi_a\rangle=c^{'}\,|\psi_a\rangle$ where $c^{'}$ is a c-number. That is $|\psi_a\rangle$ is an eigenstate of operator a(coherent state).

So we have proven that Eq. (A1) holds if and only if at least one of states $|\psi_a\rangle$ or $|\psi_b\rangle$ is a coherent state.

In the same way we find that Eq. (A2) holds under the same conditions.