

Quantum noise evolution under optical Kerr effects and two-photon absorption in a semiconductor waveguide

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Abstract: We theoretically study evolution of quantum noise of ultrashort pulsed light that propagates a semiconductor waveguide where nonlinear optical interaction occurs. Optical quantum noise is simulated by statistical (pseudo-)random distribution of phasors in a phase space with Gaussian probability weight, and each phasor evolution is governed by beam propagation method. It is shown that Kerr effects squeeze quantum noise of coherent light in a phase space such that photon-number noise is unchanged while phase noise increasing with uncertainty area invariant. However, two-photon absorption alters the photon-number statistics of light unlike Kerr effects.

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References and links

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1. Introduction

Squeezed light has been an interesting topic in nonlinear quantum optics since the first experimental realization of squeezed light [1] due to its possible application for overcoming quantum limit imposed by vacuum fluctuation. Squeezed light application includes high-precision spectroscopy, gravitational wave detection and etc [2, 3, 4].

In particular, Kerr effects have been studied for generation of quadrature squeezed light [5, 6]. During propagation of ultrashort pulsed light in a Kerr medium, the nonlinear optical phase becomes proportional to the light intensity.

Kerr effects originating from the third-order optical nonlinearity couple intensity of light with its phase by intensity-dependent refractive index, thereby converting intensity noise into phase noise. This nonlinear conversion creates correlation between amplitude and phase, and thus leads to an uncertainty ellipse with increasing phase noise in a phase space [5, 6]. This fact may be followed by possible expectation that whole photon-number statistics of light can be altered to non-Poissonian one by Kerr effects due to the expected ellipticity of quantum noise distribution in a phase space. However, the experiments show that generation of sub-Poissonian light with semiconductors where multi-photon absorption occurs as well as Kerr effects, was attributed to the multi-photon absorption [7].

In this paper, we present the evolution of phase space uncertainty distribution of ultrashort (1ps) pulsed light for various intensities, which propagates a semiconductor waveguide where Kerr effects occur. Our numerical simulation employs two-dimensional Gaussian (pseudo-)random distribution of phasors in a phase space which represents fluctuation of coherent light. Beam propagation method is used for the evolution of each phasor.

It is found that Kerr effects maintain initial uncertainty area in a phase space but shears the uncertainty circle into an ellipse during nonlinear phase shift. This kind of shearing, however, does not provide sub-Poissonian statistics of light but produces quadrature squeezed light. The fact that the simulation shows the increase in phase uncertainty with photon-number statistics invariant as a result of Kerr effects, implies that photon-number and phase are not canonical conjugate variables defining minimum uncertainty relation.

In order to see change in photon-number statistics of light, two-photon absorption (TPA) is added to the subsequent simulation with one-dimensional random distribution of phasors, given by the Gaussian photon-number distribution of a coherent state. It is shown that TPA reduces photon-number noise with nonlinear transmittance curve, via nonlinear loss mechanism, unlike the cases with Kerr effects only.

2. Simulation

We consider a semiconductor waveguide, e.g., $\text{Al}_{0.15}\text{Ga}_{0.85}\text{As}$ waveguide as a nonlinear medium where Kerr effects occur during propagation of 1 ps pulsed light at just below half the band gap energy corresponding to 1550 nm wavelength. Tuning incident photon energy across half the band gap enables TPA to turn on or off.

Quantum noise of incident coherent light is simulated by generation of an ensemble of phasors that forms uncertainty distribution with Gaussian probability weight in a complex phase space. Therefore, the probability of finding a phasor point in a phase space follows Gaussian profile with a given ensemble average of phase and photon-number that equals its photon-number variance. Each individual phasor has the well-defined amplitude and phase, or equivalently the well-defined $X1$ and $X2$ in a complex phase space, where $X1$ and $X2$ are the real and the imaginary part of a complex field envelop A , respectively. The propagation of light for each phasor is calculated by

$$\frac{\partial A(z,t)}{\partial z} = i\gamma|A(z,t)|^2A(z,t), \quad (1)$$

where $\gamma = \omega_0 n_2 / ca_{\text{eff}}$ is the Kerr coefficient. Here a_{eff} is the effective cross-sectional area for third-order nonlinearity and n_2 is the nonlinear index parameter inducing coupling of light intensity (I) with its phase via the intensity dependence of a refractive index:

$$n(I) = n_0 + n_2 I. \quad (2)$$

The parameter n_2 used in the simulation is given by the formula with the band-gap (1.61 eV) and the wavelength (1550 nm) in use [10, 11]. In this work, we assume negligible effects of group velocity dispersion of 1 ps pulsed light propagating 2 mm ($= L$) waveguide.

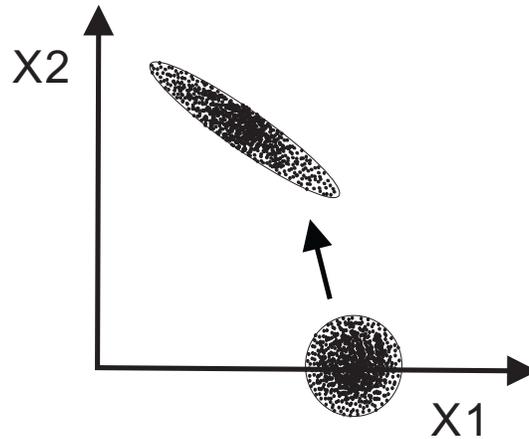


Fig. 1. Quantum noise evolution in a complex phase space

We assumed that, for incident coherent light, the center of a phasor point ensemble lies on the $X1$ axis as shown in Fig.1. This allows the fluctuations of the $X1$ and the $X2$ of an initial state to be determined by the amplitude fluctuation given by the photon-number variance of the coherent state.

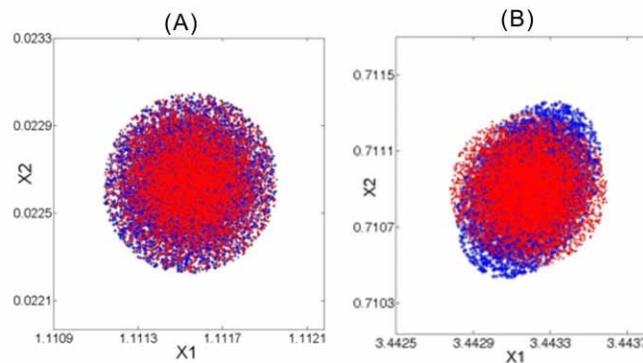


Fig. 2. Quantum noise distribution (blue dots) of an envelop $A(L, t_c)$ with respect to the reference distribution (red dots) for a 1.32 pJ pulse energy (A) and 13.2 pJ pulse energy (B). $F \simeq 1.00$ for (A) and (B). The uncertainty area ratio (R) between blue and red is $R \simeq 1.00$ for (A) and $R \simeq 1.01$ for (B). Phase uncertainty ratio $(\Delta\Phi)^2 / (\Delta\Phi_c)^2 = 1.00$ for (A) and $(\Delta\Phi)^2 / (\Delta\Phi_c)^2 = 1.19$ for (B).

Figures 2(A)-(B),3(A)-(B),4(A) show the quantum noise distribution (blue dots) of an enve-

lope function $A(t = t_c)$ at the waveguide output, for the various pulse energies of incident light. Here t_c is the temporal pulse center. For each pulse energy, the distribution denoted by blue dots is compared with the reference distribution (red dots) that represents the coherent state (C-state) with the same average photon-number as that of the squeezed state at the waveguide output. The photon-number noise change is given by the Fanofactor $F \equiv (\Delta N)^2 / (\Delta N_c)^2$, where the $(\Delta N_c)^2$ is the photon-number variance of the C-state.

Intensity dependence of a refractive index causes the phase noise $\Delta\Phi$ of the output light to increase with conversion of an uncertainty circle into an ellipse, creating correlation between fluctuations along the major and the minor axes of an ellipse. This correlation is enhanced with increasing input pulse energy. The output light phase noise (variance) $(\Delta\Phi)^2$ increases for these pulse energies with respect to that of the corresponding C-state, $(\Delta\Phi_c)^2$.

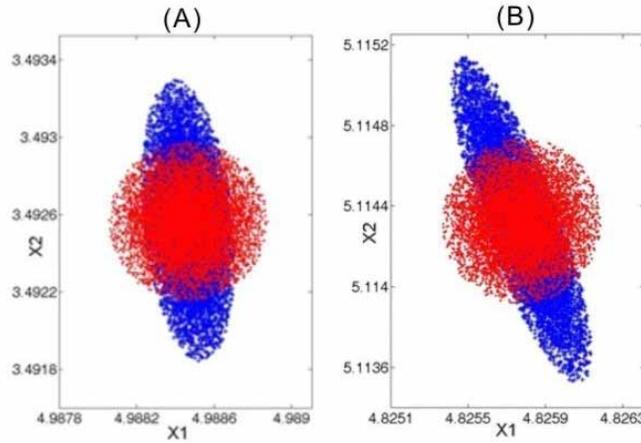


Fig. 3. Quantum noise distribution (blue dots) of an envelop $A(L, t_c)$ with respect to the reference distribution (red dots) for a 39.5 pJ pulse energy (A) and 52.6 pJ pulse energy (B). $F \simeq 1.02$ for (A) and $F \simeq 0.99$ for (B). The uncertainty area ratio (R) between blue and red is $R \simeq 0.99$ for (A) and $R = 1.00$ for (B). Phase uncertainty ratio $(\Delta\Phi)^2 / (\Delta\Phi_c)^2 = 2.55$ for (A) and $(\Delta\Phi)^2 / (\Delta\Phi_c)^2 = 3.54$ for (B).

It is also found that Kerr effects do not change the Fanofactor beyond the calculation error ($\leq 3\%$) but squeezes an uncertainty circle into an ellipse in a phase space.

Comparison of uncertainty area of the squeezed ellipse with that of the corresponding C-state shows the little change in the uncertainty area by the intensity dependence of a refractive index. This implies that photon-number and phase are not canonically conjugate variables determining minimum uncertainty relation.

We also simulate TPA effects on photon-number statistics of light [7, 8, 9]. Tuning the photon energy above half the band gap ($\lambda = 1540$ nm) enables TPA to turn on in the semiconductor waveguide. To include TPA effects in the simulation, equation (1) is extended by adding a term $-\alpha_2 |A|^2 A / 2$, where $\alpha_2 \equiv \kappa_2 / a_{\text{eff}}$ is the TPA coefficient. Here κ_2 is the TPA parameter in the unit of m/W, which is determined by the formula in [10, 11] for a given band-gap and a photon energy. This calculation utilizes an ensemble being composed of one-dimensional phasor points with Gaussian weight along a radial direction in a complex phase space.

Figure 4(B) displays the TPA reduction of photon-number variance compared to that of the coherent state having the same average photon-number as output light. The Fanofactor decreases with increasing an input pulse energy. The nonlinear transmittance curve is seen for nonlinear loss mechanism, accounting for high rate of lose for rising fluctuation of intensity.

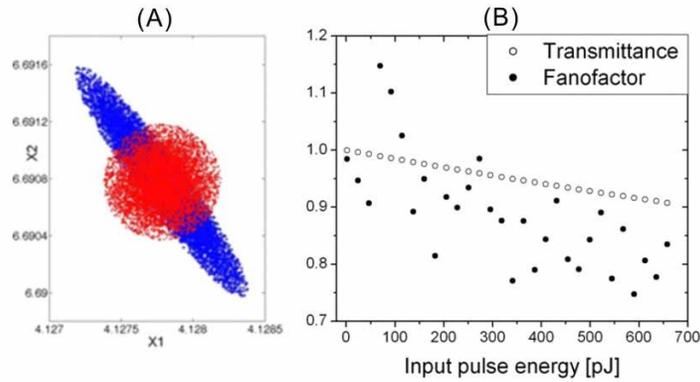


Fig. 4. (A) Quantum noise distribution of an envelop $A(L, t_c)$ for a 65.8 pJ pulse energy. $F \simeq 1.01$, the uncertainty area ratio (R) between blue and red is $R \simeq 0.99$, and Phase uncertainty ratio $(\Delta\Phi)^2/(\Delta\Phi_c)^2 = 5.11$. (B) Optical power transmittance under TPA as well as Kerr effects.

However, in a two-dimensional phase space, our calculation provides a limited information on the quantum noise evolution under both TPA and Kerr effects, due to TPA-inherent properties such as non-invariance of minimum uncertainty area [12]. This leads us to focus on one-dimensional ensemble calculation rather than two-dimensional one, and also implies that this one-dimensional calculation gives qualitative estimation of TPA effects on the photon-number variance.

3. Conclusion

Evolution of quantum noise of ultrashort pulsed light that propagates a semiconductor optical waveguide is simulated using (pseudo-)random distribution of phasors that represents a coherent state in a two-dimensional complex phase space.

Intensity dependence of a refractive index converts an uncertainty circle into an uncertainty ellipse whereas maintaining photon-number statistics unchanged in spite of increasing phase noise. It is also found that an uncertainty area in a phase space is conserved under Kerr effects, indicating that photon-number and phase may not be the canonical conjugate variables that define minimum uncertainty relation of light.

Effects of TPA on photon-number statistics is also simulated using one-dimensional (pseudo-)random distribution of phasors representing photon-number fluctuation of a coherent light. It is shown that TPA reduces photon-number noise with nonlinear loss mechanism, stemming from high loss for rising fluctuation of light intensity.

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