

Gap random-phase lattice solitons

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Abstract: We theoretically study gap random-phase lattice solitons (gap-RPLSs) in nonlinear waveguide arrays with self-defocusing nonlinearity. We find that the intensity structure and statistical (coherence) properties of gap-RPLSs conform to the lattice periodicity, while their Floquet-Bloch power spectrum is multi-humped with peaks in the anomalous diffractions regions. It is shown that a gap-RPLS can be generated when a simple incoherent beam with bell-shaped power spectrum and single-hump intensity is launched at a proper angle into the waveguide array. The input incoherent beam evolves in the lattice while shedding off some radiation, and eventually attains the features of gap-RPLS.

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1. Introduction

The behavior of light in photonic lattices is driven by interference, which crucially depends on the coherence of light. This fact motivates exploring the propagation of partially coherent light in nonlinear photonic lattices. Among the nonlinear phenomena in nonlinear photonic lattices, of particular interest are lattice solitons [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20], such as gap solitons [2, 3, 5, 6, 10, 11, 17, 18], discrete solitons [4, 7, 12], dipole-like ("twisted") solitons [8, 15], multi-band vector solitons [13, 14], and so forth. Until recently, such self-localized wavepackets were studied solely with coherent light. This situation has been changed by the prediction [19] and experimental observation [20] of random-phase

lattice solitons (RPLSs). These studies have shown that the interplay of statistical (coherence) properties of the light, nonlinearity, and lattice structure, determines the properties of RPLSs. More specifically, a partially incoherent wavepacket can excite modes across many Brillouin zones, among multiple bands, whose dynamics is determined by the degree of coherence, the sign and strength of the nonlinearity, and the underlying curvature of the bands. For example, a self-focusing (self-defocusing) nonlinearity can localize modes arising from normal (anomalous) diffraction regions of the Brillouin zones, resulting in a multi-humped Floquet-Bloch power spectrum of RPLS [19]. The propagation constants of the randomly-excited modes comprising an RPLS are located in the gap(s) of the spectrum of the linear system (the fact that self-focusing (defocusing) nonlinearity can jointly self-trap multiple modes arising from normal (anomalous) diffraction regions was shown in the context of multi-mode multi-band vector lattice solitons [13]). Particularly interesting is the dynamics of partially coherent light associated with the generation of RPLSs observed in Ref. [20]. A "simple" incoherent beam with a bell-shaped intensity structure and single-humped power spectrum was launched at a normal angle into the nonlinear waveguide array and dynamically evolved into a RPLS with a multi-humped power spectrum. Thus, the generation of RPLSs did not require engineering of the input beam to match the RPLS properties [20]. Instead, it occurred due to the energy transfer between the modes of the linear system (the Floquet-Bloch waves) induced by the self-focusing nonlinearity [20, 21]. Further exploration of incoherent light dynamics in (self-focusing and defocusing) nonlinear photonic lattices have lead to the technique for Brillouin-zone spectroscopy of such lattices [22]. Recently, it was numerically shown that incoherent gap solitons can be efficiently generated in a self-defocusing medium by engineering the input excitation [23]. In that scheme, two properly constructed incoherent beams are launched with opposite angles into the waveguide array [23] (equivalent input geometry was suggested by Feng [6] for the excitation of coherent gap solitons).

Here we pursue a different avenue, and investigate the possibility of attaining gap random-phase lattice solitons (gap-RPLSs) in experimental settings where dynamics in a (1+1)D nonlinear waveguide array naturally evolves an incoherent input beam into a beam with gap-RPLS structure. As a first step, we find gap-RPLSs and identify their features: intensity structure and coherence properties (expressed through the complex coherence factor) both conform to the lattice periodicity, while their Floquet-Bloch power spectrum is multi-humped, with humps being located mainly in the anomalous diffraction regions. Finally, we find that a "simple" incoherent beam with a bell-shaped intensity structure and a singly-humped power spectrum, when launched at a proper angle into a nonlinear waveguide array, naturally evolves (under proper self-defocusing conditions) into a beam with gap-RPLS properties.

2. Description of the physical system

Before analyzing gap-RPLSs, let us recall some concepts of incoherent light propagation in homogeneous nonlinear media, which have been extensively studied since 1996 [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38]. Of particular interest are the phenomena of incoherent solitons [24, 25, 26, 27, 28, 29, 32, 35, 37], and modulation instability with incoherent light [30, 31, 33, 34, 36]. Incoherent solitons were found [24] in noninstantaneous nonlinear medium, where an incoherent beam with a randomly fluctuating field induces a smooth multi-mode waveguide and populates (on the average) its modes self-consistently [26, 38].

The physical system studied in this paper is described as follows. A source of the light generates spatially incoherent quasimonochromatic light beam, which enters a nonlinear waveguide array with a noninstantaneous self-defocusing nonlinearity. The electric field $E(x, z, t)$ of such a beam randomly fluctuates. The characteristic time of these fluctuations (the coherence time) τ_c , is much smaller than the response time of the nonlinearity τ_m [24]. This is one of the key phys-

ical mechanisms of our system: the nonlinearity cannot follow fast fluctuations of the field but rather responds to the time-averaged intensity. This allows the induction of a spatially smooth waveguide, which enables self-guidance of the incoherent beam [26]. Incoherent light can be described with a set of coherent waves $\psi_m(x, z)$, and associated modal weights d_m . The electric field is $E(x, z, t) = \sum_m c_m(t) \psi_m(x, z)$ [26], where $c_m(t)$ are randomly fluctuating coefficients such that $\langle c_m(t) c_{m'}^*(t) \rangle = d_m \delta_{mm'}$; brackets $\langle \dots \rangle$ denote the time-average over the response time τ_m . The statistical properties of the incoherent light are contained within the mutual coherence function [28], $B(x_1, x_2, z) = \sum_m d_m \psi_m(x_1, z) \psi_m^*(x_2, z)$. The time-averaged intensity is $I(x, z) = B(x, x, z) = \sum_m d_m |\psi_m(x, z)|^2$. The evolution of the coherent waves ψ_m , and hence the evolution of correlation function $B(x_1, x_2, z)$, is governed by a set of coupled nonlinear wave equations [26]:

$$i \frac{\partial \psi_m}{\partial z} + \frac{1}{2k} \frac{\partial^2 \psi_m}{\partial x^2} + \frac{V(x, z)k}{n_0} \psi_m(x, z) = 0. \quad (1)$$

The potential $V(x, z) = p(x) + \delta n(I(x, z))$ contains both the periodic $p(x) = p(x + D)$, and the nonlinear term $\delta n(I(x, z))$; the nonlinearity is of the self-defocusing type, i.e., $\partial \delta n(I) / \partial I < 0$.

Spatial solitons occur in our system when the diffraction of a spatially localized incoherent beam is exactly balanced by nonlinear self-defocusing. This exact balance happens when the self-consistency principle holds: the incoherent beam induces (via the nonlinearity) a defect in the photonic lattice, which has several localized defect modes; the coherent waves ψ_m of the incoherent beam are the defect modes themselves. The eigenvalues of the localized defect modes reside in the gaps of the spectrum of the linear system. For this reason it is convenient to change the notation for the coherent waves: $\psi_m \rightarrow \psi_{n,l} = u_{n,l}(x) e^{i\kappa_{n,l}z}$, where $u_{n,l}(x)$ are orthonormal (real) eigenfunctions, and $\kappa_{n,l}$ are the real eigenvalues of the defect modes [19],

$$\frac{1}{2k} \frac{d^2 u_{n,l}}{dx^2} + \frac{V(x)k}{n_0} u_{n,l}(x, z) = \kappa_{n,l} u_{n,l}, \quad (2)$$

where $V(x) = p(x) + \delta n(\sum_{n,l} d_{n,l} |u_{n,l}(x)|^2)$; the index $n = 1, 2, \dots$ in the notation for the eigenmodes $u_{n,l}$ denotes that the propagation constant $\kappa_{n,l}$ is in the gap below the n th band, while the index l describes the hierarchy within a single gap: if $l < l'$ then $\kappa_{n,l} < \kappa_{n,l'}$. A gap-RPLS may be formed from several defect modes from the first gap, several modes from the second gap, and so forth. The number of modes that can be excited in different gaps, and the modal weights $d_{n,l}$ corresponding to these modes, depend on the parameters of the lattice and the nonlinearity. We solve Eq. (2) self-consistently by using the iterative procedure used for calculating multi-gap solitons [13].

3. Results

Let us present an example of a gap-RPLS in a saturable self-defocusing medium $\delta n(I) = -\gamma I / (1 + I/I_S)$. The parameters are $\gamma I_S = 0.00015$, $n_0 = 2.3$, $k = 2\pi n_0 / \lambda$, where $\lambda = 488\text{nm}$; the lattice is of the form $p(x) = p_0 \sum_i \exp(-((x - iD)/x_0)^8)$, where $x_0 = 3.7 \mu\text{m}$, $D = 10 \mu\text{m}$, and $p_0 = 6 \cdot 10^{-4}$. Fig. 1(a) shows the band-gap structure of the periodic system, and the propagation constants of the defect modes that comprise the gap-RPLS. The gap-RPLS consists of 11 coherent waves (defect modes). Eight of these modes originate from the first band, two from the second, and one from the third band. The propagation constants of these (induced) defect modes are (via the self-defocusing nonlinearity) pushed into the gaps below the first, second, and third band, respectively [see Fig. 1(a)]. The modes with larger propagation constants within each gap contain less power. The distribution of power among the modes residing within the first gap is (3.35, 4.89, 6.78, 8.93, 11.20, 13.36, 15.15, 16.34)%, among the second-gap modes it is

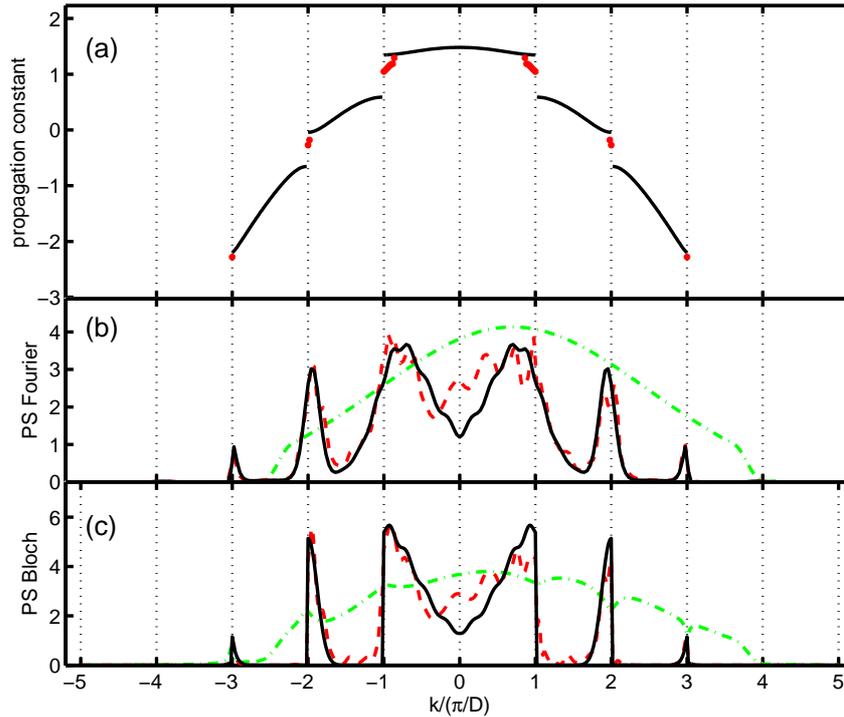


Fig. 1. The band-gap structure of the lattice, and the power spectra of the first gap-RPLS example. (a) Band-gap structure of the lattice (solid lines), and propagation constants of the RPLS modes (closed circles). (b) Fourier power spectrum of a gap-RPLS in the self-defocusing nonlinear waveguide array (solid line); the humps in the spectrum are located mainly in the anomalous diffraction regions. The Fourier power spectrum of an input incoherent beam launched at an angle is indicated by the dot-dashed line, and spectrum of the output beam is given by the dashed line (the dot-dashed line and the dashed line almost fully coincide). (c) Same as in (b) but with the Floquet-Bloch power spectrum of the beams. See text for details.

(4.14,13.86)%, and also there is 2% of the total power in the third-gap mode. The total power within the gap-RPLS is $11.0I_5D$. In contrast to the RPLSs in the self-focusing medium [19, 20], here there are no modes whose propagation constants reside in the semi-infinite gap. The solid line in Fig. 1(b) illustrates the power spectrum of the gap-RPLS in the Fourier basis, while Fig. 1(c) shows the power spectrum in the Floquet-Bloch basis (which is the natural basis for periodic systems). Both power spectra possess a multi-humped shape, with humps being located in the regions of anomalous diffraction. The FB (Fourier) power spectrum of the gap-RPLS is a sum of the power spectra of its modal constituents. It is interesting to note that, for the gap-RPLS example presented here, the FB power spectrum of the mode $u_{n,l}$ is supported only within the n th Brillouin zone [19]; for example, the localized modes from the first gap $u_{1,l}$ are orthogonal to the FB waves from the 2nd band, and so on.

The intensity profile of this gap-RPLS example, and its complex coherence factor $\mu(x_1, x_2) = B(x_1, x_2) / \sqrt{B(x_1, x_1)B(x_2, x_2)}$ are shown in Fig. 2. The intensity is located mainly on sites, but there is also a non-negligible amount of power in the interstitial regions [solid line in Fig. 2(b)]. From Fig. 2(a), i.e., from the coherence factor $\mu(x_1, x_2)$, we read the coherence properties

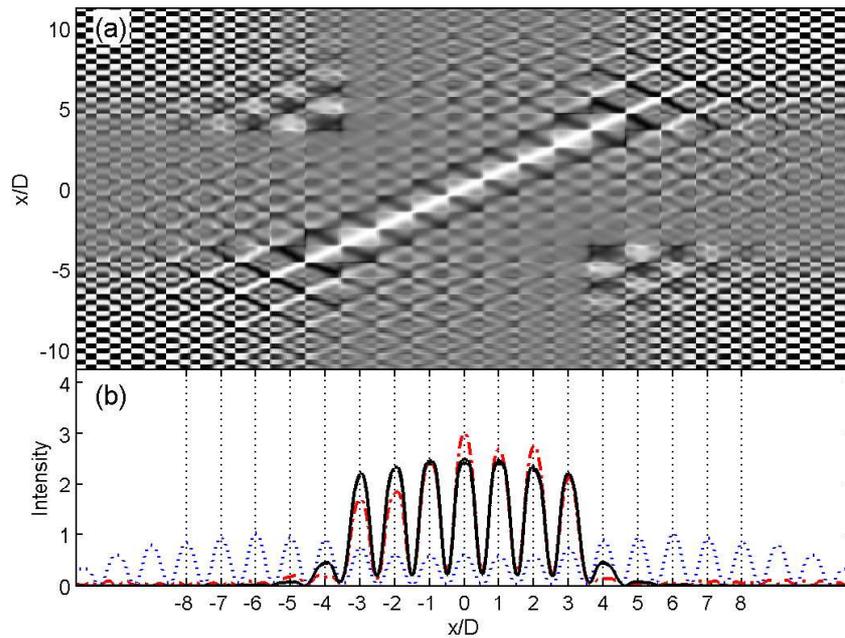


Fig. 2. The intensity structure and coherence properties of the first example of a gap-RPLS. (a) The complex coherence factor $\mu(x, x') \in [-1, 1]$ of the gap-RPLS; black (white) corresponds to a value of -1 (1). (b) The intensity profile $I(x)/k$ (solid line) of a gap-RPLS. The intensity structure of an incoherent beam launched at angle $\theta = 0.7\pi/Dk$, and propagated for 50 mm (dashed line). Dotted line illustrates the diffraction-broadened beam after $z = 30$ mm propagation in a linear lattice, for an input beam with a wavefunction identical to that of a gap-RPLS. The vertical lines represent the lattice sites.

of the gap-RPLS. For RPLSs, $\mu(x_1, x_2) \in [-1, 1]$, where $\mu(x_1, x_2) = 0$ corresponds to zero correlation between the fields at points x_1 and x_2 , whereas $\mu(x_1, x_2) = 1, -1$ corresponds to full correlation, where the field is in phase and π -out of phase, respectively. From Fig. 2(a) we see that coherence conforms to the periodicity of the lattice. The coherence increases as we move away from the soliton region, which is shown as a (high visibility) black ($\mu = -1$) and white ($\mu = 1$) checkerboard. The structure of the black and white squares at the soliton tails depends on the most slowly decaying mode(s). Most of the grey shaded regions ($\mu \approx 0$) (low correlation) are within the soliton region, where we confirm the validity of the approximate law $\mu(x_1, x_2) \approx \mu(x_1 + D, x_2 + D)$ [19]. The stability of our gap-RPLS example is checked numerically by evolving it for many diffraction lengths with small initial noise on top of the gap-RPLS; Figs. 3(a),(b), and (c) show the evolution of the intensity profile and the power spectra of the gap-RPLS, while Fig. 2(b) shows the diffraction-broadening of an input beam whose wavefunction is identical to that of the gap-RPLS, when it evolves in the lattice without the nonlinearity.

Incoherent solitons can possess a rich internal structure. This is manifested in the fact that, given the parameters of the lattice, nonlinearity, and the total power contained within an incoherent soliton, one can rearrange the power contained within the excited coherent waves (described by parameters $d_{n,t}$) to obtain different soliton solutions. For example, consider the

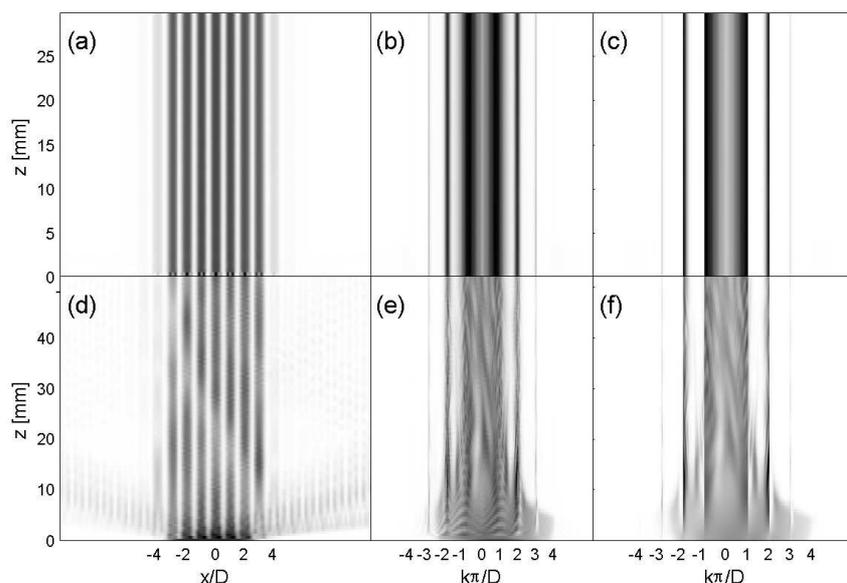


Fig. 3. The stable evolution of the (a) intensity structure, (b) Fourier power spectrum, and (c) Floquet-Bloch power spectrum of the first gap-RPLS example. The evolution of an incoherent beam launched at an angle $\theta = 0.7\pi/Dk$ into the nonlinear waveguide array; (d) intensity structure, (e) Fourier power spectrum, and (f) Floquet-Bloch power spectrum.

case where all parameters are identical to those of the previous example, but that the distribution of power within coherent waves is different: the power within the eight modes with propagation constants in the first gap is (9.94,9.96,9.98,10.00,10.01,10.03,10.03,10.04)%; the power distribution within the second gap modes is (8.97,9.03)%, and there is 0.02% of the total power within the third gap mode. The propagation constants and the power spectra of this gap-RPLS example are shown in Fig. 4, while its coherence properties and intensity structure are illustrated in Fig. 5. The Floquet-Bloch power spectrum is again multi-humped with the peaks of the humps in the anomalous diffraction regions, as in the previous example. However, the intensity structure now has a dip in the very center of the soliton, while the peaks of the Fourier power spectrum within the 1st Brillouin zone (BZ) are shifted from the edges of the 1st BZ towards its center. The differences between the two examples stem from a different choice of the modal populations; the modes with Fourier power spectrum closer to $k_x = 0$ have more power in this gap-RPLS example. Figures 6(d),(e), and (f) show the stable stationary propagation of the intensity profile, the Fourier, and Floquet-Bloch power spectrum of the second example of a gap-RPLS.

In the spirit of Ref. [20], in what follows we study the possibility of obtaining a self-trapped beam with gap-RPLS properties from a simple incoherent beam, with bell-shaped power spectrum and intensity. Because most of the Floquet-Bloch power spectrum in the gap-RPLS examples described above is within the anomalous diffraction region, we are motivated to study the dynamics of an incoherent beam launched at an angle into the waveguide array, so that the peak of the initial power spectrum is in the anomalous diffraction region of the 1st Brillouin zone.

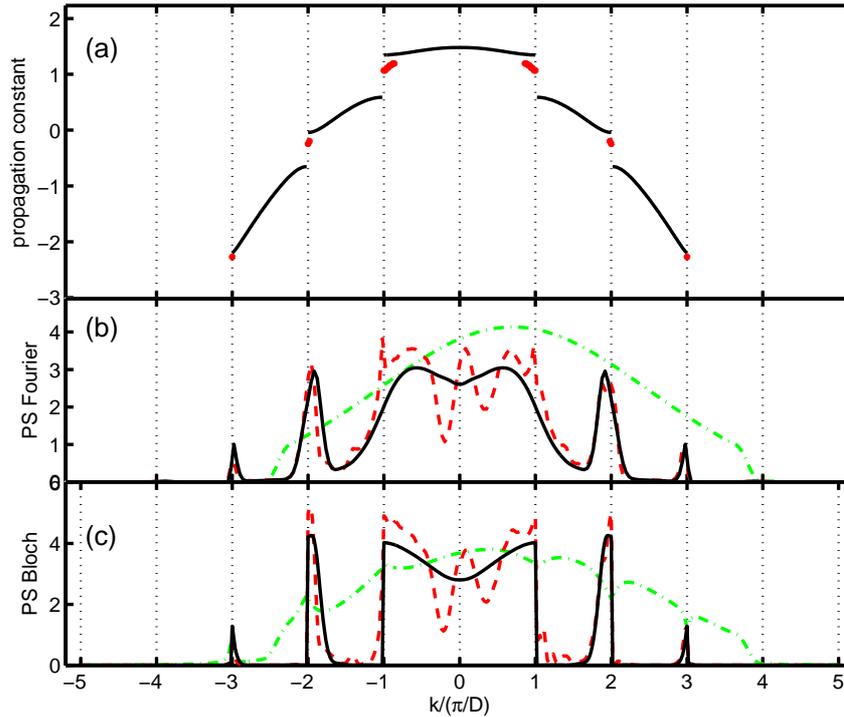


Fig. 4. Same as in Figure 1 for the second example of a gap-RPLS. See text for details.

The modal structure of the incoherent beam may in this case be described by

$$\psi_{k_x}(x, z = 0) = \sqrt{I_0(x)} G(k_x) e^{ik_x x} e^{iKx},$$

where $I_0(x) = \exp[-((x + 0.35D)/3D)^8]$ is the intensity structure of the beam; k_x corresponds to the transverse momentum of the k_x th coherent wave, while $G(k_x) \propto \exp[-(k_x D/3.5\pi)^2]$ expresses the relative power within the k_x th coherent wave. The beam enters the waveguide array at the angle $\theta = K/k = 0.7\pi/Dk$. Figures 3(d),(e), and (f) show the evolution of the intensity profile, the Fourier, and Floquet-Bloch power spectrum of such a simple beam, and its evolution into a gap-RPLS.

From these figures we notice that during the initial stages of the evolution the beam sheds off some radiation thereby filtering its spectrum. More specifically, the evolution of the Floquet-Bloch power spectrum shows that power from the normal diffraction regions is partially radiated away and partially transferred to the anomalous diffraction regions [Fig. 3(f)]. After $z = 50$ mm of propagation the intensity structure has the same width, and the output power spectra match the power spectra of the first gap-RPLS example [see Fig. 2(b) and Figs. 1(b) and (c)]. For different initial conditions, i.e., for an incoherent beam that has all parameters identical as in the previous case, but with a slightly broader initial intensity structure [$I_0(x) = \exp[-((x + 0.35D)/3.5D)^8]$] the beam evolves in its structure, becoming similar to our second example of a gap-RPLS. Figures 6(d),(e), and (f) show the evolution of the intensity profile, the Fourier, and Floquet-Bloch power spectra, respectively, of this incoherent beam. Again, we observe some radiation escaping during the initial stages of evolution and reshaping of the beam spectra. The initial beam with such a simple structure evolves into an incoherent beam with multi-humped

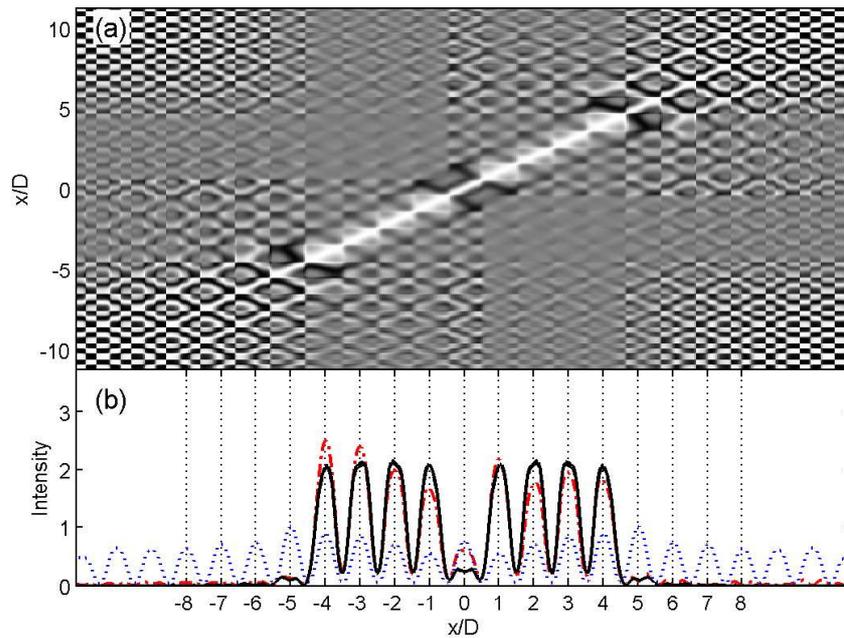


Fig. 5. Same as Figure 2 for the second example of a gap-RPLS. See text.

power spectra, with humps in the anomalous diffraction regions [see Figs. 4(b) and (c), and Figs. 6(e) and (f)].

4. Discussion

In the previous section, we have presented two examples of a gap-RPLS. However, it is clear that, by varying the parameters of the lattice and the nonlinearity, one may obtain a variety of gap-RPLS solutions. Moreover, even when the parameters of the lattice and the nonlinearity are fixed, one may find different gap-RPLSs, e.g., by different compositions of the modal weights $d_{n,l}$ of the gap-RPLS. By setting the modal weights $d_{n,l}$, one may seek for gap-RPLS solutions with several modes from the 1st gap, some from the 2nd gap, and so forth. It should be emphasized that even though there are many degrees of freedom when calculating gap-RPLSs, random-phase lattice solitons do not exist for any arbitrary choice of modal weights $d_{n,l}$. Given the parameters of the lattice and the nonlinearity, all possible sets of values $d_{n,l}$ for which gap-RPLSs exist define the existence range of gap-RPLSs. The notion of the existence range of incoherent solitons is described in Ref. [26]. A question that naturally arises is how does the existence range change while changing the lattice/nonlinearity parameters? Furthermore, one may ask whether all gap-RPLSs from a given existence range have common features? While a rigorous analysis of these questions is difficult to perform, we may use intuitive physical arguments to obtain some answers. Before doing so, we should mention that in our discussion on lattice solitons, we put forward a constraint that the defect in the lattice (induced by the beam via the nonlinearity) is a few times smaller than the lattice depth, i.e., that the nonlinearity does not completely destroy the lattice at the position of the soliton. This imposes an upper bound on the strength of the nonlinearity used, in addition to the upper bound on the strength of the

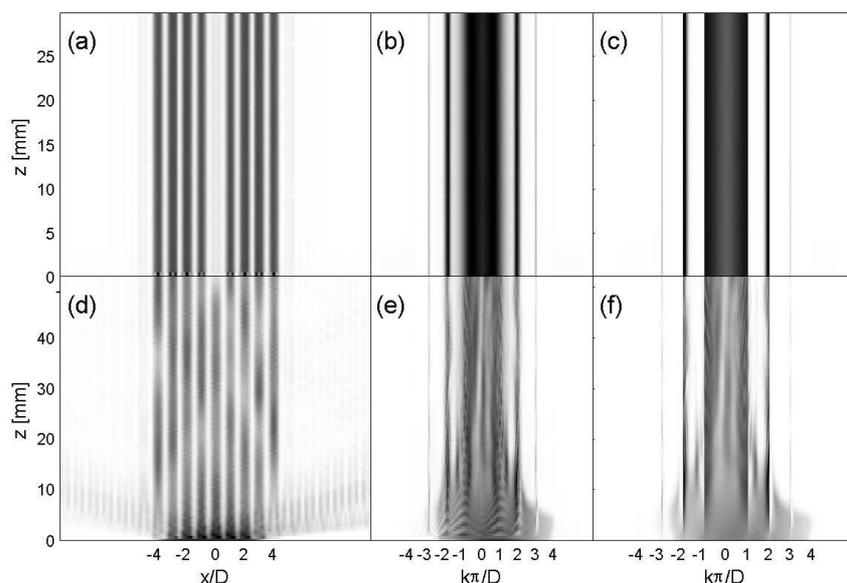


Fig. 6. Same as Figure 3 for the second example of a gap-RPLS. See text for details.

nonlinearity admissible by the material.

We may think of gap-RPLSs as solitons obtained by localizing (self-consistently) a number of FB waves by pushing their propagation constants into the gaps (through the action of the self-defocusing nonlinearity). The FB waves that are most easily pushed into the gap are those whose propagation constants are the closest to the gap, i.e., at the bottom of a band; these modes are from the edges of the BZs at the anomalous diffraction regions. That is, the modes that are most easily localized in the gap have their FB power spectrum mainly in the anomalous diffraction regions. This is underpinned by the fact that anomalous diffraction is balanced by the self-defocusing nonlinearity, and it reflects onto the FB power spectra of gap-RPLSs [see Figs. 4(c) and 4(c)]

Let us discuss the existence range(s) and features of gap-RPLSs. For lattices with narrower gaps (e.g., for lattices that are more shallow), intuition suggests that the existence range of gap-RPLSs is smaller than for lattices with broader gaps. Namely, one may excite (localize) more defect modes within a broader gap. Following the intuition that modes with FB power spectrum closer to the edges of the BZs in the anomalous diffraction regions are more easily localized with self-defocusing nonlinearity, it follows that for lattices with narrower gaps (smaller existence range), the multi-humped feature of the FB spectrum is expected to be more pronounced. For lattices with broad gaps (e.g. for deep lattices), the existence range is large. As more and more modes are being localized in a gap, the width of the FB power spectrum of the gap-RPLS within a single BZ broadens. Eventually, the FB power spectrum may fill the entire BZ [see Fig. 4(c)].

Before closing, let us add that a similar line of reasoning can be developed for the self-focusing type of nonlinearity, where modes can also be localized in the semi-infinite gap. The discussion here is restricted to (1+1)D systems, while (2+1)D systems certainly offer more possibilities and will definitely lead to exciting results. All of the above discussion points at the fact

that partially incoherent light in nonlinear photonic lattices is a rich dynamical system worthy of further exploration. One example of this is a recently developed technique for Brillouin-zone spectroscopy of nonlinear photonic lattices [22], which utilizes partially coherent light.

5. Conclusion

In conclusion, we have theoretically studied gap random-phase solitons in self-defocusing nonlinear waveguide arrays (gap-RPLSs). We have found gap-RPLS solutions and identified their features, showing that both their intensity structure and their statistical (coherence) properties conform to the lattice periodicity, while their Floquet-Bloch power spectrum is multi-humped with peaks in the anomalous diffractions regions. We have shown that a self-trapped beam with such gap-RPLS power spectrum may be obtained naturally by launching an incoherent beam with a bell-shaped power spectrum and single-hump intensity, at a proper angle into a nonlinear waveguide array. The structure of this incoherent beam evolves into a gap-RPLS structure partially due to the energy exchange between the Floquet-Bloch waves induced by the self-defocusing nonlinearity, and partially by shedding off some radiation.

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