Circumvention of noise contributions in fiber laser based frequency combs

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Abstract: We investigate the performance of an Er:fiber laser based femtosecond frequency comb for precision metrological applications. Instead of an active stabilization of the comb, the fluctuations of the carrier-envelope offset phase, the repetition phase, and the phase of the beat from a comb line with an optical reference are synchronously detected. We show that these fluctuations can be effectively eliminated by exploiting their known correlation. In our experimental scheme, we utilize two identically constructed frequency combs for the measurement of the fluctuations, rejecting the influence of a shared optical reference. From measuring a white frequency noise level, we demonstrate that a fractional frequency instability better than 1.4×10^{-14} for 1 s averaging time can be achieved in frequency metrology applications using the Er:fiber based frequency comb.

OCIS codes: 120.3930 (Metrological instrumentation); 120.4800: (Optical standards and testing); 140.3510: (Lasers, fiber); 320.7160: (Ultrafast technology).

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1. Introduction

Broadband, phase coherent frequency combs generated with femtosecond lasers have revolutionized precision frequency metrology, as they are used, e.g., as clockworks of optical atomic clocks and enable novel applications like the synthesis of ultrastable optical microwave signals [1, 2], but also new spectroscopic techniques [3, 4]. The general task in precision frequency metrological applications is to establish a phase coherent link between different frequencies while the noise contributions of the link remain negligible. In the past, this task has been solved mainly with stabilized frequency combs based on mode-locked Ti:sapphire lasers. Stabilization is, e.g., accomplished by phase locking the carrier envelope offset (CEO) beat frequency ν_{CEO} and the repetition frequency f_{rep}.

However, heading for frequency combs as a general, user-friendly laboratory tool, it is desirable to use mode-locked Er:fiber lasers instead of free-space Ti:sapphire systems due to their convenience of operation, superior long-term stability, compactness and lower cost. On the other hand, CEO beat signals generated from fiber lasers are known to exhibit more high frequency noise [5-9]. Prior to this work, it has therefore been unclear whether a level of short-term instability better than several times 10^{-13} in one second can be achieved by use of such systems [9]. Extremely fast servo elements acting on the CEO frequency with impractical bandwidths of at least several 100 kHz would be required in order to achieve a tight phase-lock for an Er:fiber based frequency comb. It should be noted that, in contrast to Ti:sapphire lasers, the CEO frequency is not amenable to changes of the pump power faster than approx. 10 kHz, owing to the long fluorescence lifetime of Erbium ions. If, however, the frequency and phase fluctuations of all comb lines are sufficiently correlated and can be described within an elastic tape model, the so-called transfer oscillator concept can be employed instead of the stabilization of the comb [10, 11]. Within this concept, noise contributions are eliminated by exploiting the correlations between the characteristic signals in a comb-based measurement, relying solely on relations which are known *a priori*. The aim of this paper is to investigate the impact of noise contributions violating the elastic tape model in a fiber-generated frequency comb. Such contributions, if present, would ultimately limit the achievable stability for high-precision metrology applications for both phase-locked frequency combs as well as transfer oscillator measurements.

2. Elastic tape model

The frequency ν_m of a line in a notionally perfect frequency comb of a mode-locked laser is given by

$$\nu_m(t) = \nu_{\mathrm{CEO}}(t) + m\, f_{\mathrm{rep}}(t), \tag{1}$$

where m is the integer order number of the comb line. Since fluctuating quantities in eq. (1) can be visualized by stretching and translating an elastic tape labeled with the equidistant comb lines, we will refer to Eq. (1) as elastic tape model. We first consider only fluctuations caused by noise processes compliant with the elastic tape model. For the special case of a single noise process, a correlation between the dilations and translations of the elastic tape

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exists, such that they cancel out against each other at a particular fixed point frequency ν_{fix} [10].

In a real-world system, fluctuations resulting from, e.g., technical noise processes or quantum noise not necessarily in accordance with the elastic tape model may be present. For this reason, we add the residual term $\delta\nu_{\text{res}}$ to Eq. (1), which accounts for these non-compliant fluctuations:

$$\nu_m(t) = \nu_{\text{CEO}}(t) + m\, f_{\text{rep}}(t) + \delta\nu_{\text{res}}(m,t). \tag{2}$$

The main goal of this paper is to investigate the properties of the residual term to estimate the quantitative impact of noise contributions not compliant with the elastic tape model, which ultimately limits the performance of the frequency comb in precision frequency metrological applications. Due to the lack of any *a priori* information about $\delta\nu_{\text{res}}(m,t)$, the non-compliant fluctuations cannot be corrected for using the transfer oscillator principle. Neither would a stabilization of the comb by phase-locking two of the quantities ν_m, ν_{CEO} and f_{rep} eliminate the fluctuations not in compliance with the elastic tape model. Integration of Eq. (2) yields the corresponding equation for the instantaneous phases:

$$\varphi_m(t) = \varphi_{\text{CEO}}(t) + m\,\varphi_{\text{rep}}(t) + \varphi_{\text{res}}(m,t), \tag{3}$$

where φ_m, φ_{CEO}, φ_{rep} and φ_{res} are the phases of ν_m, ν_{CEO}, f_{rep} and $\delta\nu_{\text{res}}$, respectively. The value of φ_{res} contains quantitative information about the impact of noise contributions not compliant with the elastic tape model. We will refer to φ_{res} as residual phase throughout this work.

The rigorous validity of Eq. (1), or a sufficiently small residual phase in Eq. (3) would open up novel possibilities for the application of fiber lasers in high precision frequency metrology by use of the transfer oscillator concept. Therefore, the purpose of this study is to determine $\varphi_{\text{res}}(m,t)$ experimentally for a single order number m by measurement of φ_m, φ_{CEO} and φ_{rep} and application of Eq. (3). This method yields an estimate for the uncorrelated phase noise contributions violating the elastic tape model.

3. Experimental setup and data analysis

According to Eq. (3), the determination of $\varphi_{\text{res}}(m,t)$ requires knowledge of the instantaneous phases φ_m, φ_{CEO} and φ_{rep}. These quantities are numerically derived from the corresponding temporal signals, which are detected with a sufficient signal to noise ratio (S/N). The phase φ_m is obtained from the beat signal between the field of a stable cw reference laser and the nearest comb line with order number *m*. The phase φ_{CEO} is obtained from the CEO beat signal measured by the f-2f self referencing technique [12, 13], and the repetition phase φ_{rep} is derived from a measurement of a high harmonic signal of the repetition rate. In a measurement with a single frequency comb, φ_m would contain noise of the optical reference and φ_{rep} would contain noise of a microwave local oscillator (LO) used in the detection electronics. To avoid these problems, we performed the measurement using two identically constructed frequency comb generators sharing the same optical reference laser and a common LO. Fluctuations of the reference laser and of the LO thus do not enter the final expression for the residual phase $\varphi_{\text{res}}(m,t)$.

The experimental setup is shown in Fig. 1. We employed two identical passively mode-locked Er:fiber lasers (TOPTICA FFS F-C SYS) as frequency comb generators. The repetition rate of the Er:fiber oscillators is tuned to approximately 107 MHz in our experiments. Since the setup is symmetric, the elements are described for one branch only in the following. The output of the fiber oscillator is split into three parts: One part is amplified in an Er:fiber

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amplifier, allowing for the generation of sub-100 fs pulses with 200 mW average power. An octave-spanning comb spectrum is generated by coupling this high power output into a highly nonlinear, dispersion shifted specialty fiber.

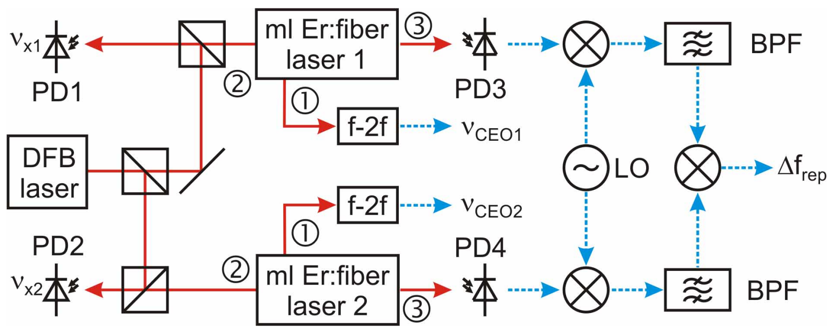

Fig. 1. Schematic drawing of the experimental setup. Red lines: optical signals, blue dashed lines: electrical signals. Details are described in the text.

This supercontinuum output ① of the laser is fed into an f-2f interferometer stage, which provides the CEO beat signal ν_{CEO1} (ν_{CEO2}), detected with a Si avalanche photodiode. Part ② of the oscillator output is used for the generation of the beat note with cw light from a DFB fiber laser (KOHERAS ADJUSTIK) common to both branches of the setup. The wavelength of the DFB laser is $\lambda = 1.5426\,\mu\text{m}$, corresponding to an order number $m \approx 19312 \times 94$ of the nearest comb line. The beat signal ν_{x1} (ν_{x2}) is detected with InGaAs photodiode PD1 (PD2). At oscillator output ③, the $h = 94$ -th harmonic of the repetition rate near 10 GHz is detected with the fast InGaAs photodiode PD3 (PD4). In order to improve the dynamic range of the photodiode, an etalon with free spectral range of approximately 10 GHz (1 cm thick fused silica plate) is placed in front of it. The microwave signals obtained from PD3 and PD4 are amplified and down-converted to intermediate frequencies (IF) of 37.9 MHz and 43.6 MHz using a joint local oscillator LO (Agilent E8257D microwave synthesizer) and double-balanced mixers. The repetition rates of laser 1 and laser 2 are deliberately detuned by a small frequency such that the intermediate frequencies differ by 5.7 MHz. The IF signals are band-pass filtered (BPF) and then mixed in a third double-balanced mixer. The output signal Δf_{rep} is low-pass filtered to obtain the signal at the difference frequency of 5.7 MHz, which is free of noise contributions from the LO [14].

Time series of the signals ν_{x1}, ν_{x2}, ν_{CEO1}, ν_{CEO2} and Δf_{rep} are recorded with a multichannel electronic sampling oscilloscope. The instantaneous phase angles φ_{x1}, φ_{x2}, φ_{CEO1}, φ_{CEO2} and $\Delta\varphi_{rep}$ are derived from these five signals by a numerical phase-retrieval algorithm. Unwanted noise contributions of the common optical reference are eliminated by subtraction of the phases for branches 1 and 2 as in the case of the repetition rate signals. Using Eq. (3) and considering that $\Delta\varphi_{rep}$ is referred to the $h = 94$ -th harmonic of the repetition rate, one arrives at the expression for the difference between the residual phases of the two combs:

$$\Delta\varphi_{res}(t) = -\left(\varphi_{CEO1}(t) - \varphi_{CEO2}(t)\right) + \left(\varphi_{x1}(t) - \varphi_{x2}(t)\right) - m/h\,\Delta\varphi_{rep}(t), \tag{4}$$

In order to demonstrate the enhancement due to application of the transfer oscillator concept, we compare the spectral noise density of the residual phase to the noise densities of the individual phases. All single-sided phase noise spectral densities shown in the following refer to a single laser and 10 GHz carrier frequency. Hence, the spectral densities of phase differences between the two branches of the setup must be divided by a factor of two, because the noise of the two lasers is not correlated. Referring to 10 GHz carrier frequency requires that all phases or phase differences except for $\Delta\varphi_{rep}$ must be divided by the ratio m/h between the order number and the order of repetition rate harmonic before determination of the spectral density. Please notice that due to the slight detuning of the repetition rates, the order numbers are slightly different in the two branches, i.e. $m_1/h = 19317$, $m_2/h = 19307$. The spectral densities are determined by a fast Fourier transform method using Hann windowing.

4. Experimental results

Figure 2 shows the experimentally determined spectral noise densities of φ_{CEO}, φ_x and φ_{rep}. In order to obtain better visibility of details at high Fourier frequencies, a frequency-adaptive moving average over $N = \frac{f}{312.5\,\mathrm{kHz}} + 1$ data points was applied for Fourier frequencies $f > 625\,\mathrm{kHz}$ before transformation of the noise densities to the logarithmic dB scale.

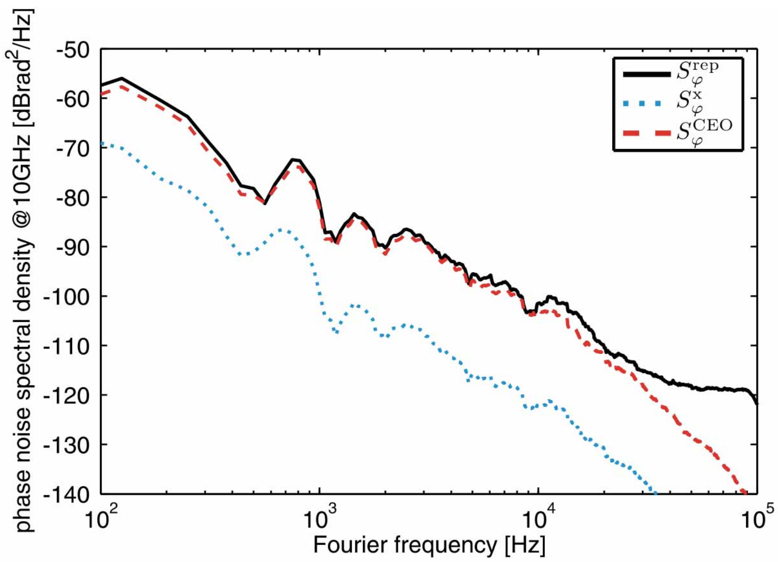

Fig. 2. Spectral noise densities of the repetition phase (black solid line), of the phase of the beat between comb and optical reference (blue dotted), and of the CEO phase (red dashed).

The repetition phase noise density S_φ^{rep} is limited above $f \approx 40\,\mathrm{kHz}$ to the noise floor of $\approx -120\,\mathrm{dBrad}^2/\mathrm{Hz}$ caused by the S/N of the photodiodes PD3 (PD4) and associated electronics like amplifiers and mixers. Furthermore, the noise floor due to digitization noise of the oscilloscope is $-125\,\mathrm{dBrad}^2/\mathrm{Hz}$. This digitization noise floor is, however, not reached in our measurements. In the case of the phases φ_{CEO} and φ_{x}, the noise floor is much lower and likewise not reached in our measurements, because these phases were determined in the optical domain, but are referred to a 10 GHz carrier, pushing the noise floor down by the factor $(m/h)^2$, i.e. by 86 dB.

It can be seen that the spectral noise densities follow a similar frequency dependence. The CEO phase noise density S_φ^{CEO} and repetition phase noise density S_φ^{rep} are of the same order of magnitude while the phase noise density S_φ^{x} of the beat with the optical reference is about 20 dB lower. From this observation, we can infer that there is a dominant technical noise contribution with a fixed point frequency ν_{fix} near the optical carrier. It is known that the fixed point frequency of pump laser current fluctuations typically shows such behavior [15] and one might argue that the origin of the main technical noise contribution are pump current fluctuations or mode-hops of the pump laser diode. By use of a scanning Fabry-Perot

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interferometer, we found that although the wavelength of all pump diodes is stabilized by fiber Bragg gratings, their emission does not occur in a single longitudinal mode.

If the noise contributions of $\varphi_{\rm CEO}$, φ_x and $\varphi_{\rm rep}$ are treated as uncorrelated, the spectral noise density of the residual phase is given by the sum $S_\varphi^\Sigma = S_\varphi^{\rm rep} + S_\varphi^{\rm CEO} + S_\varphi^{\rm x}$. This spectral density is shown as red dotted line in Fig. 3. In contrast to a picture of uncorrelated phase fluctuations within the optical frequency comb, we find that correlated fluctuations are strongly suppressed in the actual spectral noise density of the residual phase $S_\varphi^{\rm res}$ (black solid line in Fig. 3), when $\varphi_{\rm res}$ is calculated according to Eq. (4), i.e. the elastic tape model is applied. At 100 Hz, the lowest frequency measured here due to memory size limitations of the oscilloscope, an enhancement of about 55 dB due to common mode rejection of correlated noise of the comb lines is demonstrated.

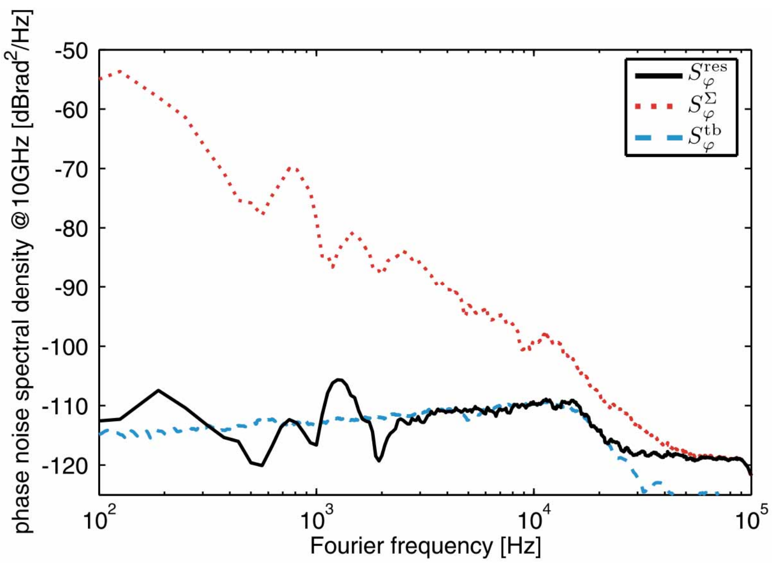

Fig. 3. Comparison between the experimentally determined spectral densities of the residual phase, treating comb fluctuations as correlated (black solid line) or uncorrelated (red dotted). The blue dashed line shows the noise floor of the measurement due to the phase noise of the oscilloscope time base.

It is important to notice that the experimentally determined residual phase noise spectral density S_φ^{res} is limited by the noise floor of the detection system in this first proof of principle and an actual enhancement due to the transfer oscillator principle even larger than 55 dB can be expected over the entire frequency spectrum. For Fourier frequencies below $f \approx 20$ kHz, the noise floor is given by the phase noise of the sampling oscilloscope time base, which is shown as a blue dashed line in Fig. 3. The noise floor above 20 kHz is caused by the S/N of the photodiodes PD3/4 due to AM-PM conversion and noise in associated amplifiers and mixers. In principle, this noise floor could be reduced by correlation-based phase noise measurements [16], in which the entire detection system is duplicated and the two identical systems share the same optical signal under test. The uncorrelated noise contributions of the detection systems can then be drastically reduced by measuring the temporally averaged cross-spectrum between the signals detected by the two systems.

With knowledge of S_φ^{res}, the resulting fractional frequency instability of the frequency comb in conjunction with a use of the transfer oscillator principle can now be estimated. The phase noise density S_φ^Σ shows a $1/f^2$ – frequency dependence between 100 Hz and 10 kHz, and the roll-off becomes steeper at frequencies above 10 kHz. It is reasonable to assume that the enhancement of at least 55 dB observed at low Fourier frequencies is also valid at higher frequencies, where it is only hidden under the noise floor of the detection system. Hence, essentially the same frequency dependence for S_φ^{res} can be assumed as for S_φ^Σ, resulting in a white frequency noise level of $S_\nu = f^2 S_\varphi^{\mathrm{res}}(f) = 4\times10^{-8}\ \mathrm{Hz}^2/\mathrm{Hz}$ between 100 Hz and 10 kHz referred to a carrier frequency of $\nu_0 = 10$ GHz. Since fluctuations in the fiber laser based frequency comb arise from a piece of fiber with finite length, the value of the residual phase φ_{res} is bounded. Therefore, the phase noise density does not diverge at low Fourier frequencies. However, assuming as a conservative estimate that the frequency noise remains white below 100 Hz, the Allan standard deviation is given by [17]:

$$\sigma_y = \frac{1}{\nu_0}\sqrt{\frac{S_\nu}{2\tau}} = 1.4\times10^{-14}\left[\tau\mathrm{Hz}\right]^{-1/2}, \tag{5}$$

where τ is the averaging time. Hence, a fractional frequency instability of 1.4×10^{-14} in 1 s averaging time results. It should be noted that this is rather a conservative estimate, mainly because the measured 55 dB enhancement due to application of the transfer oscillator principle is limited by the noise floor of the detection system. In principle, this result could also be obtained by frequency counting, which would, however, require counters with impracticable interpolation capabilities of 10^{-4} of a period, i.e. a timing resolution of 10 fs at $\nu_0 = 10$ GHz.

It should be noticed that the data post-processing procedure used here can readily be substituted by real time analogous signal processing electronics [10], i.e. high-stability frequency measurements and narrow linewidths can be accomplished by proper pre-scaling and mixing of signals detected for ν_{CEO}, f_{rep} and ν_{x}. In addition, we would like to point out that the previous finding of different linewidths for beat notes generated at various positions within a fiber based frequency comb might be explained along the lines of the results presented here. As shown above, the fluctuations of the repetition rate and the CEO frequency cancel out against each other in the optical regime close to the carrier frequency, i.e. the elastic tape is fixed at 1.5 µm. The stable position of the comb lines in this wavelength region leads to narrow-band beat signals, as observed here and by other groups before [7, 8]. For spectral regions distinct from the carrier, the elastic tape model predicts a higher amount of fluctuations in the position of the comb lines. This behavior is reflected in the higher bandwidth of beat signals generated in remote spectral regions, as described in Ref. [8]. As we have shown here, this variation of the beat linewidth is however not an indication of uncorrelated phase noise in one spectral part of the supercontinuum or another.

5. Conclusions

We have demonstrated that substantial parts of the noise contributions in fiber lasers can be circumvented by exploiting the fact that the fluctuations are correlated in compliance with the elastic tape model. This leads to a reduced frequency noise level as low as $4\times10^{-8}\ \mathrm{Hz}^2/\mathrm{Hz}$ at 10 GHz, from which a residual frequency measurement instability below $2\times10^{-14}\left[\tau\mathrm{Hz}\right]^{-1/2}$ can be inferred. To the best of our knowledge, this is the lowest value reported for a fiber laser based frequency comb generator with an octave-spanning supercontinuum. Hence, the noise of the frequency link does not impair the frequency measurement of any microwave or optical oscillator known today [1] and thus enables precision frequency metrological applications with the presented Er:fiber based system.

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