Correction of low order aberrations using continuous deformable mirrors

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Abstract: By analyzing the Poisson equation describing the static behavior of membrane and bimorph deformable mirrors and biharmonic equation describing the continuous facesheet mirror with push-pull actuators, we found that to achieve a high quality correction of low-order aberrations these mirrors should have sufficient number of actuators positioned outside the correction aperture. In particular, any deformable mirror described by the Poisson equation requires at least two actuators to be placed outside the working aperture per period of the azimuthal aberration of the highest expected order. Any deformable mirror described by the biharmonic equation, such as a continuous facesheet mirror with push-pull actuators, requires at least four actuators to be placed outside the working aperture per period of the azimuthal aberration of the highest expected order, and these actuators should not be positioned on a single circle.

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1. Introduction

Deformable mirrors (DM) are potentially applicable for precision shaping of aspherical surfaces in imaging, laser optics and astronomy. Many of these applications require low-order optical surfaces to be dynamically formed with large amplitude and very high fidelity, in many cases exceeding the Rayleigh criterion. Practice shows that the increase of the number of actuators inside the deformable mirror aperture does not always lead to a complete elimination of the approximation error, as the precision of forming of a low-order aberration is limited by the high frequency actuator print-through effect. In this article, we formally analyze the necessary conditions for a high-quality correction of low-order aberrations considering the most popular types of mechanical deformable mirrors, with special attention paid to the minimization of the approximation error.

2. Differential equations describing the DM response

Since its introduction [1, 2], the DM technology has gone through an impressive evolution. At the moment of writing we can identify three major groups of DM:

- DM formed by a stretched membrane (membrane DM) [3, 4] ;
- DM formed by a thin flexible plate, controlled by bending moments applied to the plate (bimorph DM) [5, 6];
- DM formed by a thin ($h < 10d$, where h is the thickness and d is the diameter) flexible plate, controlled by a set of forces applied orthogonally to the its surface (continuous facesheet DM) [7, 8, 9].

The deformation $\varphi(x,y)$ of a membrane DM satisfies the Poisson equation:

$$\Delta\varphi(x,y) = p(x,y)/T, \tag{1}$$

where Δ is the Laplace operator, p is the pressure caused by the actuators and T is the membrane tension. A similar equation $\Delta\varphi(x,y) = kV(x,y)$ describes the behavior of a bimorph DM [6] with a free edge, controlled by the voltage V, where k is the scaling coefficient depending on the DM design.

The deformation of a continuous facesheet DM with discrete push-pull actuators, in the thin plate approximation is described by the biharmonic equation [10]:

$$\Delta^2\varphi(x,y) = p(x,y)/D, \tag{2}$$

where D is the cylindrical stiffness, the force p describing the action of actuators satisfies the conditions of static equilibrium and $\varphi(x,y)$ satisfies to some additional boundary conditions. Equation 2 assumes the plate has tip-tilt degrees of freedom in the points of attachment to the actuators.

To find the DM controls p corresponding to the deformation required, we need to apply the differential operator Δ (Δ^2 for the continuous facesheet DM) to the function φ.

Substituting $\varphi = Z_n^m$, where Z_n^m is a Zernike polynomial with the radial order n and azimuthal order $\pm m$ as defined in [11], we find (see Appendix A) that

$$\Delta\varphi(x,y) = 0, \tag{3}$$

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for all Zernike polynomials Z_n^m with $n = |m|$ including piston, tip, tilt, astigmatism, trefoil, etc.; and

$$\Delta^2 \varphi(x,y) = 0, \tag{4}$$

for all Zernike polynomials Z_n^m with $n = |m|$ and also with $n-2 = |m|$, including piston, tip, tilt, defocus, astigmatism, trefoil, coma and some higher order aberrations. To give an example, Fig. 1 shows the shape of Zernike term Z_7^7 satisfying to both Eq. 3 and Eq. 4 and term Z_5^3 satisfying only to Eq. 4.

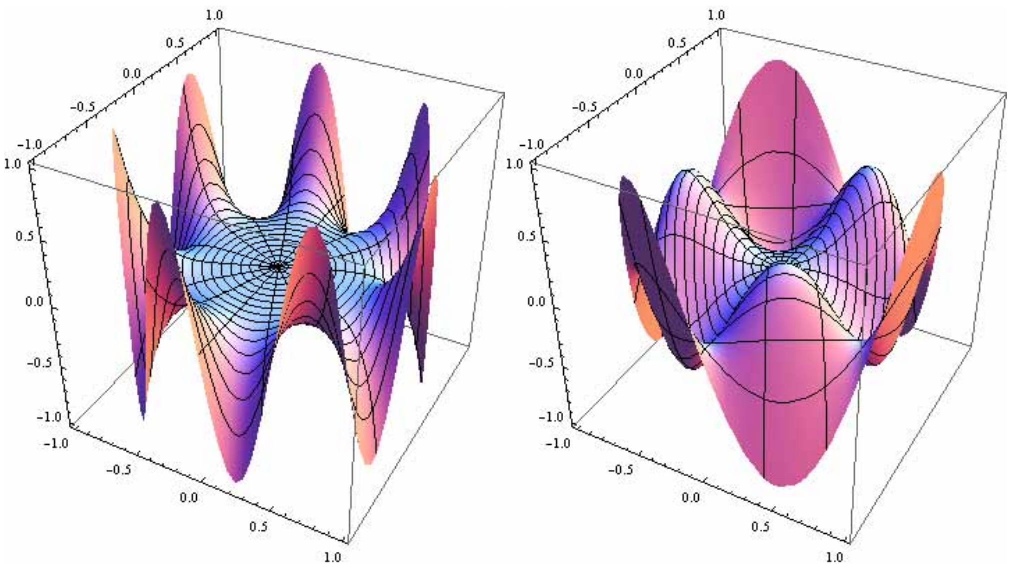

Fig. 1. Example of the DM modes that cannot be produced exactly without the actuators positioned outside the correction area: Z_7^7 (left) for both membrane and thin plate DM and Z_5^3 (right) for a thin-plate DM only.

This result can be explained by the fact that Eq. 1, 2 describe the DM shape in terms of actions p defined only within the correction aperture, while some modes can be exactly defined only by actuators positioned outside the correction aperture. These modes cannot in principle be found as an exact solution of Eq. 1 or Eq. 2 in terms of non-zero correction actions p inside the correction aperture.

3. Requirements to the geometry of actuators

Practical correction of any aberration satisfying to Eq. 3 for a membrane and bimorph DM, or Eq. 4 for a thin-plate DM, performed only by the actuators positioned within the aperture, results in a "bumpy" approximation to the desired shape. To obtain a high quality correction, a certain number of actuators should be positioned outside of the DM aperture so that the conditions Eqs. 3, 4 are satisfied inside the aperture, while the external actuators produce the required boundary corrections. The number of these external actuators should be large enough to produce a sufficient number of independent modes.

It was shown in [12] that for any DM with K_m actuators one can find a corrector with $K_z \leq K_m$ modes, described by the Zernike polynomials, such that these two correctors will have equivalent performance in the correction of aberrations with certain pre-determined statistics. It is also well known that low-order Zernike terms usually have higher statistical weights than the higher-order terms in most natural and artificial aberration sources. Therefore it is important

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Table 1. Zernike polynomials tree, the terms satisfying to the condition Eq. 3 are printed as bold capital: $\mathbf{Z}_n^m$; the terms satisfying to both conditions Eq. 3 and Eq. 4 are printed as regular capital: Z_n^m; all other terms are printed as regular: z_n^m.

$\mathbf{Z}_0^0$							
$\mathbf{Z}_1^1$	$\mathbf{Z}_1^{-1}$						
$\mathbf{Z}_2^2$	Z_2^0	$\mathbf{Z}_2^{-2}$					
$\mathbf{Z}_3^3$	Z_3^1	Z_3^{-1}	$\mathbf{Z}_3^{-3}$				
$\mathbf{Z}_4^4$	Z_4^2	z_4^0	Z_4^{-2}	$\mathbf{Z}_4^{-4}$			
$\mathbf{Z}_5^5$	Z_5^3	z_5^1	z_5^{-1}	Z_5^{-3}	$\mathbf{Z}_5^{-5}$		
$\mathbf{\dots}$	$\mathbf{\dots}$	$\dots$	$\dots$	$\dots$	$\mathbf{\dots}$	$\mathbf{\dots}$	
$\mathbf{Z}_N^N$	Z_N^{N-2}	z_N^{N-4}	$\dots$	$\dots$	z_N^{-N+4}	Z_N^{-N+2}	$\mathbf{Z}_N^{-N}$

to develop wavefront correctors with good performance of the correction of low-order Zernike terms. Counting the total number of terms in the Table 1, starting from the zero order $n = 0$ and up, we see that the total number of low-order terms with $n \leq N$ is equal to $(N^2 + 3N + 2)/2$, thus the total number of actuators K_{tot} should satisfy the condition

$$K_{tot} \geq (N^2 + 3N + 2)/2. \tag{5}$$

The number of terms satisfying the condition Eq. 3 is equal to $2N + 1$ and the number of terms satisfying to the condition Eq. 4 is equal to $4N - 2$. Therefore, the number of actuators outside the working aperture of the membrane or a bimorph DM should satisfy the condition

$$K_{ext} \geq 2N + 1, \tag{6}$$

while for a continuous facesheet DM the number of external actuators should satisfy the condition:

$$K_{ext} \geq 4N - 2. \tag{7}$$

These formulae can be used in practical designs to define the minimum requirements to the numbers of actuators inside and outside the correcting aperture of the DM. For instance, to correct all low-order terms up to the degree $N = 4$, a continuous faceplate DM should have at least $K_{total} = (4*4+3*4+2)/2 = 15$ actuators, of which at least $4*4-2 = 14$ should be placed outside the working aperture, while in the same situation a membrane or a bimorph DM should have at least $2*4+1 = 9$ actuators placed outside the working aperture.

We also need to mention that the external actuators of a continuous faceplate DM should not be positioned on a single circle. There are two conditions on the contour of the correction area to be defined: $\varphi|_S$ and $\frac{\partial\varphi}{\partial r}|_S$, where r is the radial coordinate, and S is the boundary of the correction area. A non-zero radial derivative (second condition) can be satisfied only by two push-pull actuators positioned at different radial distances from the DM center. As a practical solution we suggest to place one half of the outside actuators on a circle, with equal azimuthal spacing between the actuators. The another half of the external actuators should be placed on a circle with a different radius, also uniformly distributed by the azimuthal coordinate. Although we are not sure that this geometry is optimal for each possible application, it seems to be quite practical. Membrane and bimorph DM can have all outside actuators positioned on a single circle, as only the deflection $\varphi|_S$ should be defined.

Since our expressions define only the lower limit for the actuator count, the numbers of actuators chosen for any practical design should be somewhat larger than defined by Eqs. 5, 6 and 7.

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For a DM with relatively large number of actuators, Eqs. 6 and 7 have very simple interpretation: any DM described by the Poisson equation requires at least two actuators to be placed outside the working aperture per period of the azimuthal aberration of the highest expected order. Any DM described by the biharmonic equation requires at least four actuators to be placed outside the working aperture per period of the azimuthal aberration of the highest expected order, and these actuators should not be positioned on a single circle.

Equations. 1 and 2 describe the ideal model, while the mathematical descriptions of practical DM designs can be much more complicated. However, in a very general case, the linear elastic deformations of a solid body (excluding plastic deformations) are usually described by a linear differential operator of the second order or higher, therefore at least some basic modes will satisfy to conditions similar to Equations 3, 4. Therefore the essence of our analysis remains valid for all these cases, regardless the complexity of the mathematical description.

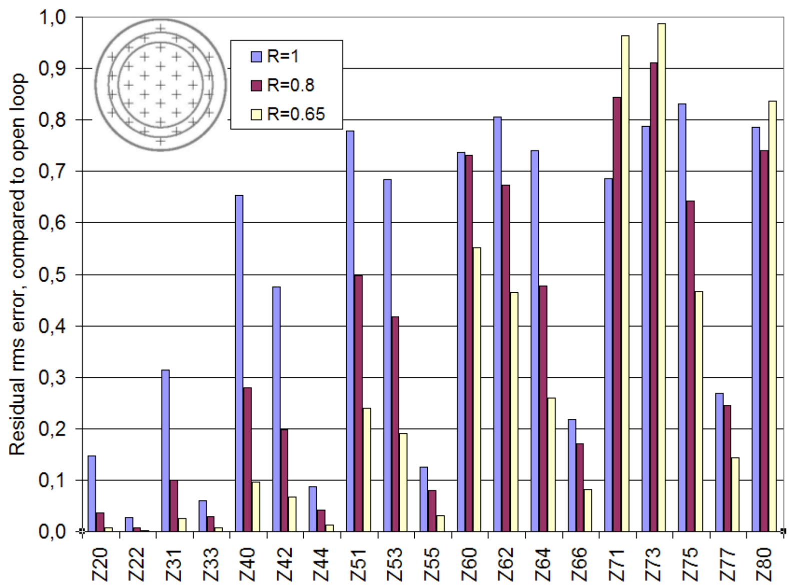

Fig. 2. Residual *rms* error for a 37-ch DM described by (2) for low order Zernike polynomials $Znm = Z_n^m$ for three different values of the correction aperture R. The actuator geometry and the correction apertures of the DM are shown in the inset.

To verify our conclusions, we have built a numerical model that calculates the correction error for a thin-plate free-edge DM described by Eq. 2 and for membrane DM described by the Eq. 1. Figure 2 shows the residual *rms* correction error for a 37-ch continuous facesheet DM with push-pull actuators for low order Zernike polynomials Z_n^m for three different values of the correction aperture $R = 1;\ 0.8;\ 0.65$. We used only positive values of the index m as the negative values correspond to a $\pi/2$ rotation of the aberration, it follows from the uniformity of the geometry of the DM actuators that the performance of the DM should be approximately the same for the positive and the negative values of m. The correction quality of low-order aberrations is poor when the full aperture $R = 1$ is used and all actuators are positioned within the correction aperture. The quality improves significantly for smaller R practically for all low-order aberrations, because the number of actuators inside the correction aperture decreases and the forces developed by these actuators decrease too, while the external actuators develop higher forces to form the modes satisfying the conditions Equations 3 and 4. Polynomials Z_7^1 and Z_7^3 do

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not satisfy to the Eq. 4, therefore the correction of these terms requires at least some actuators to be positioned within the correction aperture. For these modes, the correction quality is better for the full aperture $R = 1$ and the error increases for smaller correction apertures such as $R = 0.8$ and $R = 0.65$. For aperture $R = 0.65$, the DM provides good correction for all Zernike modes up to the 5-th order, because in this case we have 19 actuators positioned outside the correction aperture and the condition Eq. 7 is satisfied for $N = 5$.

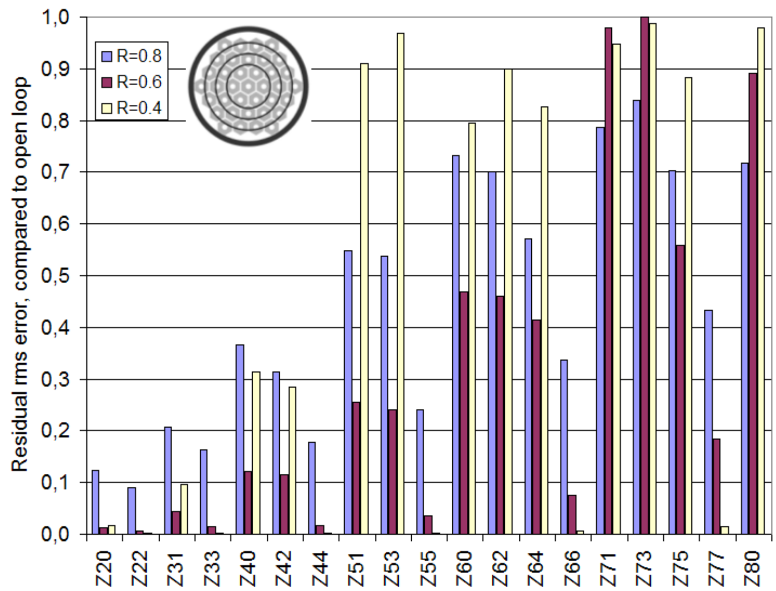

Fig. 3. Residual *rms* error for a 37-ch membrane deformable mirror described by Eq. 1 for low order Zernike polynomials $Znm = Z_n^m$ for three different values of the correction aperture R. The actuator geometry and the correction apertures of the DM are shown in the inset.

Figure 3 shows the calculated residual *rms* correction error for a 37-ch membrane DM for low order Zernike polynomials Z_n^m for three different values of the correction aperture $R = 0.8$; 0.6; 0.4. The actuator structure corresponds to the pattern used in OKO Technologies 15-mm 37-ch MMDM. The case $R = 1$ is not considered due to the fixed edge of the DM. The graph clearly shows that the correction quality improves significantly with the decrease of R for all modes Z_n^m with $n = m$, which can be explained by the increase in the number of actuators positioned outside the aperture. Correction of aberrations Z_7^1, Z_7^3 and Z_8^0 is strongly dependent on the number of internal actuators, resulting in optimal correction with $R = 0,8$ when the maximum number of actuators is positioned inside the correction aperture. All other modes require simultaneous correction with internal and external actuators, therefore the optimum correction is reached at $R = 0.6$.

4. Experimental verification

To confirm the validity of our conclusions, we experimentally measured the approximation error of a 30-mm 37-channel piezoelectric DM with a free edge for aperture values of $R = 0.92$, 0.8, 0.65 and 0.5. The DM (produced by OKO Technologies, Delft, The Netherlands) was controlled in a feedback loop by the FrontSurfer WF sensor (also produced by OKO Technologies). In the first stage, the DM was controlled to produce the best possible fit to coma with a PV am-

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plitude of 1 μm. In the second stage, the shape of the DM was measured by a high-resolution Hartmann-Shack sensor, and the *rms* error was calculated for the total surface deviation with respect to a flat δ_1, and for the surface error with respect to coma δ_2. The improvement factor δ_2/δ_1 corresponds to the theoretically calculated correction errors for Z_3^1 in the Fig. 2. Experimental and theoretical results are compared in Fig. 4. The experimental *rms* error of correction

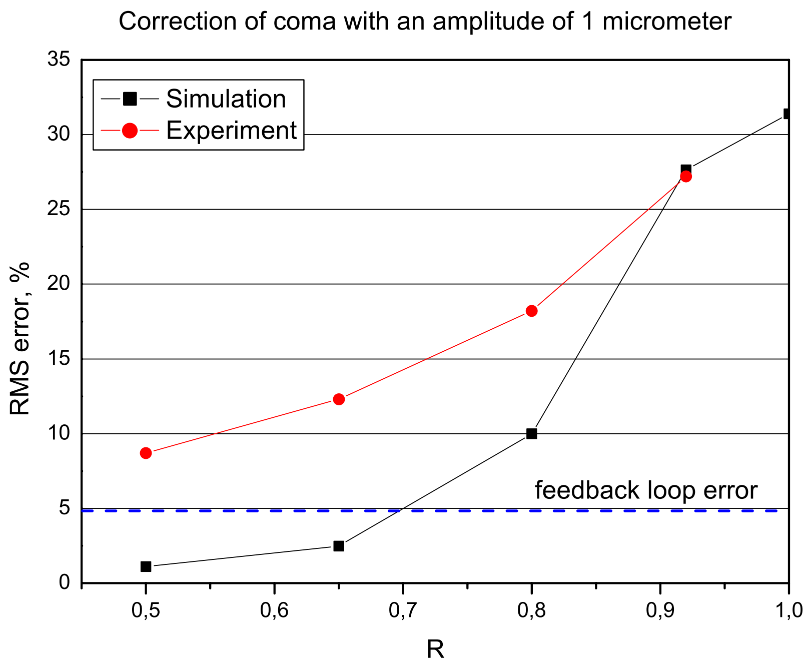

Fig. 4. Calculated and experimental residual *rms* error of compensation of coma with a 37ch DM with a geometry shown in the inset in Fig. 1. The difference between the theory and the experiment is explained by the fact that the maximum achievable precision in the experiment is limited by the feedback loop error.

was reduced by a factor of $\sim$3 when the correction aperture was reduced from $R = 0.92$ to $R = 0.5$, however this improvement is smaller than the expected theoretically factor of $\sim$18. The difference can be partly explained by the the rms error of the feedback loop δ_f depending on a combination of factors, including the sensor noise, wavefront reconstruction errors, control voltage discretisation errors etc. In our measurements the feedback rms error accounted to approximately $\delta_f \sim \lambda/40$ for the wavelength of $\lambda = 633$ nm. Another error factor is the stiffness of the DM actuators, resisting the rotations of the DM surface at the attachment points. Yet another error source is the actuator creep during the interval between the moment when the feedback loop was stopped and the moment the DM profile was measured. Accounting for these limiting factors, we conclude that our experiment demonstrated a good qualitative agreement with the theoretical model.

5. Conclusion

In conclusion, we wish to emphasize that a large number of actuators within the correction aperture of a DM does not automatically secure a high-quality correction of low-order aberrations and higher-order Zernike terms Z_n^m with $|m| = n$ and $|m| = n-2$. To achieve a high quality correction of all terms Z_n^m up to the order $n = N$, the bimorph and membrane DM should have at

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least $2N+1$ and the continuous facesheet DM should have at least $4N-2$ actuators positioned outside the correction area. For large numbers of actuators, these expressions can be translated into simple rules for the minimum number of exterior actuators: any DM described by the Poisson equation requires at least two actuators to be placed outside the working aperture per period of the azimuthal aberration of the highest expected order. Any DM described by the biharmonic equation, such as a continuous facesheet DM with push-pull actuators, requires at least four actuators to be placed outside the working aperture per period of the azimuthal aberration of the highest expected order, and these actuators should not be positioned on a single circle.

The requirement to have actuators outside the correction aperture results in a larger physical size of a deformable mirror, which may have implications in the design of portable and autonomous adaptive optical systems.

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A. Laplacian and bilaplacian of Zernike polynomials

In polar coordinates, any Zernike polynomial $\mathrm{Z}_n^{\pm m}(\rho,\theta)$ consists of terms $\rho^{l+m}{}^{\cos}_{\sin} m\theta$, where l is even and $0 \leq l \leq n-m$ (see, for instance, [11]). Thus $\mathrm{Z}_n^{\pm n}(\rho,\theta)$ consist only of one term $\rho^n {}^{\cos}_{\sin} n\theta$, and $\mathrm{Z}_n^{\pm(n-2)}(\rho,\theta)$ consists only of two terms $\rho^n {}^{\cos}_{\sin}(n-2)\theta$ and $\rho^{n-2} {}^{\cos}_{\sin}(n-2)\theta$.

Consider now Laplacian $\Delta = \nabla^2$ and biharmonic operators applied to the terms of Zernike polynomials $\rho^n {}^{\cos}_{\sin} n\theta$ and $\rho^n {}^{\cos}_{\sin}(n-2)\theta$. From the expression of Laplacian in polar coordinates:

$$\Delta f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\theta^2},$$

it immediately follows that

$$\Delta\rho^n {}^{\cos}_{\sin} n\theta = n^2\rho^{n-2} {}^{\cos}_{\sin} n\theta - n^2\rho^{n-2} {}^{\cos}_{\sin} n\theta = 0 \tag{8}$$

and

$$\begin{aligned}\Delta\rho^n {}^{\cos}_{\sin}(n-2)\theta &= n^2\rho^{n-2} {}^{\cos}_{\sin}(n-2)\theta - (n-2)^2\rho^{n-2} {}^{\cos}_{\sin}(n-2)\theta \\ &= 4(n-1)\rho^{n-2} {}^{\cos}_{\sin}(n-2)\theta,\end{aligned} \tag{9}$$

and, thus,

$$\Delta^2\rho^n {}^{\cos}_{\sin} n\theta = 0 \tag{10}$$

and

$$\Delta^2\rho^n {}^{\cos}_{\sin}(n-2)\theta = 0. \tag{11}$$

This proves that for every $n, n \geq 0$

$$\Delta\mathrm{Z}_n^{\pm n}(\rho,\theta) = 0 \tag{12}$$

and

$$\Delta^2\mathrm{Z}_n^{n}(\rho,\theta) = 0, \tag{13}$$

$$\Delta^2\mathrm{Z}_n^{\pm(n-2)}(\rho,\theta) = 0. \tag{14}$$

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