

A polarizing beam splitter using negative refraction of photonic crystals

Vito Mocella ^a, Principia Dardano ^{a,b}, Luigi Moretti ^{c,a} and I. Rendina ^a

^a*Istituto per la Microelettronica e Microsistemi – Consiglio Nazionale delle Ricerche (IMM-CNR) – Sez. di Napoli, Via P. Castellino 111, 80131 Napoli, Italy;*

^b*Dip. Scienze Fisiche, Università degli Studi di Napoli “Federico II, via Cintia, I-80126, Napoli, Italy;*

^c*Università “Mediterranea”, Località Feo di Vito, 89060 Reggio Calabria, Italy
vito.mocella@na.imm.cnr.it*

Abstract: Light passing through a photonic crystal can undergo a negative or a positive refraction. The two refraction states can be functions of the contrast index, the incident angle and the slab thickness. By suitably using these properties it is possible to realize very simple and very efficient optical components to route the light. As an example we present a passive device acting as a polarizing beam splitter where TM polarization is refracted in positive direction whereas TE component is negatively refracted.

© 2005 Optical Society of America

OCIS codes: (050.1960) Diffraction theory; (260.5430) Polarization; (290.4210) Multiple scattering; (999.9999) Photonic crystals; (999.9999) Negative refraction.

References and links

1. J.D. Joannopoulos, R.D. Mead, J.N. Winn, *Photonic crystal: Molding the flow of light*, Princeton University Press (Princeton, 1995).
2. K. Sakoda, *Optical Properties of Photonic Crystals*, Springer Verlag (2001).
3. H. Kosaka, T. Kawashima, A. Tomita, M. Notomi, T. Tamamura, T. Sato, and S. Kawakami, “Superprism phenomena in photonic crystals,” *Phys. Rev. B* **58**, R10 096-099 (1998).
4. M. Notomi. “Theory of light propagation in strongly modulated photonic crystals: Refractionlike behavior in the vicinity of the photonic band gap,” *Phys. Rev. B* **62**, 10696-10705 (2000).
5. F. Genereux, S. W. Leonard, H. M. van Driel, A. Birner and U. Gösele, “Large birefringence in two-dimensional silicon photonic crystals,” *Phys. Rev. B* **63**, 161101 (2001).
6. Lijun Wu, M. Mazilu, J.-F. Gallet and T. F. Krauss, “Dual lattice photonic-crystal beam splitters,” *Appl. Phys. Lett.* **86**, 211106, (2005).
7. T. Liu, A. R. Zakharian, M. Fallahi, J. V. Moloney and M. Mansuripur, “Design of a Compact Photonic-Crystal-Based Polarizing Beam Splitter,” *IEEE Photonics Technol. Lett.*, **17**, 1435-1437 (2005).
8. C. Y. Luo, S. G. Johnson and J. D. Joannopoulos. “All-angle negative refraction in a three-dimensionally periodic photonic crystal,” *Appl. Phys. Lett.* **81**, 2352-2354 (2002).
9. E. Cubukcu, K. Aydin, et al. “Negative refraction by photonic crystals,” *Nature* **423**, 604-605 (2003).
10. J. B. Pendry and D. R. Smith. “Reversing light with negative refraction,” *Physics Today* **57**, 37-43 (2004).
11. S Anantha Ramakrishna, “Physics of negative refractive index materials,” *Rep. Prog. Phys.* **68**, 449-521 (2005).
12. V. Mocella, “Negative refraction in Photonic Crystals: thickness dependence and Pendellösung phenomenon,” *Opt. Express* **13**, 1361-1367 (2005), <http://www.opticsexpress.org/abstract.cfm?URI=OPEX-13-5-1361>.
13. B. W. Battermann, H. Cole, “Dynamical diffraction theory of X rays by perfect crystals,” *Rev. Mod. Phys.* **36**, 681-717 (1964).
14. P.P. Ewald, “Crystal optics for visible light and X rays,” *Rev. Mod. Physics* **37**, 46-56 (1965).
15. G.S. Agarwal, D. N. Pattanyak, E. Wolf, “Electromagnetic field in spatially dispersive media,” *Phys. Rev. B*, **10**, 1447-1475 (1974).
16. K. Henneberger, “Additional Boundary Condition: an historical mistake,” *Phys. Rev. Lett.* **80**, 2889-2892, (1998).
17. J.J. Hopefield, D.G. Thomas, “Theoretical and Experimental Effects of Spatial Dispersion on the Optical Properties of Crystals,” *Phys. Rev.* **123**, 563-572 (1963).

1. Introduction

Photonic crystals (PhCs) offer new exciting opportunities in the control of light propagation [1,2]. Although the presence of a full photonic band gap (PBG) has mostly attracted the interest toward PhCs, other important properties related to the dispersion, anisotropy, and polarization characteristics of the photonic bands do not require a PBG [3-5]. Using these properties one could for instance fabricate very efficient PhC devices, which are extremely compact, overcoming the limit of conventional devices which require relatively large structures, such as beam splitters and polarizing beam splitters [6-7].

In particular, exploiting the PhC negative refraction behavior, a refracted beam can be obtained going in the “wrong” direction, compared to the direction expected from classical refraction laws. [8-9]. PhCs, in which a periodical modulation of the refractive index with periodicity comparable to the incident light wavelength is introduced, represent another way to achieve negative refraction, respect to metamaterials, where the inhomogeneities introduced are much smaller than the wavelength of the incoming radiation [10-11].

In a previous work we showed that it is possible to obtain a negative or a positive refraction at the exit surface of a PhC as a function of the slab width [12], following a classical derivation of the of x-rays diffraction [13-14]

The effect is due to a periodic exchange of energy inside the crystal between the components refracted in the positive and the negative direction, respectively. This result, well known in x-ray diffraction, is immediately applicable to low-contrast PhCs [12]. However, also high contrast PhCs exhibit the same effect, as it will be shown in section 2 of this work by an analysis based on equi-frequency surfaces. It will be proved that, in general, for a PhCs, many different wavevectors coexist and interference between such wavevectors inside the PhC is the key for understanding the occurrence of the physical phenomena observed.

We shall show how to make use of the interference to control the light propagation at the exit surface of the PhC sample. Indicating the period of the above modulation by Λ_0 (Pendellösung length) it can be shown that the energy transmitted at the exit surface of the PhC is at a maximum in the positive (i.e., forward diffracted) direction when the thickness is equal to an even multiple of $\Lambda_0 / 2$, whereas it exhibits a maximum in the negative (i.e., diffracted) direction when the thickness is an odd multiple of half the period, $\Lambda_0 / 2$.

In the following we discuss the origin of the Pendellösung effect and will extend the results of [12] to high contrast PhCs, in particular to the case of a square lattice of air holes in silicon ($\epsilon=11.9$ in the infrared region). Afterwards, we will show how to determine the relevant parameters of the PhC (normalized frequency, air hole radius and slab thickness) in such a way that the Pendellösung effect gives a maximum intensity for either positive or negative direction. Making use of this procedure we will illustrate the design of a particularly efficient polarizing beam splitter that works properly across the whole C-band of telecommunication (wavelength in the range: 1.3-1.65 μm).

2. Pendellösung in high contrast photonic crystals

In order to study the wave propagation in a medium, it is fundamental to establish the dispersion characteristics, i.e. the link between frequency and wavevectors in such a medium.

The dispersion relation determines all the allowed wavevectors corresponding to a given frequency, i.e. $\vec{k}(\omega)$. The classical approach for PhCs provides, for a given wavevector (in general along a path within the irreducible Brillouin zone), the eigenfrequencies, i.e. $\omega(\vec{k})$. In vacuum, as in any homogeneous material, the dispersion relation in reciprocal space represents a sphere of radius, $k = 2\pi/\lambda$, where λ is the wavelength in the medium (in vacuum $k = \omega/c$).

Considering spatial dispersion in a more general form of the dispersion relation (for transverse waves) in a non-magnetic medium, one can write:

$$\vec{k}^2 = \vec{k} \cdot \vec{k} = (\omega/c)^2 \tilde{\epsilon}(\vec{k}, \omega) \quad (1)$$

where the term $\tilde{\epsilon}(\vec{k}, \omega)$, indicating the Fourier transform of $\epsilon(\vec{r}, t)$, is dependent on \vec{k} . Relation (1) includes the simplest case of a homogeneous, spatially non dispersive medium, where $\tilde{\epsilon}(\vec{k}, \omega)$ does no longer depend on \vec{k} , therefore (1) becomes a simple quadratic relation in k , for a given frequency ω . In such a case, taking into account also the temporal dispersion, i.e. $\tilde{\epsilon}(\omega)$, and considering \vec{k} as a real quantity, (1) describes a simple sphere of constant radius $|k| = (\omega/c) \sqrt{\tilde{\epsilon}(\omega)} = 2\pi/\lambda$ in \vec{k} -space.

The dispersion relation (1) is in general of higher order than 2 in k , because of the dependence of $\tilde{\epsilon}(\vec{k}, \omega)$ on \vec{k} . According to (1) many monochromatic plane waves of the same frequency can propagate inside the medium, with different phase and group velocities. This is the origin of a number of particular features exhibited by dispersive media, well explained by Maxwell theory [15-16].

PhCs, like real crystals in x-ray diffraction, are a special case of a spatially dispersive medium, wherein a periodic spatial variation of the dielectric properties has to be considered. To a first approximation, a medium can be considered homogeneous when the wavelength is much longer than the typical size of inhomogeneities; in the case of a PhC the inhomogeneities are of the order of the lattice periodicity, hence the dispersion surfaces occur to be spheres centered in each reciprocal lattice point, Fig. 1(a).

When the wavelength decreases the PhC cannot be considered homogeneous any more because spheres approach each others. The sphere split results into a more complex dispersion surface and a Bragg gap appears around the point in k -space indicated by L_0 , where the Bragg law is satisfied, see Fig. 1(b) and also Fig. 3.

For a low contrast PhC, the dispersion, or equivalently the equi-frequency curve can be approximated by a hyperbola close to the point L_0 . One has to consider also the corresponding point on the other branch of the curve, L'_0 in Fig. 1(b), since the dispersion relation is a fourth-order curve in \vec{k} , thus implying terms like $1/k^2$ in $\tilde{\epsilon}(\vec{k}, \omega)$, similar to those obtained in cases next to an excitonic resonance in microscopic models, having taken into account spatial dispersion (s. e.g. Eq. 1.23 in [16]).

As mentioned above, an important consequence of the dispersion relation (1) for spatially dispersive media is that monochromatic plane waves of the same frequency ω propagate with different phase velocities (the usual definition of phase velocity being $v_p = c / \sqrt{\text{Re}[\tilde{\epsilon}(\vec{k}, \omega)]}$), even if they propagate in the same direction. This is the origin of the Pendellösung phenomenon, due to the phase modulation between coexisting plane wave components, propagating in the same direction.

We consider here only the components corresponding to O and H (see Fig. 1) in the Bloch expansion of the wave field inside the PhC and assume the incident wavevector to lie on a dispersion surface of the external medium (i.e. silicon for our investigations). We consider the case that the Bragg law applies for the \overline{OH} vector of the reciprocal space (see Fig 1c). Owing to the conservation of the tangential components of the wavevectors across the surface boundary, the wavevectors inside the PhC ($\overline{P_1O}, \overline{P_2O}, \overline{P_1H}, \overline{P_2H}$) are thus determined.

It can be noticed that they exhibit a path difference $2\pi/\Lambda_0$ along the normal direction \vec{n} . This implies a phase difference between coexisting plane wave components, so that Λ_0 is the modulation period along the normal to the sample surface in real space.

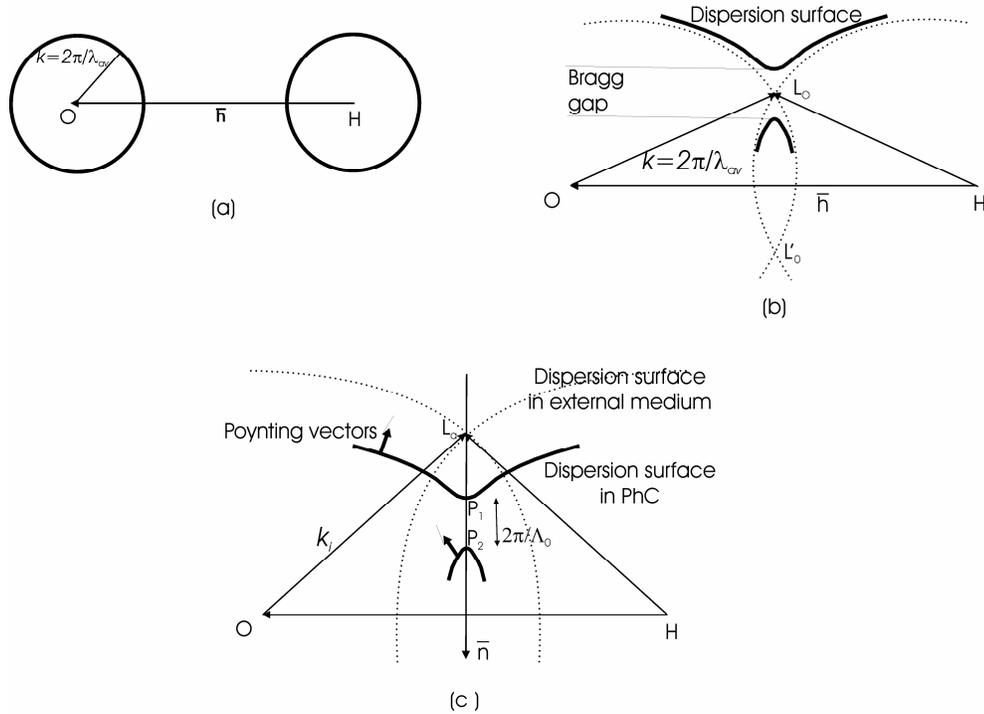


Fig. 1. When the incoming wavelength is larger than the lattice period of the PhC the medium can be considered homogeneous and the dispersion surfaces are spheres (a). When the wavelength decreases the spheres approach one another and a Bragg gap appears (b). The conservation of the tangential component of the wavevector in the external medium, \vec{k}_i , determines the wavevectors in the PhC: $\vec{P}_1\vec{O}$, $\vec{P}_2\vec{O}$, $\vec{P}_1\vec{H}$, $\vec{P}_2\vec{H}$ (c).

In [12] it was shown that, for low contrast PhCs, it is possible to obtain outgoing intensity by modulating the thickness on a period Λ_0 , either in positive or in negative direction. This is a direct consequence of the interference between plane wave components propagating inside the crystal (Pendellösung¹), in analogy with a well known equivalent effect described by the dynamical diffraction of x-rays, where, however, the refractive index contrast is extremely low ($\sim 10^{-6}$).

However low contrast is not necessary and Pendellösung occurs also for high contrast PhCs. In the following we will analyze the case of a 2D square lattice PhC composed by air holes in silicon ($\epsilon=11.9$).

In such case a numerical solution of dispersion surface is necessary, since the analytical derivation in [12] apply only for low-contrast PhCs. The corresponding dispersion (equi-frequency) surface for the lower (valence) 0-band is shown in Fig. 2(a) for a sample with $r/a=0.195$, where r is the hole radius and a is the lattice parameter, for TM polarization (E field out of plane, along the holes' axis). It appears that for large wavelengths (i.e. normalized frequencies $\omega_n = \omega a/2\pi c$) the dispersion surfaces are circles and the medium acts as a homogenous one. When the wavelength decreases the circles centered in adjacent Brillouin zones intersect (approximately at $\omega_n=0.15$). The dispersion surface becomes more complex, in particular there is an overlap of the upper zone of the valence band with the lower zone of the

¹ The German name Pendellösung has its origin in the analogous mechanical system of energy exchange between coupled pendulums [14]. It is interesting to state a very close mechanical analogy for the excitons' mass in [17].

first upper band. The dispersion surface at $\omega_n=0.177$ for both bands is shown in Fig. 2(b), where the Pendellösung parameter Λ_0 is also underlined.

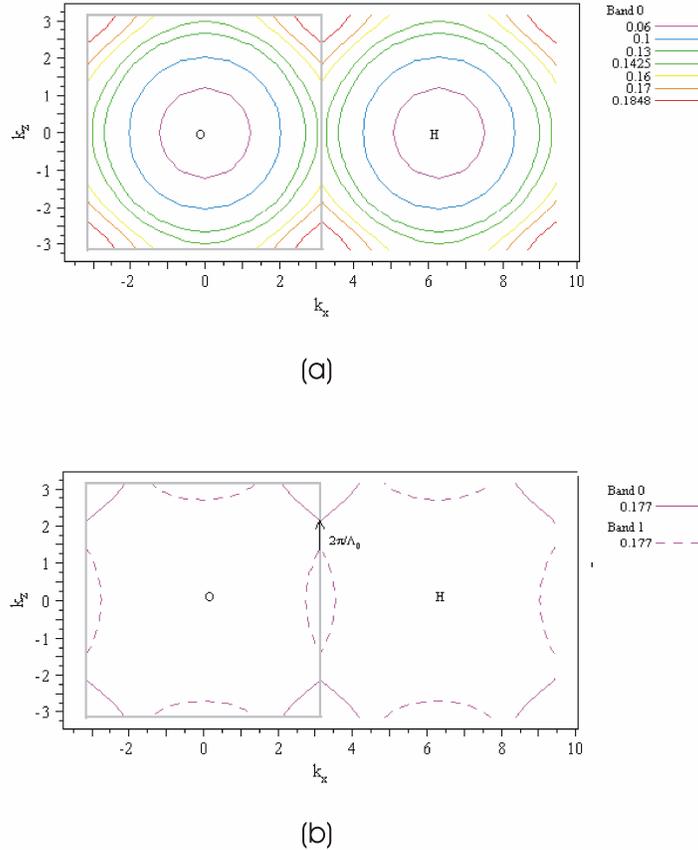


Fig. 2. Dispersion (equi-frequency) surface for an air hole square lattice PhC in silicon ($\epsilon=11.9$), $r/a=0.195$, in two adjacent Brillouin zones along the ΓX direction for TM polarization (E along the holes' axis). By increasing the normalized frequency from 0.06 to 0.1848, the dispersion surfaces intersect over the lower 0-band (a). In such a case the upper 1-band has to be considered as well to get the complete dispersion surface, where the Bragg gap $2\pi/\Lambda_0$ occurs (b).

The coexistence of many wavevectors can be seen also by looking at the band diagram, Fig. 3(a). If we just consider the XM direction it is evident that for a given frequency in the ranges $0.157 < \omega_n < 0.205$, $0.241 < \omega_n < 0.325$, $0.365 < \omega_n < 0.458$, labeled as α, β, γ , respectively, in Fig. 3(a) there are two allowed wavevectors, as it is confirmed by the dispersion surfaces in Fig. 3(b) and Fig. 3(c).

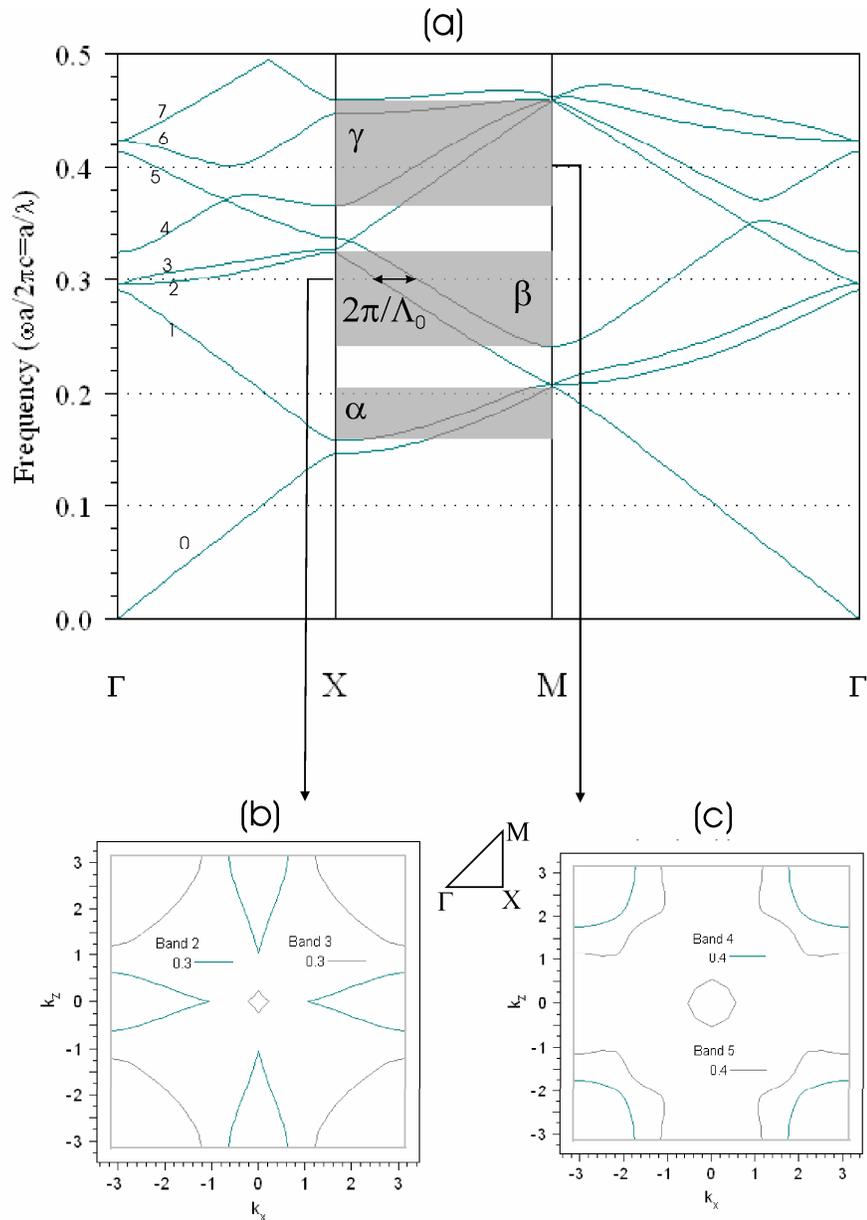


Fig. 3. Band diagram of the PhC of Fig. 2 for TM polarization (a). The regions along the XM direction, where many wavevectors are allowed for a given frequency, are highlighted in gray and labeled as α - β - γ . The dispersion surfaces corresponding to a frequency in region β (b) and region γ (c) show the overlap of different bands.

The same applies in frequency ranges along the other symmetry directions Γ X and Γ M. Intensity will hence have a maximum in positive (+) direction when the thickness t of the PhC is an integer multiple of Λ_0 ; the intensity will have a maximum in negative (-) direction when t is an odd multiple of the half-period $\Lambda_0/2$, i.e.:

$$I_{+\max} \rightarrow t = 2m \frac{\Lambda_0}{2}$$

$$I_{-\max} \rightarrow t = (2m-1) \frac{\Lambda_0}{2} \quad m = 1, 2, \dots \quad (2)$$

3. Polarizing Beam Splitter based on the Pendellösung effect

From the analysis of the previous section we know that the intensity at the exit of a PhC sample with thickness t can be a function of the Pendellösung parameter Λ_0 , following the relations (2).

For a given geometry the parameter Λ_0 is a function of polarization, of the normalized frequency ω_n , as well as the index contrast and the ratio r/a . Using this dependence it is possible to design a PhC sample that exhibits certain suitable properties.

In particular Λ_0 is different for the TE and TM polarizations. From (2) it is clear that if, for $m=1, 2, \dots$ the requirement:

$$t = 2m \frac{\Lambda_{0TE}}{2} = (2m-1) \frac{\Lambda_{0TM}}{2} \quad (3)$$

is met, a PhC sample of thickness t becomes a polarizing beam splitter (PBS), where TE is positively and TM is negatively refracted. In order to determine the parameters which fulfil the condition (3) we study the dependence of $\Lambda_{0TE, TM}$ as a function of ω_n , for a given r/a .

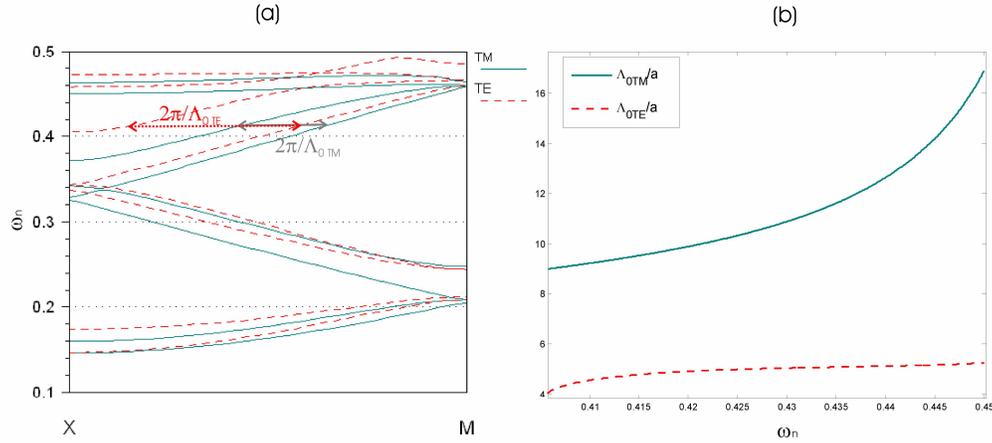


Fig. 4. Band diagram for TM and TE polarization referring to the same PhC as in Fig. 2, along the XM direction (a). Λ_0/a as a function of ω_n (b). The difference between the two polarization states is apparent.

The band structure in the XM direction, calculated for both TE and TM polarization, displays the different behaviours of $\Lambda_{0TE, TM}$, Fig. 4(a). Plotting $\Lambda_{0TE, TM}$ as a function of ω_n , Fig. 4(b) one sees that for $\omega_n=0.4125$ $\Lambda_{0TE}=5a=1/2\Lambda_{0TM}$. The condition (3) is satisfied for $m=1$, so that using a thickness $t=5a$, a 2D PhC square lattice in Silicon with $r/a=0.195$ can be used as a PBS. Indeed when an incident wave with frequency $\omega_n=0.4125$ impinges at an angle $\theta_B=20.57^\circ$ with respect to the surface normal (oriented along XM), so that the extremity of the incident wavevector is along XM, the TE polarized wave is positively refracted whereas the TM polarized wave is negatively refracted.

In Figs. 5(a) and 5(b) a movie versus the time of simulation generated using a FDTD code for both polarizations shows clearly how simply and efficiently the PBS works. For the movie we fixed $a = 0.64 \mu\text{m}$ and $\omega_n = 0.4125$ and hence $\lambda = 1.55 \mu\text{m}$.

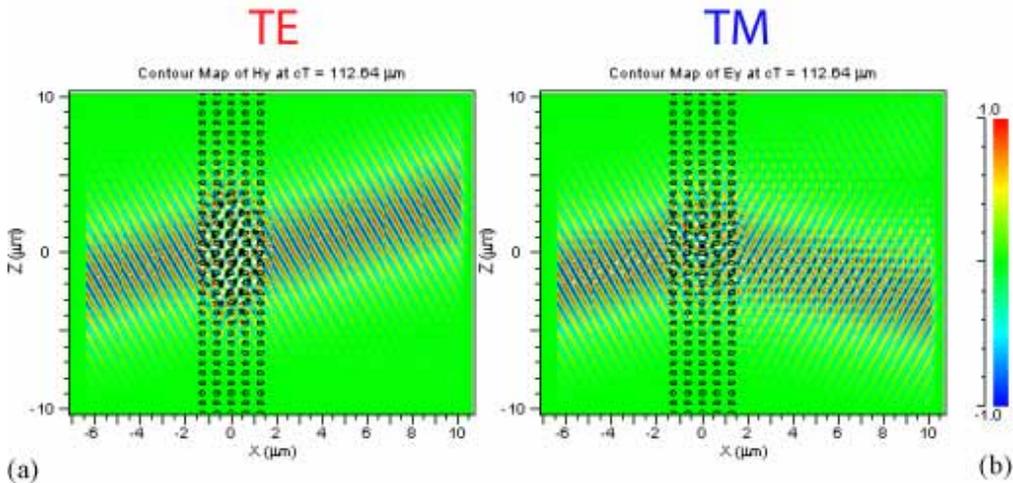


Fig. 5. (2.42 Mb) Movie versus time of FDTD simulation for TM (a) and TE polarization (b). The incident wave ($\lambda = 1.55 \mu\text{m}$) has a Gaussian profile with $4 \mu\text{m}$ FWHM and impinges at an angle 20.57° over a PhC square lattice of air holes in silicon with $r/a = 0.195$, $a = 0.64 \mu\text{m}$. The grid size in calculation is 15 nm , in x and z direction.

The efficiency of this device is very high also varying the frequency. A broad working range is anticipated, with a tolerance of few degree to the variations in incidence angle. In Fig. 6 the power flow of the refracted beams in positive and in negative directions, together with the reflected beam is displayed when the device is designed to work around $\lambda = 1.55 \mu\text{m}$. As it is shown in Fig. 6 the efficiency is as high as roughly 90 % in all the C-band ensuring an effectiveness of the device for practical use in infrared telecommunication.

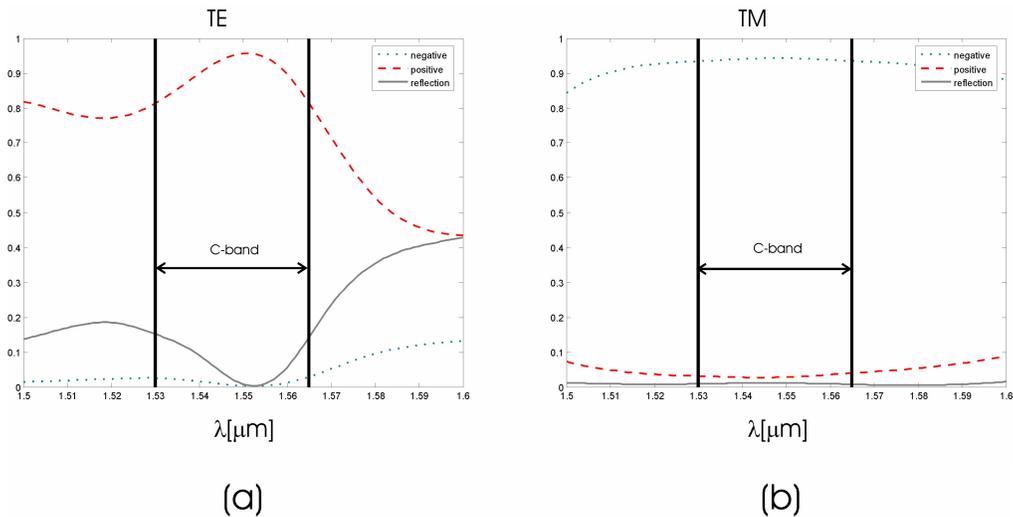


Fig. 6. The power flows of the refracted beams and of the reflected beam as a function of the wavelength for TE (a) and TM (b) polarization.

4. Conclusions

We have shown that interference between the positively and negatively refracted components of the incident wave gives rise to a Pendellösung effect for high contrast PhCs. It is possible to make use of this effect to design very simple and efficient devices, where the exit wave is either positively or negatively refracted, following the design parameters. The procedure for designing an appropriate device has been illustrated for the case of a PBS, which has proven to be efficient over all the telecommunication infrared C-band. Devices based on the Pendellösung effect are therefore a valid alternative to other devices based on more usual properties of PhCs.

Acknowledgments

The authors gratefully acknowledges Claudio Ferrero for helpful comments. This research has been supported by the project 156-DM 1105/2002.