The expansion coefficients of arbitrary shaped beam in oblique illumination

Yiping Han, Huayong Zhang and Guoxia Han

School of Science, Xidian University, Xi'an, 710071, P. R. China,
yphan@xidian.edu.cn

Abstract: In Generalized Lorenz-Mie theories, (GLMTs), the most difficult task concerns the description of the illuminating beam. We provide an approach for expansions of the incident arbitrary shaped beam in spherical and spheroidal coordinates in the general case of oblique illumination. The representations for shaped beam coefficients are derived by using addition theorem for spherical vector wave functions under coordinate rotations. For the special cases of the particle center located on shaped beam axis and plane wave, the simplified expressions are given. The convergence of the beam shape coefficients is discussed.

© 2007 Optical Society of America

OCIS codes: (290.4020) Mie theory; (140.3430) Laser theory; (290.5850) Scattering, particles; (260.2110) Electromagnetic theory.

References and links

1. G. Gouesbet, B. Maheu, and G. Gréhan, "Light scattering from a sphere arbitrarily located in a Gaussian beam, using a Bromwich formulation," J. Opt. Soc. Am. A **5**, 1427-1443 (1988).
2. G. Gouesbet, G. Gréhan, and B. Maheu, "Computations of the g_n coefficients in the generalized Lorenz-Mie theory using three different methods," Appl. Opt. **27**, 4874-4883 (1988).
3. G. Gouesbet, G. Gréhan, and B. Maheu, "Localized interpretation to compute all the coefficients g_n^m in the generalized Lorenz-Mie theory," J. Opt. Soc. Am. A **7**, 998-1003 (1990).
4. K. F. Ren, G. Gréhan, and G. Gouesbet, "Scattering of a Gaussian beam by an infinite cylinder in the framework of generalized Lorenz-Mie theory formulation and numerical results," J. Opt. Soc. Am. A **14**, 3014-3025 (1997).
5. Z. S. Wu, L. X. Guo, K. F. Ren, G. Gouesbet, and G. Gréhan, "Improved algorithm for electromagnetic scattering of plane waves and shaped beams by multilayered spheres," Appl. Opt. **36**, 5188-5198 (1997).
6. A. Doicu and T. Wriedt, "Computation of the beam-shape coefficients in the generalized Lorenz-Mie theory by using the translational addition theorem for spherical vector wave functions," Appl. Opt. **36**, 2971-2978 (1997).
7. J. P. Barton, D. R. Alexander, and S. A. Schaub, "Internal and near-surface electromagnetic fields for a spherical particle irradiated by a focused laser beam," J. Appl. Phys. **64**, 1632-1639 (1988).
8. J. P. Barton, "Internal and near-surface electromagnetic fields for a spheroidal particle with arbitrary illumination," Appl. Opt. **34**, 5542-5551 (1995).
9. E. E. M. Khaled, S. C. Hill, and P. W. Barber, "Scattered and internal intensity of a sphere illuminated with a Gaussian beam," IEEE Trans. Antennas Propagat. **41**, 259-303 (1993).
10. F. M. Schulz, K. Stamnes, and J. J. Stamnes, "Scattering of electromagnetic wave by spheroidal particles: a novel approach exploiting the T matrix computed in spheroidal coordinates," Appl. Opt. **37**, 7875-7896 (1998).
11. H.-E. Albrecht, M. Borys, N. Damaschke, and C. Tropea, "The imaging properties of scattering particles in laser beams," Meas. Sci. Technol. **10,** 564-574 (1999).
12. T. Evers, H. Dahl, and T. Wriedt, "Extension of the program 3D MMP with a fifth order Gaussian beam," Electron. Lett. **32**, 1356-1357 (1996).
13. A. Doicu and T. Wriedt, "Formulations of the extended boundary condition method for incident Gaussian beams using multiple-multipole expansions," J. Modern. Opt. **44**, 785-801 (1997).
14. Y. Han and Z. Wu, "Scattering of a spheroidal particle illuminated by a Gaussian beam," Appl. Opt. **40**, 2501-2509 (2001).
15. Y. Han and Z. Wu, "The expansion coefficients of a Spheroidal Particle illuminated by Gaussian Beam," IEEE Trans. Antennas Propagat. **49**, 615-620 (2001).
16. H. Y. Zhang and Y. P. Han, "Scattering by a confocal multilayered spheroidal particle illuminated by an axial Gaussian beam," IEEE Trans. Antennas Propagat. **53**, 1514-1518 (2005).

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007

17. Y. Han, H. Zhang, and X. Sun, "Scattering of shaped beam by an arbitrarily oriented spheroid having layers with non-confocal boundaries," Appl. Phys. B. **84**, 485-492 (2006).
18. B. Friedman and J. Russek, "Addition theorems for spherical waves," Q. Appl. Math. **12**, 13-23 (1954).
19. S. Stein, "Addition theorems for spherical wave functions," Q. Appl. Math. **19**, 15-24 (1961).
20. O. R. Cruzan, "Translational addition theorems for spherical vector wave functions," Q. Appl. Math. **20**, 33-40 (1962).
21. S. Asano and G. Yamamoto, "Light scattering by a spheroid particle," Appl. Opt. **14**, 29-49 (1975).
22. J. Dalmas and R. Deleuil, "Translational addition theorems for prolate spheroidal vector wave functions $\mathbf{M}^r$ and $\mathbf{N}^r$," Q. Appl. Math. **38,** 143-158 (1980).
23. A. R. Edmonds, *Angular momentum in quantum mechanics*, (Princeton University Press, Princeton, N. J, 1957), Chap.4.
24. B. P. Sinha and R. H. Macphie, "Translational addition theorems for spheroidal scalar and vector wave functions," Q. Appl. Math. **44,** 213-222 (1986).
25. L. W. Davis, "Theory of electromagnetic beam," Phys. Rev. A **19**, 1177-1179 (1979).
26. C. Flammer, *Spheroidal wave functions*, (Stanford University Press, Stanford, California, 1957).
27. J. A. Stratton, *Electromagnetic Theory*, (New York: McGraw-Hill, 1941).

1. Introduction

There is a rather large literature on the interaction of the incident shaped beam with some kinds of regular scatterers which has been of interest in such areas as particle sizing, optical tweezers, nonlinear optics, laser beam aerosol penetration, and so on. There are a number of mathematical theories for arbitrary beam scattering. Each of these theories relies on the decomposition of the incident beam into an infinite series of elementary constituents, such as partial waves or plane waves, with amplitudes and phases given by a set of beam-shape coefficients [1-10]. The generalized Lorenz-Mie theory (GLMT) developed by Gouesbet et al, one fundamental problem in it concerning the expansion of the incident shaped beam as a series of spherical vector wave functions, is effective to describe the electromagnetic scattering of a shaped beam by a spherical particle, in which the beam shape coefficients g_n^m are obtained when the shaped beam propagates parallel to the coordinate axis [1-3]. It has been extended to the case of multilayered spheres and of infinitely long circular cylinders [4, 5]. Barton et al have calculated the intensity distributions internal and external to a sphere illuminated with a focused TEM_{00} mode laser beam by using spherical harmonic expansions of the scalar potential for the electric and magnetic fields and by matching the electromagnetic fields at the surface [7]. Such a procedure has been applied to the study of arbitrary beam scattering by a spheroidal particle, but Barton didn't give the expressions of the beam shape coefficients of the shaped beam [8]. An approach presented by Khaled et al uses the angular spectrum of plane waves to model the shaped beam and the T-matrix method (TMM) or extended boundary condition method (EBCM) to compute the fields inside and outside a sphere [9]. Some numerical methods, such as Fourier Lorenz-Mie theory, mutiple multiple method (MMP), discrete dipole approximation (DDA), etc., have been used to study scattering of Gaussian beam by particles [11, 12, 13].

A strong effort has recently been devoted by us to the study of arbitrary beam scattering by spheroids, due to the fact that a large number of real objects can be modeled by spheroids with appropriate axial ratios and to the possibility of subsequent applications of exact analytical methods. In our previous papers, we studied the scattering of a spheroidal particle illuminated by a Gaussian beam propagating along the axis, and the expansion coefficients of the shaped beam in terms of spheroidal vector wave functions are determined [14-17]. In this paper, the approach to expand an arbitrary shaped beam in spherical and spheroidal coordinates in the general case of oblique illumination is presented. Once the beam-shape coefficients are determined, the solution of scattering for arbitrary shaped beam in oblique illumination by a spherical and spheroidal particle can be obtained by means of the method of separation of variables.

The paper is organized as follows. Section 2 provides a procedure to expand the incident arbitrary shaped beam in oblique illumination, and the beam shape coefficients $g_n^{ms}, g_n'^{ms}$ and

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

$G_n^m, G_n'^m$, corresponding to spherical and spheroidal coordinates respectively, are given. In section 3, for particle center located on Gaussian beam axis and plane wave, the simplified representations for beam shape coefficients are obtained. The convergence of the beam shape coefficients is discussed in section 4. Section 5 is a conclusion.

2. Expansion of incident shaped beam with respect to spherical and spheroidal coordinates

2.1. Shaped beam coefficients in spherical coordinates

Figures 1 and 2 show the geometry of the scatterer. The shaped beam propagates in free space and from the negative z' to the positive z' of $O'x'y'z'$, with the middle of beam waist located at origin O'. The system $Ox''y''z''$ is parallel to $O'x'y'z'$ and the Cartesian coordinates of O' in $Ox''y''z''$ are x_0, y_0, z_0. The center of the scatterer is located at the point O of the Cartesian coordinate system $Oxyz$ which is obtained by rotating the system $Ox''y''z''$ through Euler angles α, β, γ as defined in Ref. [23]. For spheroidal particle, the major axe of the spheroid is along the z axis. In this paper, we assume that the time-dependent part of the electromagnetic fields is $e^{-i\omega t}$.

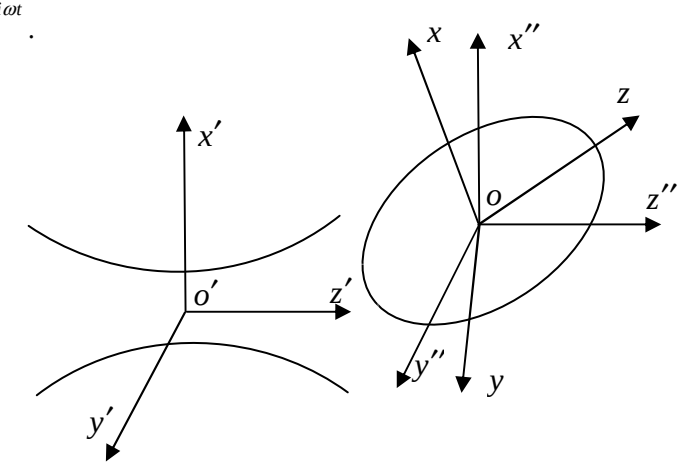

Fig. 1. The center of an arbitrarily oriented scatterer is located at origin O of the Cartesian coordinate systems $Oxyz$ and $Ox''y''z''$. (For spheroid, the major axe along the z axis of $Oxyz$). The xyz axes are obtained by a rigid-body rotation of the $x''y''z''$ axes through Euler angles α, β, γ. The scatterer is illuminated by a shaped beam propagating along the z' axis with the middle of its waist located at origin O'. The oz'' is parallel to $o'z'$, and with similar conditions for the other axes. The Cartesian coordinates of O' in the system $Ox''y''z''$ are (x_0, y_0, z_0).

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

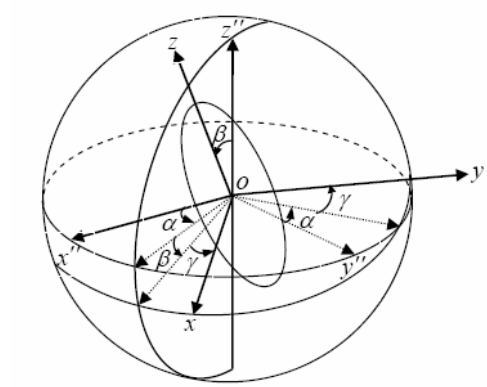

Fig. 2. Geometry of the scattering description for a scatterer in the Cartesian coordinate system $Oxyz$. The rotation axis of the spheroid is the z axis and its orientation in space is specified by the Euler angles α,β,γ of the xyz axes with respect to the $x''y''z''$ axes.

The incident shaped beam can be expanded in terms of spherical vector wave functions $\mathbf{m}_{\substack{e\\o}mn}^{r(1)}(kr,\theta'',\phi'')$ and $\mathbf{n}_{\substack{e\\o}mn}^{r(1)}(kr,\theta'',\phi'')$ natural to the system $Ox''y''z''$ [1, 14] (TE Mode)

$$\mathbf{E}^i = E_0\sum_{m=0}^{\infty}\sum_{n=m}^{\infty}\left[\overline{g_{n,TE}^{m}}\mathbf{m}_{emn}^{r(1)}(kr,\theta'',\phi'') + \overline{g_{n,TE}^{\prime m}}\mathbf{m}_{omn}^{r(1)}(kr,\theta'',\phi'') + i\overline{g_{n,TM}^{\prime m}}\mathbf{n}_{emn}^{r(1)}(kr,\theta'',\phi'') + i\overline{g_{n,TM}^{m}}\mathbf{n}_{omn}^{r(1)}(kr,\theta'',\phi'')\right] \tag{1}$$

where $\overline{g_{n,TE}^{m}}$, $\overline{g_{n,TE}^{\prime m}}$, $\overline{g_{n,TM}^{m}}$ and $\overline{g_{n,TM}^{\prime m}}$ are the beam shape coefficients in spherical coordinates for the incident shaped beam propagating parallel to the z' axis. They can be derived from those results obtained by the three computational methods given by Gouesbet et al, which are quadratures, finite series technique and localized approximations [1-3], and by an approach presented by A. Doicu using the translational addition theorem for spherical vector wave functions [6] (see Appendix).

We have found it convenient to describe the addition theorem for spherical scalar wave functions under coordinate rotations by referring to the common forms in quantum mechanics presented by Edmonds [23].

$$P_n^m(\cos\theta'')e^{im\phi''} = \sum_{s=-n}^{n}\rho(m,s,n)P_n^s(\cos\theta)e^{is\phi} \tag{2}$$

The coordinates θ,ϕ are the angle coordinates of a point in space defined in the usual manner with respect to the xyz axes, and θ'',ϕ'' to the $x''y''z''$ axes. Here the associated Legendre functions $P_n^m(\cos\theta)$ will be taken following Stratton [27].

The coefficients $\rho(m,s,n)$ are given by

$$\rho(m,s,n) = (-1)^{s+m}e^{is\gamma}\left[\frac{(n+m)!(n-s)!}{(n-m)!(n+s)!}\right]^{1/2}\mathrm{u}_{sm}^{(n)}(\beta)e^{im\alpha} \tag{3}$$

$$\mathrm{u}_{sm}^{(n)}(\beta) = \left[\frac{(n+s)!(n-s)!}{(n+m)!(n-m)!}\right]^{1/2}\sum_{\sigma}\binom{n+m}{n-s-\sigma}\binom{n-m}{\sigma}(-1)^{n-s-\sigma}(\cos\frac{\beta}{2})^{2\sigma+s+m}(\sin\frac{\beta}{2})^{2n-2\sigma-s-m} \tag{4}$$

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

Let $\rho(m,s,n)=\rho_1(m,s,n)+i\rho_2(m,s,n)$. $\rho_1(m,s,n)$ and $\rho_2(m,s,n)$ are the real and imaginary parts of $\rho(m,s,n)$ respectively.

Substituting $\rho(m,s,n)$ into Eq. (2) and having the real and imaginary parts on both sides of the equality be equal respectively, we can obtain

$$P_n^m(\cos\theta'')\begin{pmatrix}\cos(m\phi'')\\ \sin(m\phi'')\end{pmatrix}=\sum_{s=-n}^{n}\Bigg[\ \rho_1(m,s,n)P_n^s(\cos\theta)\begin{pmatrix}\cos(s\phi)\\ \sin(s\phi)\end{pmatrix} + \begin{pmatrix}-1\\ 1\end{pmatrix}\rho_2(m,s,n)P_n^s(\cos\theta)\begin{pmatrix}\sin(s\phi)\\ \cos(s\phi)\end{pmatrix}\Bigg] \tag{5}$$

Since the vector operator ∇ is invariant to a transformation of the coordinate system and $\mathbf{r}$ is the same position vector for both due to the common origin O of Cartesian coordinate systems $Oxyz$ and $Ox''y''z''$, by using Eq. (5) multiplied by the spherical Bessel functions $j_n(kr)$ and the definitions of $\mathbf{m}_{\substack{e\\o}mn}^{r(1)}(kr,\theta,\phi)$ and $\mathbf{n}_{\substack{e\\o}mn}^{r(1)}(kr,\theta,\phi)$ [27], we have

$$\mathbf{w}_{\substack{e\\o}mn}^{r(1)}(kr,\theta'',\phi'')=\sum_{s=-n}^{n}\rho_1(m,s,n)\mathbf{w}_{\substack{e\\o}sn}^{r(1)}(kr,\theta,\phi)\mp\sum_{s=-n}^{n}\rho_2(m,s,n)\mathbf{w}_{\substack{o\\e}sn}^{r(1)}(kr,\theta,\phi) \tag{6}$$

where $\mathbf{w}$ stands for the spherical vector wave functions $\mathbf{m}$ or $\mathbf{n}$,

Taking into account the following relation

$$P_n^{-m}(\cos\theta)=(-1)^m\frac{(n-m)!}{(n+m)!}P_n^m(\cos\theta)\qquad m>0 \tag{7}$$

one can easily obtain from Eq. (5)

$$\mathbf{w}_{\substack{e\\o}-mn}^{r(i)}(kr,\theta,\phi)=\pm(-1)^m\frac{(n-m)!}{(n+m)!}\mathbf{w}_{\substack{e\\o}mn}^{r(i)}(kr,\theta,\phi)\qquad m>0 \tag{8}$$

Substituting Eq. (6) into Eq. (1) and using Eq. (8), we can rewrite Eq. (1) or expand the incident beam in terms of the spherical vector wave functions $\mathbf{m}_{\substack{e\\o}mn}^{r(1)}(kr,\theta,\phi)$ and $\mathbf{n}_{\substack{e\\o}mn}^{r(1)}(kr,\theta,\phi)$ natural to the system $Oxyz$, as follows (TE Mode):

$$\mathbf{E}^i=E_0\sum_{m=0}^{\infty}\sum_{n=m}^{\infty}\sum_{s=0}^{n}\ [g_{n,TE}^{ms}\mathbf{m}_{esn}^{r(1)}(kr,\theta,\phi)-g_{n,TE}'^{ms}\mathbf{m}_{osn}^{r(1)}(kr,\theta,\phi) + ig_{n,TM}'^{ms}\mathbf{n}_{esn}^{r(1)}(kr,\theta,\phi)+ig_{n,TM}^{ms}\mathbf{n}_{osn}^{r(1)}(kr,\theta,\phi)] \tag{9}$$

where $g_{n,TE}^{ms}$, $g_{n,TE}'^{ms}$, $g_{n,TM}^{ms}$ and $g_{n,TM}'^{ms}$ are the expansion coefficients for arbitrary shaped beam in the spherical coordinates at oblique incidence

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

$$\begin{pmatrix} g_{n,TE}^{ms} \\ g'^{ms}_{n,TE} \\ g_{n,TM}^{ms} \\ g'^{ms}_{n,TM} \end{pmatrix} = \begin{pmatrix} \overline{g_{n,TE}^{m}} \\ \overline{g'^{m}_{n,TE}} \\ \overline{g_{n,TM}^{m}} \\ \overline{g'^{m}_{n,TM}} \end{pmatrix} \left[\begin{pmatrix} \rho_1(m,s,n) \\ \rho_2(m,s,n) \\ \rho_1(m,s,n) \\ \rho_2(m,s,n) \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} (1-\delta_{0s})(-1)^s \frac{(n-s)!}{(n+s)!} \begin{pmatrix} \rho_1(m,-s,n) \\ \rho_2(m,-s,n) \\ \rho_1(m,-s,n) \\ \rho_2(m,-s,n) \end{pmatrix} \right]$$

$$+ \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} \overline{g'^{m}_{n,TE}} \\ \overline{g'^{m}_{n,TE}} \\ \overline{g'^{m}_{n,TM}} \\ \overline{g'^{m}_{n,TM}} \end{pmatrix} \left[\begin{pmatrix} \rho_2(m,s,n) \\ \rho_1(m,s,n) \\ \rho_2(m,s,n) \\ \rho_1(m,s,n) \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} (1-\delta_{0s})(-1)^s \frac{(n-s)!}{(n+s)!} \begin{pmatrix} \rho_2(m,-s,n) \\ \rho_1(m,-s,n) \\ \rho_2(m,-s,n) \\ \rho_1(m,-s,n) \end{pmatrix} \right] \tag{10}$$

2.2. Shaped beam coefficients in spheroidal coordinates

The following derivations are for the prolate spheroidal coordinate system, since the derivations for the oblate case are very similar. In fact, the results for the oblate system are obtained from those of the prolate system by the transformations [26]

$$c \to -ic\,,\ \zeta \to i\zeta$$

The spherical wave functions can be expanded in terms of spheroidal ones[26].

$$P_n^m(\cos\theta)\, j_n(kr) = \frac{2(n+m)!}{(2n+1)(n-m)!} \sum_{l=m,m+1}^{\infty}{}' \frac{i^{l-n}}{N_{ml}} d_{n-m}^{ml}(c) S_{ml}(c,\eta) R_{ml}^{(1)}(c,\zeta) \tag{11}$$

As similar derivation for Eq. (6) from Eq. (5), it follows immediately from Eq. (11) that the spherical and spheroidal vector wave functions are related by

$$\mathbf{w}_{\substack{e\\o}mn}^{r(1)}(kr,\theta,\phi) = \sum_{l=m,m+1}^{\infty}{}' \frac{2(n+m)!}{(2n+1)(n-m)!} \cdot \frac{i^{l-n}}{N_{ml}} d_{n-m}^{ml}(c) \mathbf{W}_{\substack{e\\o}ml}^{r(1)}(c,\zeta,\eta,\phi) \tag{12}$$

in which $\mathbf{W}$ stands for the spheroidal vector wave functions $\mathbf{M}$ or $\mathbf{N}$.

Substituting Eq. (12) into Eq. (9) and considering that the summation of the series $\sum_{n=0}^{\infty}\sum_{m=0}^{n} a_{mn}\mathbf{W}_{mn}^{r(i)}$ is equal to that of the series $\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} a_{mn}\mathbf{W}_{mn}^{r(i)}$, with a_{mn} representing the expansion coefficients, we can obtain

$$\begin{aligned} \mathbf{E}^i = E_0 \sum_{s=0}^{\infty}\sum_{l=s}^{\infty}\sum_{n=s,s+1}^{\infty}{}'\sum_{m=0}^{n} \frac{2(n+s)!}{(2n+1)(n-s)!} \frac{i^{l-n}}{N_{sl}} d_{n-s}^{sl}(c) [& g_{n,TE}^{ms} \mathbf{M}_{esl}^{r(1)}(c,\zeta,\eta,\phi) \\ & - g'^{ms}_{n,TE} \mathbf{M}_{osl}^{r(1)}(c,\zeta,\eta,\phi) + i g'^{ms}_{n,TM} \mathbf{N}_{esl}^{r(1)}(c,\zeta,\eta,\phi) + i g_{n,TM}^{ms} \mathbf{N}_{osl}^{r(1)}(c,\zeta,\eta,\phi)] \end{aligned} \tag{13}$$

Let $n-s=r$. One replaces s by m, l by n and m by s, then Eq. (13) or the expansion of the incident beam in terms of spheroidal vector wave functions attached to an arbitrarily oriented spheroid can be written as follows (TE Mode):

$$\begin{aligned} \mathbf{E}^i = E_0 \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n [& G_{n,TE}^{m} \mathbf{M}_{emn}^{r(1)}(c,\zeta,\eta,\phi) - G'^{m}_{n,TE} \mathrm{M}_{omn}^{r(1)}(c,\zeta,\eta,\phi) \\ & + i G_{n,TM}^{m} \mathbf{N}_{omn}^{r(1)}(c,\zeta,\eta,\phi) + i G'^{m}_{n,TM} \mathbf{N}_{emn}^{r(1)}(c,\zeta,\eta,\phi)] \end{aligned} \tag{14}$$

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

where $G_{n,TE}^{ms}$, $G_{n,TE}^{\prime ms}$, $G_{n,TM}^{ms}$ and $G_{n,TM}^{\prime ms}$ are the expansion coefficients for arbitrary shaped beam at oblique incidence in the spheroidal coordinates.

$$\begin{bmatrix} G_{n,TE}^{m} \\ G_{n,TE}^{\prime m} \\ G_{n,TM}^{m} \\ G_{n,TM}^{\prime m} \end{bmatrix} = \sum_{r=0,1}^{\infty}{}' \sum_{s=0}^{r+m} \frac{2(r+2m)!}{(2r+2m+1)r!} \frac{i^{-r-m}}{N_{mn}} d_r^{mn}(c) \begin{bmatrix} g_{r+m,TE}^{sm} \\ g_{r+m,TE}^{\prime sm} \\ g_{r+m,TM}^{sm} \\ g_{r+m,TM}^{\prime sm} \end{bmatrix} \tag{15}$$

Once the beam-shape coefficients in oblique illumination are determined in spherical and spheroidal coordinates, the incident, scattered and internal fields can be expanded in terms of spherical and spheroidal vector wave functions. For spheroidal particle, as an example, incident fields are given in Eq. (14) for TE mode, and the internal and scattered fields can be expressed as following [17]:

$$\begin{aligned} \mathbf{E}^{w} = E_0 \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} i^n [& \delta_{mn} \mathbf{M}_{emn}^{r(1)}(c',\zeta,\eta,\phi) + i\gamma_{mn} \mathbf{N}_{omn}^{r(1)}(c',\zeta,\eta,\phi) \\ & + \delta'_{mn} \mathbf{M}_{omn}^{r(1)}(c',\zeta,\eta,\phi) + i\gamma'_{mn} \mathbf{N}_{emn}^{r(1)}(c',\zeta,\eta,\phi)] \end{aligned} \tag{17}$$

$$\begin{aligned} \mathbf{E}^{s} = E_0 \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} i^n [& \beta_{mn} \mathbf{M}_{emn}^{r(3)}(c,\zeta,\eta,\phi) + \beta'_{mn} \mathbf{M}_{omn}^{r(3)}(c,\zeta,\eta,\phi) \\ & + i\alpha_{mn} \mathbf{N}_{omn}^{r(3)}(c,\zeta,\eta,\phi) + i\alpha'_{mn} \mathbf{N}_{emn}^{r(3)}(c,\zeta,\eta,\phi)] \end{aligned} \tag{18}$$

The corresponding magnetic fields can be obtained with the following relations:

$$\mathbf{H} = \frac{1}{iw\mu} \nabla \times \mathbf{E} \qquad \mathbf{M}_{\substack{e\\o}mn} = \frac{1}{k} \mathbf{N}_{\substack{e\\o}mn} \qquad \mathbf{N}_{\substack{e\\o}mn} = \frac{1}{k} \mathbf{M}_{\substack{e\\o}mn} \tag{19}$$

The unknown coefficients (γ_{mn}, δ_{mn}, γ'_{mn}, δ'_{mn} α_{mn}, β_{mn}, α'_{mn} β'_{mn}) are determined by applying the boundary conditions of continuity of the tangential electromagnetic fields over the surface of the particle. Thus, the solution of scattering for arbitrary shaped beam by a homogeneous spheroidal particle can be obtained

3. Expressions of beam shape coefficients for particle center located on Gaussian beam axis or plane wave illumination

Equations (10) and (15) enable us to compute beam shape coefficients for sphere and spheroid at oblique illumination in all case. It may be interesting to calculate these coefficients for the special cases.

For particle center located on Gaussian beam axis $(x_0 = 0, y_0 = 0)$ or plane wave illumination $(x_0 = 0, y_0 = 0, z_0 = 0, w_0 \to \infty)$, we have [1] [6]

$$\left.\begin{aligned} \overline{g_{n,TE}^{m}} = \overline{g_{n,TM}^{m}} = g_n, \quad \overline{g_{n,TE}^{\prime m}} = \overline{g_{n,TM}^{\prime m}} = 0 \qquad m = 1 \\ \overline{g_{n,TE}^{m}} = \overline{g_{n,TE}^{\prime m}} = \overline{g_{n,TM}^{m}} = \overline{g_{n,TM}^{\prime m}} = 0 \qquad m \neq 1 \end{aligned}\right\} \tag{20}$$

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

$$\begin{pmatrix} g_{n,TE}^{1m} \\ g_{n,TE}^{\prime 1m} \\ g_{n,TM}^{1m} \\ g_{n,TM}^{\prime 1m} \end{pmatrix} = (-1)^{m-1} \frac{(n-m)!}{(n+m)!} g_n \left[(2-\delta_{m0}) \frac{dP_n^m(\cos\beta)}{d\beta} \begin{pmatrix} \cos(m\gamma)\cos\alpha \\ \sin(m\gamma)\cos\alpha \\ -\sin(m\gamma)\sin\alpha \\ \cos(m\gamma)\sin\alpha \end{pmatrix} \right.$$
$$\left. + 2m \frac{P_n^m(\cos\beta)}{\sin\beta} \begin{pmatrix} -\sin(m\gamma)\sin\alpha \\ \cos(m\gamma)\sin\alpha \\ \cos(m\gamma)\cos\alpha \\ \sin(m\gamma)\cos\alpha \end{pmatrix} \right] \tag{21}$$

By substituting Eq. (21) into Eq. (15), we obtain the simpler form of the beam shape coefficients for particle center on beam axis and plane wave (taking $w_0 \to \infty$ in g_n) as follows

$$\begin{pmatrix} G_{n,TE}^{m} \\ G_{n,TE}^{\prime m} \\ G_{n,TM}^{m} \\ G_{n,TM}^{\prime m} \end{pmatrix} = \frac{2}{N_{mn}} \sum_{r=0,1}^{\infty}{}' \frac{i^{-r-m}}{2r+2m+1} d_r^{mn}(c) g_{r+m} (-1)^{m-1}$$
$$\times \left[(2\text{-}\delta_{m0}) \frac{dP_{r+m}^m(\cos\beta)}{d\beta} \begin{pmatrix} \cos(m\gamma)\cos\alpha \\ \sin(m\gamma)\cos\alpha \\ \text{-}\sin(m\gamma)\sin\alpha \\ \cos(m\gamma)\sin\alpha \end{pmatrix} + 2m \frac{P_{r+m}^m(\cos\beta)}{\sin\beta} \begin{pmatrix} -\sin(m\gamma)\sin\alpha \\ \cos(m\gamma)\sin\alpha \\ \cos(m\gamma)\cos\alpha \\ \sin(m\gamma)\cos\alpha \end{pmatrix} \right] \tag{22}$$

When $\alpha = \gamma = 0$, consider $g_{r+m} = -i^{r+m} \dfrac{2r+2m+1}{(r+m)(r+m+1)}$ for an incident plane wave, Eq. (14) and Eq. (22) can be reduced to

$$\hat{e}_y E_0 e^{ikr(-\sin\theta\cos\varphi\sin\beta + \cos\theta\cos\beta)} = E_0 \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} i^n \left[G_{n,TE}^m \mathbf{M}_{emn}^{r(1)}(c,\zeta,\eta,\phi) + i G_{n,TM}^m \mathbf{N}_{omn}^{r(1)}(c,\zeta,\eta,\phi) \right] \tag{23}$$

where

$$\begin{pmatrix} G_{n,TE}^{m} \\ G_{n,TM}^{m} \end{pmatrix} = -\frac{2}{N_{mn}} (-1)^{m-1} \sum_{r=0,1}^{\infty}{}' \frac{d_r^{mn}(c)}{(r+m)(r+m+1)} \begin{pmatrix} (2-\delta_{m0}) \dfrac{dP_{r+m}^m(\cos\beta)}{d\beta} \\ 2m \dfrac{P_{r+m}^m(\cos\beta)}{\sin\beta} \end{pmatrix} \tag{24}$$

Eq. (24) agrees with Eqs. (9.2.18) and (9.2.1) for plane wave in Flammer's book [26, p72], with which to validate expressions for beam shape coefficients given in Section 2.

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

4. Numerical results

The expansion coefficients of a shaped beam in oblique illumination are given by Eqs. (10), (15), and those for particle center on beam axis and plane wave by Eq. (22). Numerical computations of the beam shape coefficients of a Gaussian beam are performed on the spheroid having the ratio of the semimajor axis a to the semiminor axis b of $a/b=2$, and the size parameter $2\pi a/\lambda=5$.

Figures 3 and 4 respectively show the convergence with n of the beam shape coefficients $G_{n,TM}^{m}$, $G_{n,TE}^{m}$ of particle center located on Gaussian beam axis with $w_0=2\lambda$, given $\alpha=\pi/4, \beta=\pi/6, \gamma=0$, $x_0=0, y_0=0, z_0=0$. Computational results indicate that $G_{n,TM}^{m}$, $G_{n,TE}^{m}$ converge rapidly for increasing n and m.

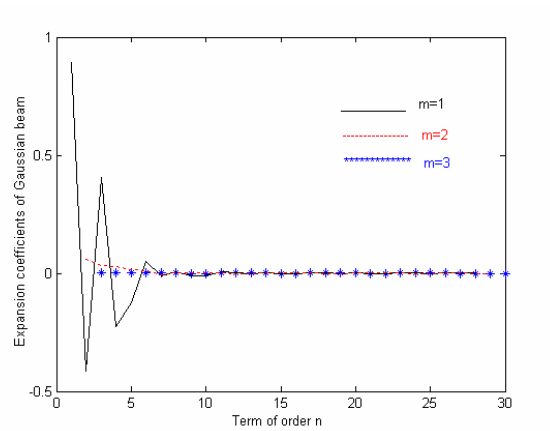

Fig. 3. The convergence of the beam shape coefficients $G_{n,TM}^{m}$

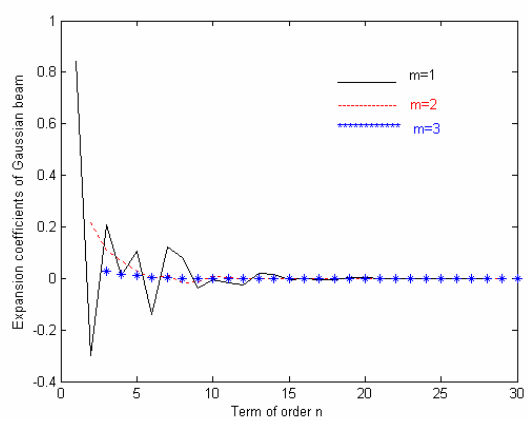

Fig. 4. The convergence of the beam shape coefficients $G_{n,TE}^{m}$

5. Conclusion

A description of arbitrary shaped beam in the general case of oblique illumination is given by using addition theorem for spherical vector wave functions under coordinate rotations and

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

relations between spheroidal vector wave functions and spherical ones. The representations for shaped beam coefficients are derived in spherical and spheroidal coordinates. For the special case of the plane wave, the simplified expressions are given, which agree with that in Flammer's book. As a result, this approach provides a useful practical tool for calculating scattering of an arbitrary shaped beam by particles.

Appendix

At order L of approximation in the description of the Gaussian beam, the field components in the spherical coordinates (r,θ'',ϕ'') with respect to $Ox''y''z''$ are obtained as follows [1] [25]

$$E_r = E_0\psi_0[\cos\phi''\sin\theta''(1 - \frac{2Q}{l}r\cos\theta'') + \frac{2Q}{l}x_0\cos\theta'']\exp(K) \quad \text{(A1)}$$

$$E_\theta = E_0\psi_0[\cos\phi''(\cos\theta'' + \frac{2Q}{l}r\sin^2\theta'') - \frac{2Q}{l}x_0\sin\theta'']\exp(K) \quad \text{(A2)}$$

$$E_\phi = -E_0\psi_0\sin\phi''\exp(K) \quad \text{(A3)}$$

$$H_{r,TE} = H_0\psi_0[\sin\phi''\sin\theta''(1-\frac{2Q}{l}r\cos\theta'') + \frac{2Q}{l}y_0\cos\theta'']\exp(K) \quad \text{(A4)}$$

$$H_\theta = H_0\psi_0[\sin\phi''(\cos\theta'' + \frac{2Q}{l}r\sin^2\theta'') - \frac{2Q}{l}y_0\sin\theta'']\exp(K) \quad \text{(A5)}$$

$$H_\phi = H_0\psi_0\cos\phi''\exp(K) \quad \text{(A6)}$$

in which

$$\psi_0 = iQ\exp[\frac{2iQ}{w_0^2}r\sin\theta''(x_0\cos\phi'' + y\sin\phi'')\times \exp(-iQ\frac{r^2\sin^2\theta''}{w_0^2})\exp(-iQ\frac{x_0^2+y_0^2}{w_0^2}) \quad \text{(A7)}$$

$$K = ik(r\cos\theta - z_0) \quad \text{(A8)}$$

$$Q = \frac{1}{i + (z - z_0)/l} \quad \text{(A9)}$$

where $l = kw_0^2$, and w_0 is the beam's electric-field half-width in the focal plane.

E_0 and H_0 are linked by

$$E_0 / H_0 = (\mu/\varepsilon)^{1/2} \quad \text{(A10)}$$

When we replace $\mathbf{E}$ by $-\mathbf{H}$, $\mathbf{H}$ by $\mathbf{E}$, ε by μ, μ by ε, from Eqs. (A1)-(A6) order L of approximation can also be expressed by

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

$$E_{r,TE} = E_0\psi_0[\sin\phi''\sin\theta''(1-\frac{2Q}{l}r\cos\theta'')+\frac{2Q}{l}y_0\cos\theta'']\exp(K) \tag{A11}$$

$$E_\theta = E_0\psi_0[\sin\phi''(\cos\theta''+\frac{2Q}{l}r\sin^2\theta'')-\frac{2Q}{l}y_0\sin\theta'']\exp(K) \tag{A12}$$

$$E_\phi = E_0\psi_0\cos\phi''\exp(K) \tag{A13}$$

$$H_r = -H_0\psi_0[\cos\phi''\sin\theta''(1-\frac{2Q}{l}r\cos\theta'')+\frac{2Q}{l}x_0\cos\theta'']\exp(K) \tag{A14}$$

$$H_\theta = -H_0\psi_0[\cos\phi''(\cos\theta''+\frac{2Q}{l}r\sin^2\theta'')-\frac{2Q}{l}x_0\sin\theta'']\exp(K) \tag{A15}$$

$$H_\phi = H_0\psi_0\sin\phi''\exp(K) \tag{A16}$$

The description of the Gaussian beam by Eqs. (A1-A6) is TM mode, and that of Eqs. (A11-A16) is TE mode

The incident shaped beam can be expanded in terms of spherical vector wave functions $\mathbf{m}_{mn}^{r(1)}(kr,\theta'',\phi'')$ and $\mathbf{n}_{mn}^{r(1)}(kr,\theta'',\phi'')$ as in [1] [6]

$$\mathbf{E}^i = E_0\sum_{n=1}^{\infty}\sum_{m=-n}^{n}C_{nm}\left[ig_{n,TE}^m\mathbf{m}_{mn}^{r(1)}(kr,\theta'',\phi'')+g_{n,TM}^m\mathbf{n}_{mn}^{r(1)}(kr,\theta'',\phi'')\right] \tag{A17}$$

in which the C_{nm} coefficients are normalized factors for negative values of the index m

$$C_{nm} = \begin{cases} C_n & m \ge 0 \\ (-1)^{|m|}\dfrac{(n+|m|)!}{(n-|m|)!}C_n & m < 0 \end{cases} \tag{A18}$$

$$C_n = i^{n-1}\frac{2n+1}{n(n+1)} \tag{A19}$$

and

$$\begin{pmatrix}\mathbf{m}_{mn}^{r(1)}(kr,\theta'',\phi'')\\ \mathbf{n}_{mn}^{r(1)}(kr,\theta'',\phi'')\end{pmatrix} = \begin{pmatrix}\mathbf{m}_{emn}^{r(1)}(kr,\theta'',\phi'')\\ \mathbf{n}_{emn}^{r(1)}(kr,\theta'',\phi'')\end{pmatrix} + i\begin{pmatrix}\mathbf{m}_{omn}^{r(1)}(kr,\theta'',\phi'')\\ \mathbf{n}_{omn}^{r(1)}(kr,\theta'',\phi'')\end{pmatrix} \tag{A20}$$

By substituting Eq. (A20) into Eq. (A17) and by considering Eq. (8), we can obtain Eq. (1), in which

#74838 - $15.00 USD Received 11 September 2006; revised 1 January 2007; accepted 4 January 2007
(C) 2007 OSA

$$\begin{pmatrix} \overline{g_{n,TE}^{m}} \\ \overline{g'^{m}_{n,TE}} \\ \overline{g_{n,TM}^{m}} \\ \overline{g'^{m}_{n,TM}} \end{pmatrix} = i^{n} \frac{2n+1}{n(n+1)} \frac{1}{(1+\delta_{0m})} \begin{pmatrix} 1 \\ i \\ -i \\ -1 \end{pmatrix} \begin{pmatrix} (g_{n,TE}^{m} + g_{n,TE}^{-m}) \\ (g_{n,TE}^{m} - g_{n,TE}^{-m}) \\ (g_{n,TM}^{m} - g_{n,TM}^{-m}) \\ (g_{n,TM}^{m} + g_{n,TM}^{-m}) \end{pmatrix} \tag{A21}$$

Acknowledgments

This work is supported by NCET-04-0949 of China and by Nation Nature Science Foundation of China (grant no: 60371019).