

Role of the anomalous self-steepening effect in modulation instability in negative-index material

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Abstract: In negative-index materials (NIMs), the self-steepening (SS) effect is proven to be anomalous in two aspects: First, it can be either positive or negative, with the zero SS point determined by the size of split-ring resonator circuit elements. Second, the negative SS parameter can have a very large value compared to an ordinary positive-index material. We present a theoretical investigation on modulation instability (MI) to identify the role of the anomalous SS effect in NIM. We find that the first anomaly of SS doesn't influence MI, yet the controllable zero SS point can be used to manipulate MI, and thus manipulate the generation of solitons. The second anomaly, however, leads to significant changes in the MI condition and property, compared with the case of an ordinary positive-index material. Numerical simulations confirm the theoretical results and show that negative SS moves the center of generated pulse toward the leading side, and shifts a part of energy of the generated pulse toward the red side, opposite to the case of positive SS.

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1. Introduction

Modulation instability (MI) is one of the most fundamental processes in nonlinear wave systems in nature. In a nonlinear dispersive material, it stems from the interaction between the nonlinear effect and group velocity dispersion (GVD), and in general, is characterized by instability experienced by a continuous wave (cw) when it propagates together with a weak noise [1]. Because of MI, small amplitude perturbations that originate from noise on top of a homogeneous wave front grow rapidly under the combined effects of nonlinearity and dispersion. As a result, a quasi-cw pulse breaks up into a train of short pulses. It is important to note that MI is an issue closely related to the existence of both bright and dark solitons; it can be considered as a precursor to soliton formation.

Over the years MI in nonlinear dispersive materials such as optical fibers, has been systematically investigated in connection with numerous nonlinear processes [1]. All the investigations, however, are for the ordinary positive-index materials. Various higher-order effects, such as higher-order dispersions, self-steepening and time-delayed Raman effects, have been found to strongly influence MI. It has been found that third-order dispersion (TOD) contributes none to instability[2, 3], while fourth-order dispersion (FOD) plays a major role in the critical MI frequency and may lead to additional instability regions [4, 5]. SS narrows the MI frequency region [3, 5, 6] and the time-delayed Raman nonlinearity alters MI fundamentally [3].

In this paper, we investigate MI in negative-index material (NIM) with Kerr nonlinearity. NIMs' most impressive property is their ability to refract light in the opposite way with respect to what an ordinary material does [7, 8, 9, 10]. NIMs were realized primarily in the microwave range. Nowadays, NIMs in the near IR and optical range have also been experimentally demonstrated [11, 12, 13]. Additionally, nonlinear NIMs can also be created [14, 15, 16]. For example, Zharov et al. [14] demonstrated that a two dimensional periodic structure created by arrays of wires and split-ring resonators embedded into a nonlinear dielectric takes on a Kerr-type dielectric permittivity. Very recently, authors have investigated the nonlinear propagation of ultrashort pulse in NIM [17, 18, 19, 20]. In Ref.[18] Lazarides and Tsironis derived a system of coupled nonlinear Schrödinger equations (NLSEs) for the envelopes of the propa-

gating electric and magnetic fields in an isotropic, homogeneous, quasi-one-dimensional NIM under the conditions of nonlinear polarization and nonlinear magnetization. Based on the coupled NLSEs, Kourakis and Shukla have investigated the nonlinear stability of electromagnetic waves in NIM, and obtained the modulational stability profile of the coupled plane-wave solutions [19]. Scalora et al. [20] investigated the propagation of pulses at least a few tens of optical cycles in duration in NIM with only a nonlinear polarization. They also derived a system of coupled nonlinear equations. By eliminating the magnetic field, the system leads to a new generalized NLSE in which the linear and nonlinear coefficients can be tailored through the linear properties of the medium to attain any combination of signs unachievable in ordinary matter, showing significant potential to realize a wide class of solitary waves. The aim of this paper is to disclose the new MI properties in NIM. Especial attention is paid to the role played by the anomalous SS effect in NIMs. First, we derive a propagation model for ultrashort pulse in NIM characterized by a Drude model of both ε and μ , under the condition of a nonlinear polarization, and compare it to that in an ordinary material. Unlike Refs.[18, 20], we eliminate the magnetic field at the very start. This makes it possible for us to derive the propagation equation by the same way as in ordinary materials. Compared to Ref.[20], we obtain a more general NLSE suitable for few-cycle pulse propagation. By using the same approximations, our equation is reduced to that of Ref.[20]. Second, we present a linear stability analysis to identify the role of the anomalous SS effect in MI. Third, we give a numerical simulation to confirm the obtained results and to further demonstrate that NIMs provide new choice for soliton generation.

2. Modelling the pulse propagation in NIM

We assume that the pulse is propagating in uniform, bulk materials under the condition of a nonlinear polarization [14]. For simplicity we consider one longitudinal spatial coordinate and time. We thus start with the following wave equation

$$\frac{\partial^2 E(x,t)}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E(z,t)}{\partial t^2} + \mu \frac{\partial^2 P_{nl}(z,t)}{\partial t^2}. \quad (1)$$

The electric field E propagates along the z direction. Both E and the nonlinear polarization P_{nl} are assumed to be polarized parallel to the x axis. The dielectric susceptibility ε and magnetic permeability μ are dispersive in a NIM, otherwise the energy density could be negative [7]. Their frequency dispersion can be described by a lossy Drude model [8]:

$$\varepsilon(\omega) = \varepsilon_0 \left(1 - \frac{\omega_{pe}^2}{\omega(\omega + i\gamma_e)} \right), \mu(\omega) = \mu_0 \left(1 - \frac{\omega_{pm}^2}{\omega(\omega + i\gamma_m)} \right), \quad (2)$$

where ω is frequency, ω_{pe} and ω_{pm} are the respective electric and magnetic plasma frequencies, γ_e and γ_m are the respective electric and magnetic loss terms, which are very small and are neglected in the following analysis for simplicity, and ε_0 and μ_0 are the respective vacuum susceptibility and magnetic permeability.

We can transform Eq. (1) into frequency space in order to expand $\varepsilon(\omega)$ and $\mu(\omega)$ in powers of ω , thus enabling us to treat the material parameters as a power series which we can truncate to an appropriate order. However for simplicity it is better to expand $\omega\varepsilon(\omega)$ and $\omega\mu(\omega)$ about a suitable ω_0 instead,

$$\omega\varepsilon(\omega) = \sum_{m=0}^{\infty} \left\{ \frac{\partial^m [\omega\varepsilon(\omega)]}{m! \partial \omega^m} \Big|_{\omega=\omega_0} (\omega - \omega_0)^m \right\} \equiv \sum_{m=0}^{\infty} \Theta_m (\omega - \omega_0)^m, \quad (3)$$

$$\omega\mu(\omega) = \sum_{m=0}^{\infty} \left\{ \frac{\partial^m [\omega\mu(\omega)]}{m! \partial \omega^m} \Big|_{\omega=\omega_0} (\omega - \omega_0)^m \right\} \equiv \sum_{m=0}^{\infty} \Xi_m (\omega - \omega_0)^m. \quad (4)$$

We can now write the frequency space version of Eq. (1) as

$$\begin{aligned} \frac{\partial^2 \tilde{E}(z, \omega)}{\partial z^2} = & - \sum_{m=0}^{\infty} [\Theta_m(\omega - \omega_0)^m] \sum_{l=0}^{\infty} [\Xi_l(\omega - \omega_0)^l] \tilde{E}(z, \omega) \\ & - \omega \sum_{l=0}^{\infty} [\Xi_l(\omega - \omega_0)^l] \tilde{P}_{nl}(z, \omega). \end{aligned} \quad (5)$$

We introduce an envelope and carrier form for the field in the usual way, $E(z, t) = (1/2)A(z, t) \exp(ik_0 z - i\omega_0 t) + c.c.$, where $k_0 = n(\omega_0)\omega_0/c$ is wave number in medium at the carrier frequency ω_0 , $n(\omega_0)$ is the refractive index of medium at ω_0 , and assume that, $P_{nl}(z, t) = \epsilon_0 \chi^{(3)} |E(z, t)|^2 E(z, t)$, which characterizes a Kerr nonlinearity. With these envelope-carrier substitutions, taking the inverse Fourier transform of the obtained equation, we have

$$\left(\frac{-i}{2k_0} \frac{\partial^2}{\partial z^2} + \frac{\partial}{\partial z} + \frac{1}{V} \frac{\partial}{\partial t} \right) A = i \sum_{m=2}^{\infty} \frac{i^m \delta_m}{m!} \frac{\partial^m A}{\partial t^m} + i \sum_{m=0}^{\infty} \frac{i^m \Upsilon_m}{m!} \left(1 + \frac{i}{\omega_0} \frac{\partial}{\partial t} \right) \frac{\partial^m}{\partial t^m} (|A|^2 A), \quad (6)$$

where $V = 2k_0 / (\Theta_0 \Xi_1 + \Theta_1 \Xi_0)$ is the group velocity of the pulse, $\delta_m = m! \sum_{l=0}^m \Theta_l \Xi_{m-l} / (2k_0)$, $\Upsilon_m = m! \omega_0 \epsilon_0 \chi^{(3)} \Xi_m / (2k_0)$. Introducing co-moving variables, $\tau = t - z/V$, $\xi = z$, Eq. (6) is transformed to the following generalized NLSE

$$\begin{aligned} \frac{\partial A}{\partial \xi} = & - \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial \tau^2} + \sum_{m=3}^{\infty} \frac{i^{m+1} \delta_m}{m!} \frac{\partial^m A}{\partial \tau^m} + \sum_{m=0}^{\infty} \frac{i^{m+1} \Upsilon_m}{m!} \left(1 + \frac{i}{\omega_0} \frac{\partial}{\partial \tau} \right) \frac{\partial^m}{\partial \tau^m} (|A|^2 A) \\ & + \frac{i}{2k_0} \left(\frac{\partial^2 A}{\partial \xi^2} - \frac{2}{V} \frac{\partial^2 A}{\partial \tau \partial \xi} \right), \end{aligned} \quad (7)$$

where

$$\beta_2 = \delta_2 - 1 / (k_0 V^2), \quad (8)$$

is the GVD which can be easily proven to satisfy the definition of GVD, $\beta_2 = \partial^2 k / \partial \omega^2 |_{\omega_0}$.

In deriving Eq. (7), we have not made any further approximations. Thus it is suitable for the propagation of ultrashort pulse with few optical cycles in NIMs. It is formally similar to the models for few-cycle pulse propagation in ordinary positive-index materials [22]. In addition, by using the same approximations, Eq. (7) is reduced to Eq. (12) of [20]. The most notable characteristic of the propagation equation for NIM is the anomalous self-steepening (SS) parameter. To exclusively demonstrate the role of SS in MI in NIM, we keep the dispersion coefficients to second order and neglect the higher order derivatives with respect to the nonlinearity, and use the same estimations as [20]: $\partial^2 A / \partial \xi^2 = i\Upsilon_0 \partial(|A|^2 A) / \partial \xi$, $\partial^2 A / \partial \tau \partial \xi = i\Upsilon_0 \partial(|A|^2 A) / \partial \tau$. Thus we obtain the following NLSE

$$\frac{\partial A}{\partial \xi} = - \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial \tau^2} + iC_{nl} \left(1 + i \frac{C_s}{\omega_0} \frac{\partial}{\partial \tau} \right) (|A|^2 A), \quad (9)$$

where $C_{nl} = \Upsilon_0$ and $C_s = 1 + \omega_0 \Upsilon_1 / \Upsilon_0 - \omega_0 / (k_0 V)$ are the nonlinear and SS coefficients, respectively. For lossless Drude model, the expressions for β_2 , C_{nl} and C_s are

$$\beta_2 = \frac{1}{2cn\omega_0} \left(1 + \frac{3\omega_{pe}^2 \omega_{pm}^2}{\omega_0^4} \right) - \frac{(1 - \omega_{pe}^2 \omega_{pm}^2 / \omega_0^4)^2}{cn^3 \omega_0}, \quad (10)$$

$$C_{nl} = \frac{\chi^{(3)} \omega_0}{2nc} \left(1 - \frac{\omega_{pm}^2}{\omega_0^2} \right), \quad (11)$$

$$C_s = 1 + \frac{\omega_{pm}^2 \omega_{pe}^2 - \omega_0^4}{n^2 \omega_0^4} - \frac{\omega_0^2 + \omega_{pm}^2}{\omega_{pm}^2 - \omega_0^2}. \quad (12)$$

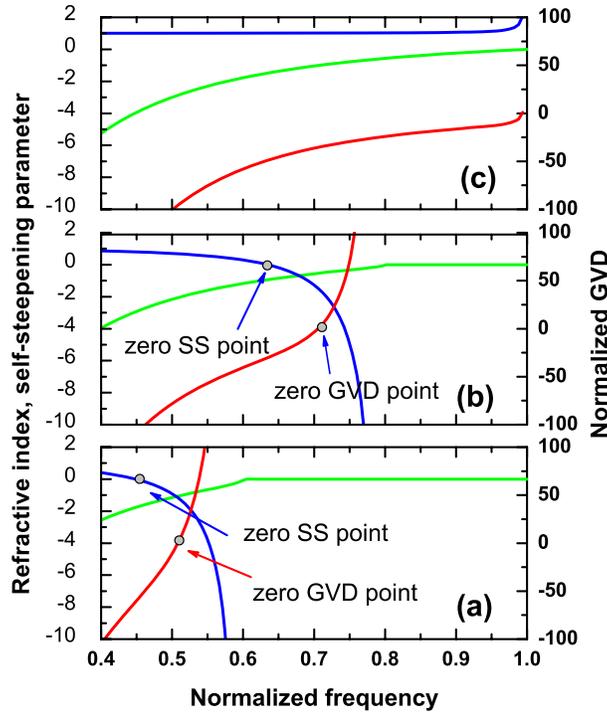


Fig. 1. Refraction index n (green lines), SS parameter C_s (blue lines), and GVD parameter β_2 (red lines), versus ω/ω_{pe} for $\omega_{pm}/\omega_{pe} = 0.6$ (a), 0.8 (b) and 1.0 (c). β_2 is calculated in units of $1/(2\pi c\omega_{pe})$.

Equation (9) is formally the same as that for nonlinear pulse propagation in ordinary positive-index materials. The noticeable anomaly in Eq.(9) is the SS parameter C_s . In its peer in ordinary positive-index materials, $C_s = 1$. To disclose the anomalous properties of the SS parameter, we plot the variation of n , C_s , and β_2 with ω/ω_{pe} for different values of the ratio ω_{pm}/ω_{pe} for $\gamma_e = \gamma_m = 0$ in Fig. 1. We see that, first, like GVD [10, 21], SS can also be engineered by choosing the size of split-ring resonator circuit elements. Second, the SS parameter can be negative under the condition of $\omega_{pm}/\omega_{pe} < 1$, with the zero SS point located at

$$\omega/\omega_{pe} = \left[1 - \left(1 - \omega_{pm}^2/\omega_{pe}^2 \right)^{1/2} \right]^{1/2}. \quad (13)$$

Third, in the positive SS region, the normalized SS parameter is less than 1; while in the negative SS region, its value can be far greater than 1, especially in the normal GVD regime.

The possibility of controlling the zero GVD point of a NIM can be exploited to obtain dispersion-free propagation in spectral regions otherwise inaccessible using ordinary positive-index materials [10, 21]. Similarly, by controlling the SS in NIM, we believe that we can obtain some new properties of the nonlinear pulse propagation, such as solitons and MI.

3. Role of the anomalous self-steepening in MI in NIM

In this Section we identify the role of the anomalous self-steepening in MI in NIM. For simplicity, we first introduce the normalized variables $T = \tau/\tau_p$, $Z = \xi/l_d$, and $U = A/A_0$, where τ_p is the width of the input pulse, $l_d = \tau_p^2/|\beta_2|$ the dispersion length, $l_{nl} = 1/(|C_{nl}|A_0^2)$ the nonlinear length, and A_0 the amplitude of the input field, to transform Eq. (9) into the normalized form:

$$\frac{\partial U}{\partial Z} = -i\frac{\delta}{2}\frac{\partial^2 U}{\partial T^2} + i\vartheta N\left(1 + iS\frac{\partial}{\partial T}\right)(|U|^2 U), \quad (14)$$

where $N = l_d/l_{nl}$, $S = C_s/(\tau_p\omega_0)$ is the normalized self-steepening coefficient, $\delta = \pm 1$ stands for normal and anomalous GVD respectively, and $\vartheta = \pm 1$ for focusing and defocusing nonlinearity respectively.

We then study MI in NIM based on Eq. (14). MIs in ordinary materials based on the similar nonlinear Schrödinger equation has been well understood [1, 3]. By a standard linear stability analysis, we can obtain the MI growth rate

$$g(\varpi) = \sqrt{-\frac{\varpi^2}{2}\left(2\delta\vartheta b + \frac{\varpi^2}{2}\right) - S^2 b^2 \varpi^2}. \quad (15)$$

where ϖ is frequency for perturbation, $b = NU_0^2$, U_0 is the normalized initial amplitude of the wave. Eq. (15) shows that MI in NIM appears for focusing nonlinearity ($\vartheta = 1$) and anomalous dispersion ($\delta = -1$), or defocusing nonlinearity ($\vartheta = -1$) and normal dispersion ($\delta = 1$), as in ordinary materials[1, 3]. The fact that negative refraction doesn't change the conditions for MI to appear is due to that although negative refraction alters the sign of wavenumber k_0 , it doesn't alter the sign of nonlinearity coefficient determined by the combination of signs of μ , n and $\chi^{(3)}$, as Eq.(11) shows. For NIM, μ and n are simultaneously negative, and thus the sign of nonlinearity coefficient is only determined by $\chi^{(3)}$, as in ordinary materials. From Eq.(15), we can obtain the critical frequency for MI to occur, and the fastest growing frequency for occurring MI

$$\varpi_c = \sqrt{4b - 4S^2 b^2}, \varpi_{\max} = \sqrt{2b - 2S^2 b^2}. \quad (16)$$

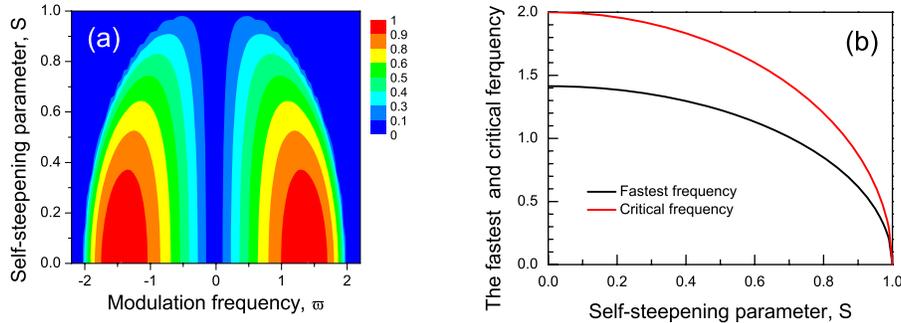


Fig. 2. (a) MI gain spectrum with different SS parameter S . (b) Variations of the critical frequency and the fastest frequency with self-steepening parameter S .

The role of the anomalous SS effect in MI in NIMs is clearly seen in Eqs.(15) and (16). Firstly, although the SS parameter can be either positive or negative, MI is irrespective of the sign of SS parameter. Secondly, for some frequencies, the value of SS parameter is very large, as Fig. 1 shows, possibly enough to make the MI growth rate and the critical frequency be

imaginary, meaning MI can't occur in NIM. Figure 2(a) is the contour plot of MI growth rate versus ϖ and SS parameter; and Fig.2(b) is the variation of the critical frequency and the fastest growing frequency with SS parameter. We see that the SS effect leads to the reduction of the gain and movement of the positions of the critical frequency and the fastest growing frequency to lower frequencies. As the SS parameter increases further to exceed a critical value, MI can't appear. As stated before, SS effect can be engineered, thus the MI can be manipulated. These results illustrate not only the unusual nonlinear effects that can be seen in NIMs, but also the new ways of manipulating solitons.

4. Numerical simulations

To further disclose the role of SS in MI and to confirm the obtained properties of MI in NIMs we numerically solve Eq.(14) using the standard split-step Fourier method for $\omega_{pm}/\omega_{pe} = 0.8$ in the case of focusing nonlinearity and anomalous dispersion. In this case, the SS parameter can vary from 0.867 to -1.5 , with the zero SS point located at $\omega/\omega_{pe} = 0.632$, as Fig. 1(b) shows. The initial field distribution is a cosinoidally modulated plane wave,

$$U(T, 0) = U_0 [1 + u_0 \cos(\varpi T)], \quad (17)$$

where U_0 is the initial amplitude of background wave which is set to be 1 in the following simulations, $u_0 = 0.05$ is the initial amplitude of modulation wave. In a laboratory experiment, noise in the system provides the seed from which instability frequencies begin to develop. However, for the numerical simulations, it is necessary to explicitly provide an initial seed at the frequency ϖ to stimulate a response (induced MI). In the following simulations, ϖ is chosen such that it is the fastest growing frequency in the case of $S = 0$.

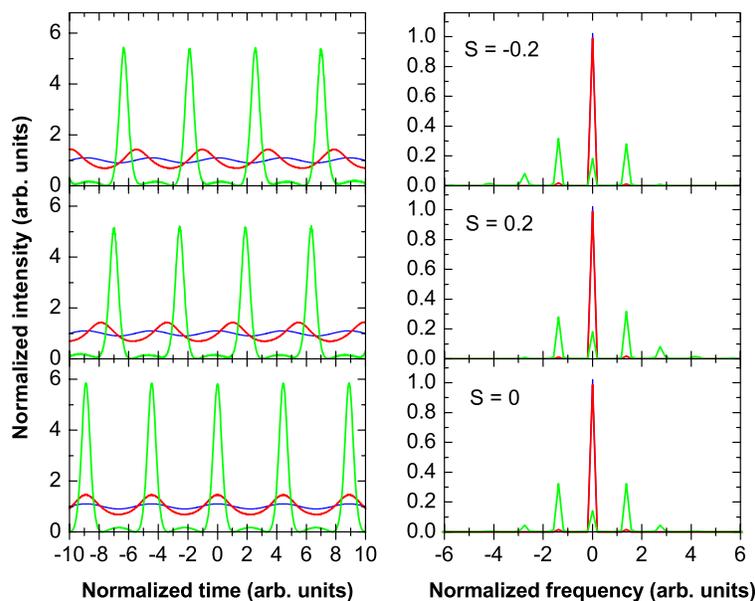


Fig. 3. Temporal (left column) and spectral (right column) distributions of the field intensity at different propagation distances for different SS parameters: bottom row: $S = 0$, medium row: $S = 0.2$, and top row: $S = -0.2$. The blue, red and green lines are for $Z = 0, 2$ and 4 , respectively.

Figure 3 shows the temporal and spectral distribution of the field intensity in the cases of

zero SS ($S = 0$), positive SS ($S = 0.2$) and negative SS ($S = -0.2$), at different propagation distances into the NIM. It is shown that in all the three cases, the cosinoidally modulated plane wave evolves into a train of pulses with much higher amplitude than the initial modulation. The period of the train equals $2\pi/\omega$. In addition, we can find, (i) compared to the zero SS case, both positive and negative SS slow down the growth rate of the modulation, in accordance with the prediction of the linear stability theory, and (ii) as the propagation distance increases, the negative SS moves the center of modulated wave or generated pulse toward the leading side, and shifts a part of energy of the generated pulse toward the red side, opposite to the case of positive SS. As is well known, MI is closely related to the phenomenon of soliton. The important difference between negative and positive SS effects in MI also manifests in solitons, as Fig. 4 shows. It clearly shows that, as the propagation distance increases, the second order soliton moves to the leading sides due to the role of the negative SS effect, opposite to the case of positive SS.

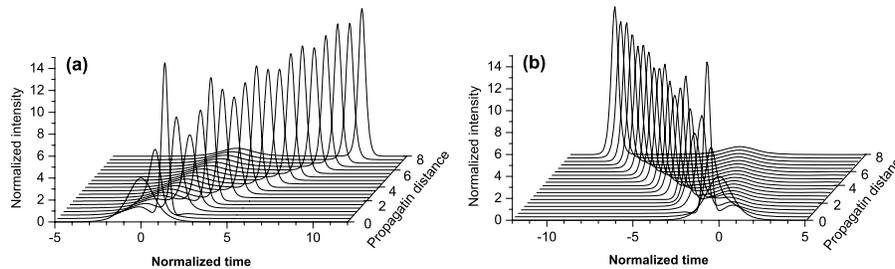


Fig. 4. Evolution of the 2nd-order soliton in (a) ordinary positive-index material for $S = 0.2$ and (b) NIM for $S = -0.2$.

5. Conclusion

We have derived a NLSE for ultrashort pulse propagation in NIM with Kerr nonlinearity. The derived equation is formally similar to the case of ordinary positive-index materials, but has an anomalous SS parameter: it can be either positive or negative, and can have a very large negative value. Based on the obtained NLSE, we have analyzed MI in NIM using the standard linear stability analysis. It is shown that the conditions for MI to occur in NIM are the same as in ordinary materials if SS effect are neglected. The sameness is due to the fact that although negative refraction reverses the phase velocity and thus the sign of the wavenumber, it doesn't alter the signs of dispersion and nonlinearity. The most notable property of MI in NIM is that it can be manipulated by engineering SS effect by choosing the size of split-ring resonator circuit elements. Numerical simulations confirms the linear analysis results and show that negative SS moves the center of generated pulse toward the leading side, and shifts a part of energy of the generated pulse toward the red side, opposite to the case of positive SS. The important difference between negative and positive SS effects in MI suggests that we can manipulate solitons in a new way.

About a month later after we had submitted this manuscript, we were reminded that Marklund et al. also studied the MI in metamaterials [23].

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