

Correlation properties and drift phenomena in the dynamics of vertical-cavity surface-emitting lasers with optical feedback

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Abstract: We investigate experimentally the polarization dynamics of vertical-cavity surface-emitting lasers with isotropic optical feedback operating in the long-cavity regime. By means of an analysis of the correlation properties in the time domain and in the frequency domain a connection between a drift phenomenon and frequency components that deviate from the harmonics of the external cavity round-trip frequency is revealed. The latter frequency components are shown to result from an interaction of external cavity dynamics and relaxation oscillations. An analogy to the carrier-envelope effect in mode-locked lasers is drawn. Similar drift phenomena are observed also for other laser systems with delay.

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References and links

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1. Introduction

In the past two decades, the dynamics of semiconductor lasers with optical feedback has attracted enormous interest (for an introduction and review see, e.g., [1, 2] and the contributions in [3]). This is on the one hand due to the technological importance of semiconductor lasers. On the other hand, a semiconductor laser with optical feedback is also a prominent example of a non-linear dynamical system with delay.

Whereas the majority of investigations was performed on edge-emitting semiconductor lasers, the dynamics of vertical-cavity surface-emitting lasers (VCSELs) with optical feedback is investigated only since a few years (see, e.g., [4, 5, 6, 7, 8, 9]). In contrast to edge-emitting lasers, VCSELs operate in only one longitudinal mode, but can exhibit two orthogonally polarized modes, assuming operation in only the fundamental transverse mode (see,

e.g., [10]). In the presence of feedback, these two polarization modes can exhibit rich dynamics [4, 5, 6, 7, 8, 9], where the specific dynamics that is observed depends on parameters like, e.g., feedback strength, operation current and the type of the feedback to that the VCSEL is subjected.

In one of the contributions mentioned above, namely in Ref. [8], a drift phenomenon in the dynamics of a VCSEL with polarized feedback has been found in experiments. The drift has been observed in the auto correlation function of the temporal dynamics of the dominant polarization mode. Drift phenomena are also reported for other systems with feedback for which the time scale of the delay is considerably larger than the intrinsic time scale of the system without feedback [8, 11, 12].

In this contribution, *isotropic* and delayed optical feedback is applied to a VCSEL. Whereas in Ref. [8] the polarized feedback has resulted in a strong suppression of one of the two orthogonally polarized fundamental transverse modes, in this contribution the presence of strong polarization dynamics will be beneficial for new insights into the drift phenomenon. More precisely, it will be shown that the drift is related to the existence of frequency components in the power spectra that deviate from the harmonics of the characteristic frequency of the external cavity. Such spectral components have already been observed for a VCSEL with isotropic feedback in previous investigations [5, 13] and also for edge-emitting lasers [14, 15, 16, 17]. However, in the latter publications this observation has not been commented on explicitly. It is common to all of the above investigations and also to the experiment reported here that the round-trip time τ_{ext} in the external cavity, which is composed of the output facet of the laser and the external source of reflection, is longer than the period of the relaxation oscillations (ROs), i.e., they are all systems with 'long' delay. Hence, it can be expected (although it will not be rigorously proven) that the results obtained here are relevant also for other semiconductor lasers with long feedback cavities.

In addition, we will demonstrate an analogy between the drift phenomena in lasers with feedback and the carrier-envelope slippage effect in mode-locked lasers.

2. Experimental results

The experiments have been performed on commercial gain-guided VCSELs from EMCORE Corp. (model 8085-2010) that operate in the fundamental transverse mode up to two times the threshold current. The lasers are subjected to isotropic feedback. To this purpose, a fraction of the light output is directed by use of a non-polarizing beam splitter towards a highly reflecting mirror. The light is focussed onto the mirror surface with an anti-reflection coated collimation lens. The longitudinal position of the lens and the tilt adjustments of the mirror are adjusted in an iterative procedure such that the threshold of the laser under study is minimized. The strength of the feedback is adjusted with neutral density filters. The external cavity frequency is $\nu_{ext} = 1/\tau_{ext} \approx 300$ MHz. The VCSEL output is split into its linear polarization components by use of a half-wave-plate and a Wollaston prism. The temporal dynamics of each polarization component is recorded simultaneously with avalanche photo diodes (1.8 GHz analogue bandwidth) and a digital oscilloscope (sampling interval 125 ps, 1 GHz analogue bandwidth). In some of the experiments an oscilloscope with 6 GHz bandwidth and a sampling interval of 50 ps was available. This oscilloscope was also used in combination with fast photo diodes of 10 GHz bandwidth but poor sensitivity. Therefore, the photo diode signals were amplified by 20 dB by the cost of cutting off frequency components below 10 MHz. Unintended back reflections into the laser are prevented by optical isolators that are located in each of the two beam paths that emerge from the Wollaston prism.

The first device that is investigated operates in the regime of the well known low-frequency fluctuations (see, e.g., [3]) for currents close to the threshold value of the solitary laser [cf.

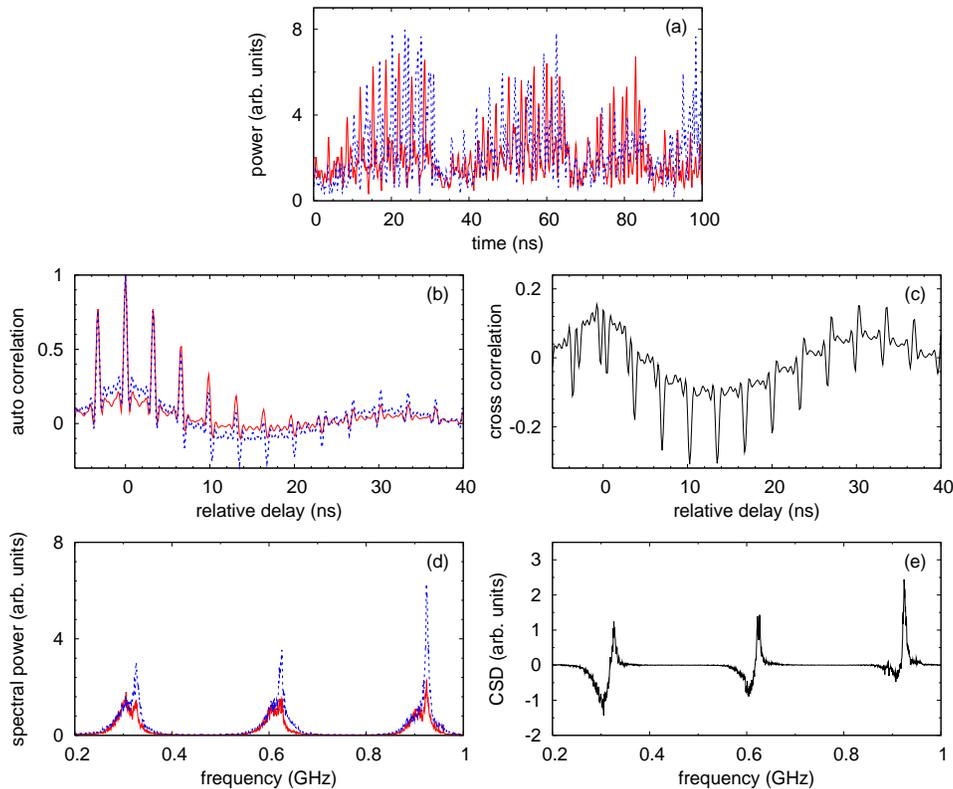


Fig. 1. Characterization of the dynamics of a VCSEL with isotropic optical feedback: (a) measured time traces, (b) auto correlation functions, (c) cross correlation function, (d) power spectra of the polarization modes, (e) cross spectral density (CSD). In panels displaying data for both polarization modes, the red and blue lines denote the data corresponding to the two orthogonal polarization modes. The injection current is set to the threshold of the solitary laser. The threshold reduction with feedback is 6%. The detection bandwidth is 1 GHz.

Fig. 1(a) and Refs. [7, 9, 13]]. Due to the weak intrinsic polarization anisotropies, both polarization modes participate in the dynamics with almost equal power. The temporal correlation functions are obtained numerically from the recorded time series by use of the Wiener-Chintchine theorem [18]. The spectral cross correlation or cross spectral density (CSD) is obtained from the two time traces $I_{x,y}(t)$ by Fourier transformation (denoted in the formula below by a tilde) and use of the relationship $CSD(|f|) = \tilde{I}_x(f) \cdot \tilde{I}_y^*(f) + \tilde{I}_x(-f) \cdot \tilde{I}_y^*(-f)$. This results in a real number for the CSD at a certain frequency of modulus f with a positive (negative) number indicating correlation (anticorrelation).

The auto correlation functions (ACFs) of both modes [Fig. 1(b)] exhibit a slow background modulation, which can be attributed to the presence of the low-frequency fluctuations [9]. A similar conclusion was made in Ref. [8] for the case of polarized feedback. Superimposed on the slow modulation are peak structures that are roughly located at multiples of τ_{ext} . The shape of these structures changes continuously, if the time delay in the ACFs is increased continuously: For small values of the delay, the structure is predominantly pointing upwards. With increasing delay time, the structure is first transformed such that it resembles the form of a

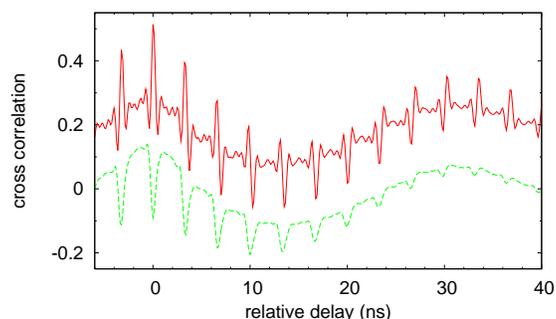


Fig. 2. Cross correlation function for the same parameters as in Fig. 1(c) but after elimination of the correlated (green line) and anticorrelated (red line) frequency components from the cross spectral density (see text for further explanations). The red line is raised by 0.2 units for better visibility.

dispersion curve [e.g., about 10 ns in Fig. 1(b)], until at approximately 20 ns the structure is predominantly pointing downwards. If the delay time is increased even further, the process is continued.

A slow transformation process of the peak structures at multiples of τ_{ext} was also observed recently for the dominant polarization mode in the case of polarized feedback [8]. There, this observation was associated with a drift phenomenon of systems with delayed feedback. It has also been visualized by applying a two-dimensional rearrangement of the ACF, following a procedure also used for other systems (e.g., [11]). An application of this procedure leads to comparable results in the case of the data presented here.

A drift is also observed in the cross correlation function (CCF) [Fig. 1(c)]. Here, seemingly two downward pointing peaks that are superimposed on the small envelope are observed close to zero delay. If the relative delay between the two time series is increased, these two peaks are transformed first into a single peak and then into a correlated peak, as it is observed also for the ACFs.

The observations made so far are best understood by analyzing the spectral properties. In the power spectra of the modes, two pronounced frequency components are observed at every harmonic of ν_{ext} and at slightly larger frequencies [cf. Fig. 1(d)]. These components are also present in the CSD [Fig. 1(e)]. The striking observation is that the dynamics at the exact harmonics of ν_{ext} is anticorrelated whereas it is correlated at the shifted frequency. This observation suggests the following interpretation of the CCF: At zero delay, a superposition of an upward pointing and a downward pointing peak is observed. Since the correlated part of the dynamics corresponds to the larger frequencies (the shorter time scale), the upward peak (correlated dynamics) and the downward peak (anticorrelated dynamics) run apart for increasing delay. Thus, the downward pointing peak is observed alone for a sufficient increase of the delay [about 17 ns in Fig. 1(c)]. If the delay is increased further, the positive peak moves continuously back into the negative peak from the direction of larger delay and a superposition of a correlated and an anticorrelated peak is observed again. For the ACFs of the modes, a similar argumentation is applicable.

The point of view taken here is confirmed by a comparison with the correlation properties close to the reduced threshold. There, the correlated frequency components are much weaker in comparison to the previous case (see also Ref. [13]) and a drift phenomenon cannot be resolved in the correlation functions.

The results obtained above strongly suggest that the observed drift phenomenon is related

to the presence of a comb of frequencies that deviates from the harmonics of ν_{ext} . As a further test of this hypothesis, the correlated components have been eliminated from the CCF displayed in Fig. 1(c). The procedure is as follows: The complex CSD is calculated by use of $CSD_{complex}(f) = \tilde{I}_x(f) \cdot \tilde{I}_y^*(f)$. The resulting function is set to zero at the frequency components near multiples of ν_{ext} which have a positive real part. Next, the complex CSD is Fourier transformed in order to obtain the modified temporal CCF. The result of this procedure is presented in Fig. 2 (green line). A double peak structure at zero delay and a transformation of the peak structure is not observed anymore, i.e., there is no drift. On the contrary, if the same procedure is applied to the components with a negative real part (anticorrelated components at the harmonics of ν_{ext}), the drift phenomenon is fully preserved (see red line in Fig. 2).

An analysis of the spectral components for different values of the injection current reveals that the frequency of the peak that deviates from the harmonics of ν_{ext} depends on the injection current (for details see also Fig. 3 of [13]). Furthermore, these shifted frequency components do only exist in a limited current interval, where the limiting values depend on the order of the harmonic. Similar observations are also made for the device investigated in Fig. 4, which suggests that the shifted, i.e., the correlated frequency components are related to some current dependent mechanism. In particular, the square of the frequency of the envelope maximum of the spectra is proportional to the injection current. The latter behaviour is typical for ROs [19] in free running semiconductor lasers and has been reported also for edge-emitting lasers with feedback [20].

ROs are well known to induce positively correlated dynamics of different laser modes in free running class-B lasers (see figures in Refs. [21, 22] and Ref. [23]) as well as in the presence of optical feedback [24]. Therefore, it is assumed that the shifted frequency components which correspond to the correlated dynamics of polarization modes are related to the ROs.

This conjecture has been tested in a measurement with a detection bandwidth of 6 GHz. The experiment was performed with another VCSEL that has an intermediate intrinsic polarization anisotropy and that exhibits strong polarization mode competition in the presence of feedback. Also in this case the observed spectra consist of a series of peaks that are separated by ν_{ext} (see Fig. 3). At several harmonics of ν_{ext} , again, the peaks have a doublet structure and coexistence of correlated and anticorrelated dynamics is observed. Each of the two kinds of dynamics is represented in the CSD by a frequency comb, where the frequency comb corresponding to the correlated dynamics is shifted to frequencies slightly larger than the exact harmonics of ν_{ext} .

Thanks to the large bandwidth also an envelope with a pronounced maximum is recognized [e.g., at 1.8 GHz in Fig. 3(a),(b)]. The (local) maximum of the envelope moves to higher frequencies [5.4 GHz in Fig. 3(c),(d)], if the current is increased. Furthermore, a pronounced 'low' frequency tail survives (frequencies less than 2 GHz).

The frequency of the (local) maximum of the envelope is associated with the RO frequency (similar findings about the shape of the power spectrum can be found for the case of an edge-emitter with feedback in [1]). As it is evident, the correlated components are strongest close to the RO frequency and the anticorrelated components are weakest there. The latter components are strong below the RO frequency [best seen in Fig. 3(d)] and far above this frequency [best seen in Fig. 3(b)]. Moreover, the positively correlated components are not detected at all at frequencies far from the RO frequency [high frequencies in Fig. 3(a),(b), low frequencies in Fig. 3(c),(d)]. This is a clear indication that the correlated, i.e., the shifted components near the harmonics of ν_{ext} are due to an interaction of external cavity dynamics and the ROs.

Finally, we mention that the observation of drift phenomena in the auto correlation functions is not depending on the occurrence of dynamics in both of the two polarization modes of the VCSEL, but occurs also in devices (or for parameter settings) in which one of the polarization modes is strongly favoured or even only one mode is lasing [cf. Fig. 4(a)]. This is not

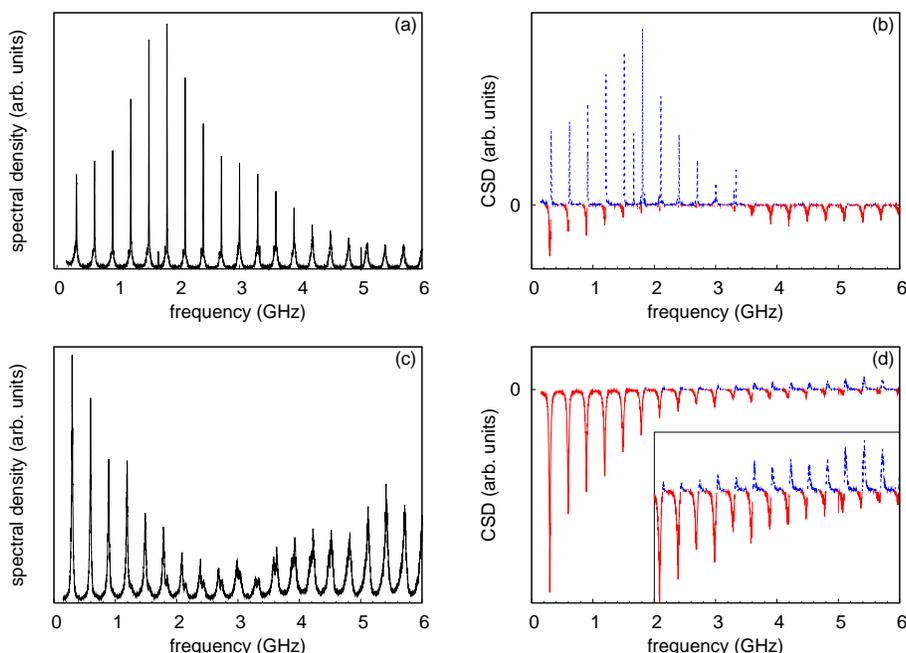


Fig. 3. Power spectrum of the dominant polarization mode and cross spectral density (CSD) of the dynamics of the both polarization modes for another device with a threshold reduction of 18% for injection currents 4% below (a,b) and 32% above (c,d) the threshold of the solitary laser, respectively. Red (blue) colour denotes anticorrelated (correlated) components. The inset in panel (d) is a magnification of the spectral components for frequencies larger than 2 GHz. The detection bandwidth is 6 GHz. The corresponding time series are amplified by 20 dB and the DC-component is cut off. The features near 1.7 GHz and 3.4 GHz in panel (b) are artefacts that are captured by the setup.

unexpected, since the drift was reported for operation in a single polarization mode for VCSELs with polarized feedback [8]. Our measurements show that also in these cases the power spectrum exhibits more than one frequency component at the harmonics of ν_{ext} [cf. Fig. 4(b)].

However, the multiplet at each harmonic has a different shape than in the case of pronounced polarization dynamics: A frequency component at the exact multiples of ν_{ext} is not discernable in the power spectrum. Instead, one weak component at slightly lower frequencies and a strong component at slightly larger frequencies is observed [see Fig. 4(b)]. If the injection current is changed, both peaks move further apart from each other, i.e., the deviation from ν_{ext} increases [see the inset in Fig. 4(b)]. At the reduced threshold (current value 0.92 in the inset of the figure), only one peak with a centre frequency of approximately ν_{ext} is discernable.

3. Discussion

The discussion in the preceding section has been restricted to cases where the multiplet at the harmonics of ν_{ext} consists of two peaks. There are parameter regimes and/or devices for which even more peaks are observed at the harmonics of ν_{ext} . Also in these cases, a coexistence of correlated and anticorrelated dynamics at nearby time scales is observed if polarization dynamics occur and the same conclusions on the relation of drift phenomena in the correlation functions and the components of the power spectra can be drawn. Therefore, the simplest situations that

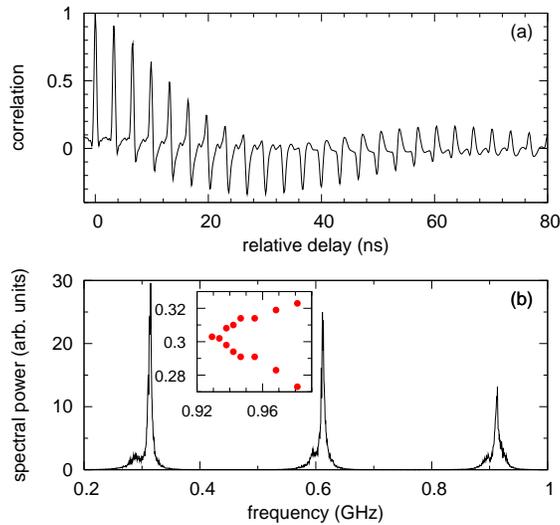


Fig. 4. Auto correlation function (a) and power spectrum (b) of the dominant polarization mode of another device with isotropic feedback. The inset in (b) displays the frequencies (in GHz) of the spectral components at the first harmonic of the external cavity frequency as a function of the injection current normalized to the threshold of the solitary laser. The threshold reduction is 8%. The current is set to 0.95 times the threshold of the laser without feedback. The detection bandwidth is 1 GHz.

have been found in the experiments have been chosen for presentation for the sake of clarity.

The observation of the drift phenomenon is not restricted to the dynamical regime of the low-frequency fluctuations, i.e., the drift in the correlation functions is also observed if the injection current is increased and the devices operate in the regime of fully developed coherence collapse (see also Refs. [7, 9]). However, one has to take care of the fact that the frequency range in which the multiplets are observed in the power spectra moves to higher values if the injection current is increased [see also Fig. 3(b)]. Therefore, the drift phenomena might easily be missed in an experiment, if the detection bandwidth is too low.

It has to be noted that multiplets at the harmonics of ν_{ext} have also been observed in experiments on edge-emitters with optical feedback with a *short* external cavity [25], where τ_{ext} is shorter than the RO period. There, the splitting between the multiplet components coincided with a pronounced low-frequency component of the observed dynamics, which indicates a wave mixing relation. However, such a relation can be excluded in the experiments reported here, since (i) the low frequency component does not exactly match the splitting of the multiplet components and (ii) the multiplets are also observed if no pronounced low-frequency components are present in the dynamics at all (see above and Ref. [13]). Furthermore, to the best knowledge of the authors there are no reports on drift phenomena in feedback systems with delay times shorter than the typical time scale of the system (which is the RO period in the case of a class-B laser).

The obtained results can also be interpreted within the framework of ultra-short laser pulses that are created by mode-locking (see, e.g., Ref. [26, 27, 28]): The pulses in the pulse train emitted by a mode-locked laser are separated by a certain period τ_{rep} , which is the round-trip time of the pulse in the laser cavity. In the frequency domain, there is a frequency comb with a separation $\nu_{rep} = 1/\tau_{rep}$ and an envelope, which is centred at the optical carrier frequency. It turns out that the actual value of the frequency components of the comb is not given by $m \cdot \nu_{rep}$

but by $\nu_{CEO} + m \cdot \nu_{rep}$, where ν_{CEO} denotes the so-called carrier-envelope offset frequency. The name originates from the fact that there is a phase slippage between the optical carrier wave and the pulse envelope, if $\nu_{CEO} \neq 0$.

There is a close analogy between these observations in mode-locked lasers and our findings for the VCSEL with delayed feedback: The harmonics of ν_{ext} in frequency space correspond to a frequency comb at $m \cdot \nu_{ext}$, m integer, that produces a train of pulses with a repetition period of τ_{ext} in the correlation functions. The dynamics that underlies the pulse envelope consists of oscillations at the frequency of the envelope maximum of the frequency comb. This envelope maximum is given by the RO frequency in the case of the correlated dynamics, i.e., the shifted frequency components that deviate from the harmonics of ν_{ext} . Also these components are separated by ν_{ext} . Thus, these frequencies constitute a frequency comb with frequencies $\nu_0 + m \cdot \nu_{ext}$, where ν_0 is the shift from the exact harmonic [e.g., $\nu_0 \approx 25$ MHz in the case of Fig. 1(e) and $\nu_0 \approx 21$ MHz in the case of Fig. 3(b)]. However, the fact that $\nu_0 \neq 0$ implies that the fast oscillations that underlie the pulse envelope (i.e., the *carrier signal* at the frequency of the ROs) suffer a continuous phase slip with respect to the repetition frequency ν_{ext} of the pulses [26]. This phase slip results in the observed drift phenomenon, i.e., in the continuous change of the pulse structure in the correlation functions.

We mention that in the mode-locked laser the difference between phase velocity and group velocity is at the origin of the phase slippage. Here, the velocity of the 'slower' pulses (at ν_{ext}) is still given by the group velocity of the light in the external cavity, where the frequency of the 'faster' pulsing (the RO frequency) is determined by an interplay of carrier and field dynamics. Hence, it is probably not surprising that there is an offset frequency, in general.

We caution that in contrast to - usually well-behaved - mode-locked lasers the dynamics in semiconductor lasers with feedback has strong irregular components [see, e.g., the time series in Figs. 1(a)] and that we are discussing the robust statistical features in long-time averages. Nevertheless, the results provide a further connection between mode-locking and feedback-induced dynamics, after mode-locking of external cavity modes has been proposed some time ago in order to explain the shortness of the pulses [29].

An important question is the question of the generality of the obtained results. In several theoretical works, components of the power spectrum that deviate from exact integer multiples of ν_{ext} have been predicted for coherent [30] as well as incoherent or opto-electronic feedback [31, 32, 33]. Since furthermore from a theoretical point of view the dynamics of class-B lasers with feedback is to some extent independent of the kind of the feedback [34], the results obtained here may be applied for other class-B lasers with feedback of a long delay time. However, definitive statements are difficult to obtain, since in none of the contributions cited here both the correlation functions *and* power spectra are discussed.

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