

# Design of a coarse WDM bandpass filter using the Thelen bandpass design method

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**Abstract:** A starting design for a prototype bandpass is modeled as a periodic structure, following the teachings of Thelen. A sample prototype bandpass contains twenty-one cavities and, after computer optimization, manifests a quasi-Chebyshev transmissive response in its passband. When the prototype is converted to a thin film multilayer bandpass, its spectral bandwidth is 12.70 nm at the  $-0.5$  dB transmittance level and 12.99 nm at the  $-25$  dB level.

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**OCIS codes:** Thin films (310.0310), Multilayer (230.4170), Fiber optics components (060.2340), Couplers, switches, and multiplexers (060.1810)

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## 1. Introduction

A coarse bandpass for Wavelength Division Multiplexing (WDM) typically manifests a spectral bandwidth that is usually at least ten times the bandwidth of the ITU 100 GHz channels. Its transmittance should descend precipitously to low levels at the spectral edges of its passband. It has been shown that the steepness of the aforementioned descent increases with the number of cavities.[1] A twenty-one cavity bandpass is chosen for that reason. The goal is a bandpass with a spectral bandwidth in the range of 12 nm to 13 nm, which corresponds to 0.8% at a central wavelength of 1550 nm.

The keystone of the design method is the prototype bandpass filter. It was invented four decades ago to design microwave bandpass filters.[2] As depicted in Fig. 1, the refractive indices of the prototype alternate from a fractional value to a value greater than one. Each layer (i.e., cavity) has a halfwave optical thickness. Once a prototype is established, the

standing wave ratio  $V_i$  at each of the  $i^{\text{th}}$  interfaces is known and it is a straightforward task to design a multilayer reflector that produces that  $V_i$ . The standing wave ratio is defined as

$$V = \frac{1 + \sqrt{R}}{1 - \sqrt{R}} \quad (1)$$

where  $R$  is the radiant reflectance. The bandpass design is complete after layers of halfwave optical thickness (i.e., the spacers) have been inserted between the reflectors.

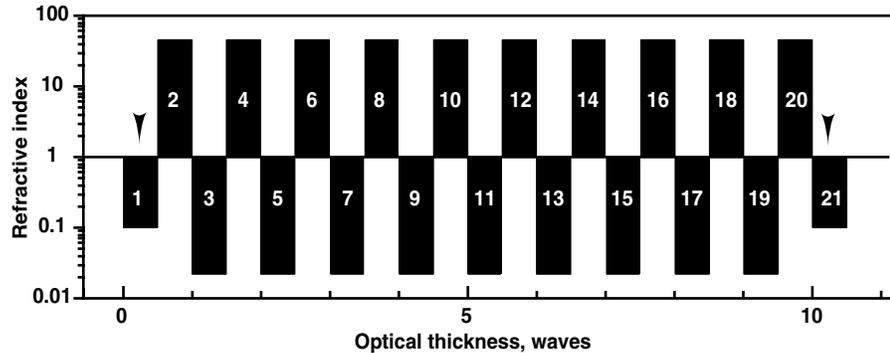


Fig. 1. Versus optical thickness in waves, refractive index (on log scale) of a starting design for a prototype bandpass. The antireflection layers are arrowed.

## 2. Starting design for optimization

More than three decades ago Thelen[3] developed a procedure for bandpass design in which a periodic structure is surrounded by antireflection layers. The stopband spectral region of that structure provides the rejection and a selected spectral region outside of the stopband is the passband. A quarterwave stack, which is discussed in standard texts on optical thin films.[4,5,6] is used as the periodic structure for the prototype bandpass. The aforementioned texts show that if a quarterwave stack is centered at a normalized frequency  $f$  of 1.0, the high frequency edge of the stopband extends to a frequency  $f_1$  of

$$f_1 = 1 + \frac{2}{\pi} \arcsin\left(\frac{V - 1}{V + 1}\right) \quad (2)$$

where  $V$  is the standing wave ratio at the interface between the low index and high index layers. The passband center is at a normalized frequency of 2.0. The fractional bandwidth BW (a dimensionless parameter) is

$$\text{BW} = (2 - f_1) = 1 - \frac{2}{\pi} \arcsin\left(\frac{V - 1}{V + 1}\right). \quad (3)$$

Inversion of the foregoing equation results in

$$V = \frac{1 + \cos\left(\frac{\pi}{2} \text{BW}\right)}{1 - \cos\left(\frac{\pi}{2} \text{BW}\right)}. \quad (4)$$

The phase dispersion[7] of the multilayer bandpass causes its bandwidth to be 3.5 times smaller than the bandwidth of the prototype bandpass. This factor of 3.5 is justified in a subsequent section. Given our target fractional bandwidth of 0.008, this requires that the prototype bandpass manifest a bandwidth of  $(3.5) \times (0.008) = 0.028$ . Substitution of this value into Eq. (4) shows that the standing wave ratio at the interfaces should be 2067. Figure 1 depicts the refractive index versus optical thickness of the prototype of the starting design. It is clearly a periodic medium—a quarterwave stack. The refractive indices alternate between a

maximum of 45 and a minimum value of 0.022, which produces a standing wave ratio at the interface of  $45/0.022 = 2045$ , which is reasonably close to the 2067 cited above.

The final step in the Thelen design method is to add two antireflection layers (arrowed in Fig. 1) to the exterior of the stack. Figure 1 shows that an index of 0.1 is used, which is not distant from the square root of 0.022.

### 3. Optimization

The refractive indices of the foregoing starting design are then adjusted by an optimization process, with the objective of improving the passband transmittance. The optical thicknesses remain constant. The first step is to locate the edge of the stopband. This is done by starting in the stopband frequency spectral region (which is lower than the frequency of the passband) and increasing the frequency until the derivative of the transmittance changes sign. This establishes the pseudo-edge of the stopband. Given the ripple in the transmittance in the passband, targets may be selected such that the transmittance emulates a Chebyshev polynomial of the first kind.

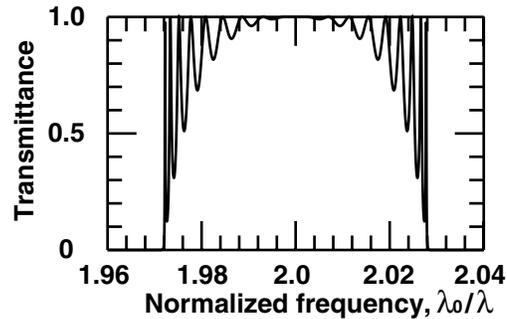


Fig. 2 Versus normalized frequency, transmittance of the prototype bandpass whose design appears in Fig. 1. At a frequency of 2.0, each layer is a halfwave in optical thickness.

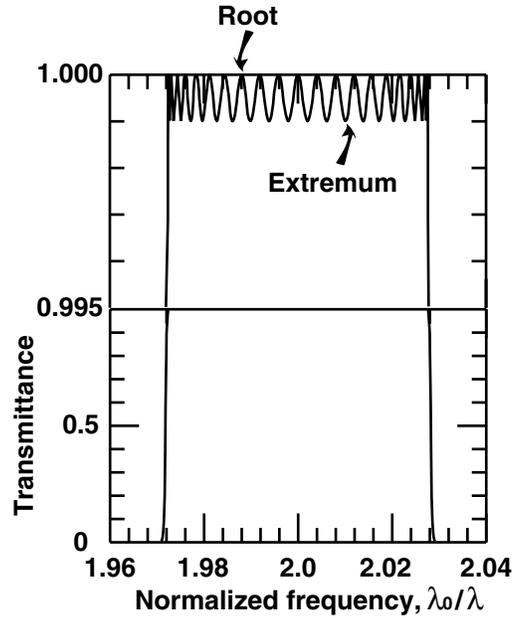


Fig. 3 Versus normalized frequency, transmittance of design  
*air A B C D E F G H J K L K J H G F E D C B A air*,  
 where the refractive indices of A, B, C, D, E, F, G, H, J, K and L are 0.05775, 32.935, 0.02422, 41.093,  
 0.02223, 42.815, 0.02174, 43.375, 0.02157, 43.568 and 0.02153, respectively. At a frequency of 2.0,  
 each layer is a halfwave in optical thickness.

Figure 2 depicts the transmittance of the starting design prototype and Fig. 3 the transmittance after a commercial software package[8]had optimized it. The arrows in Fig. 3 identify the optimization targets, which are the roots and extrema of a Chebyshev polynomial of order 21.

The foregoing procedure is repeated until a satisfactory design is achieved.

#### 4. Results

Figure 4 depicts the refractive indices of the prototype after optimization. The interior of the stack still resembles quite closely a periodic medium.

The final task is to convert the prototype into an actual multilayer design. The refractive indices at the  $i^{th}$  interface are known for the prototype. The standing wave ratio  $V_i$  at that interface may then be calculated. The task is to produce a multilayer stack that simulates  $V_i$ . The stack contains mostly layers of quarterwave optical thickness that are high index (symbol: **H**) and low index (symbol: **L**), with refractive indices of 2.065 and 1.47, respectively. The spacer layers of halfwave optical thickness are low index.

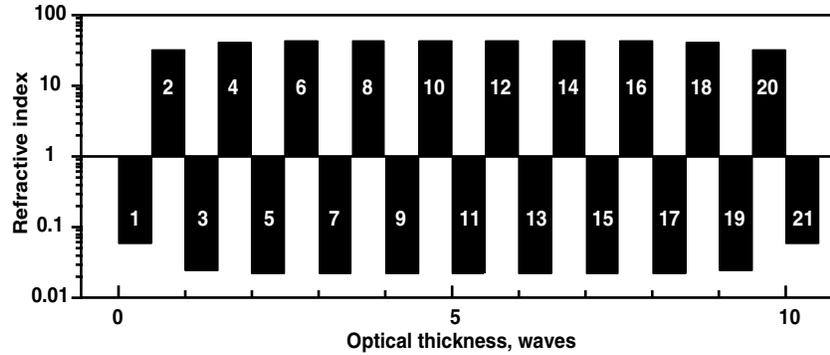


Fig. 4. Versus optical thickness in waves, refractive index (on log scale) of an optimized design for a prototype bandpass. Its design is captioned in Fig. 3.

An example is the outer reflector. Its required  $V_i$  is 17.3. The incident medium is *air* and the emergent medium index is 1.47, i.e., that of the low index spacer. Consider the following seven layer design

$$\text{air E (L H)}^3 \text{ low-index-pseudo-substrate}$$

where **E** is a layer of quarterwave optical thickness (or odd multiple thereof). The standing wave ratio  $V$  of this stack is

$$V = 17.3 = n_E^2 n_L^{-7} n_H^6 (1.0003)^{-1} \quad (5)$$

where the index 1.0003 for *air* is retained for pedagogical reasons. Manipulation of Eq. (7) shows that the refractive index of the **E** layer is 1.819. Standard computer codes for thin film design enable the user to synthesize an equivalent layer of that index. The result is the three layer sequence 0.278L 0.425H 0.278L. The target standing wave ratio for the outermost inner reflector is 570. Its design is

$$\text{low-index-pseudo-incident-medium (H L)}^4 \text{ H E H (L H)}^4 \text{ low-index-pseudo-substrate} .$$

The stack is bounded on its incident and emergent sides by media of refractive index 1.47. The standing wave ratio of this reflector is

$$V = 570 = n_E^{-2} n_L^{-18} n_H^{20} \quad (6)$$

The foregoing equation obtains when the **E** layer index is 1.842. As in the previous example, this **E** layer is replaced with an equivalent layer.

The aforementioned procedure is repeated until all 22 reflectors have been designed. Low index layers of halfwave optical thickness are inserted between each reflector. The design of the entire multilayer bandpass, containing 482 layers, appears in the next section. Figure 5 graphs its transmittance. Its spectral bandwidth of 12.70 nm may be narrowed by the use of second order spacers.

### 5. Effect of phase dispersion upon the bandwidth

The final step is to quantify the effects of phase dispersion on bandwidth of a bandpass filter. Consider a Fabry-Perot interferometer in which the reflectors (assumed to be identical) manifest a phase shift upon reflection  $\delta$ . The passband of the bandpass is centered at a wavelength  $\lambda_0$  and the spacer thickness is assumed to be first order. Abelès[9] showed that for coating materials in which the dispersion of the refractive index may be neglected, the slope of the phase shift of a quarterwave stack is

$$\frac{d\delta}{d\sigma} \approx \frac{-\pi \lambda_0 n_0}{n_H - n_L} \quad (7)$$

where  $n_0$ ,  $n_H$  and  $n_L$  are the refractive index of the medium in which the phase shift is measured, refractive index of the high index coating material and refractive index of the low index coating material, respectively.  $\sigma$  is the wave number. The foregoing equation obtains when the number of layers is substantial. The full spectral width of the passband  $\Delta\sigma$  of the Fabry-Perot is [10]

$$\Delta\sigma = F(R) (\pi \lambda_0 - d\delta/d\sigma)^{-1} \quad (8)$$

where  $F(R)$  depends upon the radiant reflectance of the reflectors. Substitution of  $n_L$  for  $n_0$  and combining Eqs. (7) and (8) leads to

$$\Delta\sigma = F(R) (\pi \lambda_0)^{-1} [1 + n_L/(n_H - n_L)]^{-1} . \quad (9)$$

Insertion of the refractive indices of 1.47 and 2.065 leads to the factor of 3.5 cited above. Although the foregoing equation obtains for a Fabry-Perot, empirical studies show that it may be applied to a multiple cavity bandpass, as well.

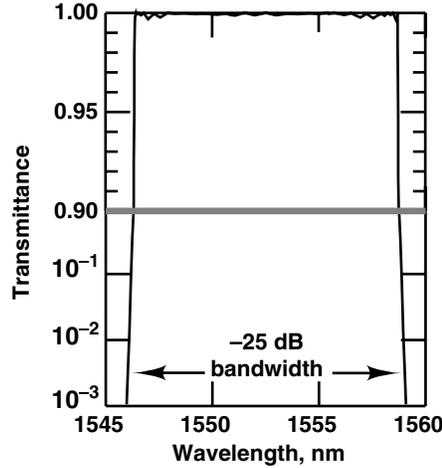


Fig. 5 Transmittance of the coating described in the section "Twenty-one cavity bandpass design." The scale of the ordinate changes from linear to log at 0.9.

## 6. Twenty-one cavity bandpass design

**H** and **L** represent layers of optical thickness  $\lambda_0/4$  at  $\lambda_0$  of 1552.5 nm and their refractive indices are 2.065 and 1.47, respectively. The substrate index is 1.50. The design is

*air* 0.278L 0.425H 0.278L (L H)<sup>3</sup> 2L (H L)<sup>4</sup> 1.387H 0.214L 1.387H (L H)<sup>4</sup> 2L (H L)<sup>5</sup> 0.284L 0.415H 0.284L (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 0.107L 0.774H 0.107L (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 0.522H 1.959L 0.522H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.458H 0.079L 1.458H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.447H 0.1L 1.447H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.441H 0.112L 1.441H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.437H 0.119L 1.437H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.435H 0.123L 1.435H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.434H 0.125L 1.434H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.434H 0.125L 1.434H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.435H 0.123L 1.435H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.437H 0.119L 1.437H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.441H 0.112L 1.441H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.447H 0.1L 1.447H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 1.458H 0.079L 1.458H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 0.522H 1.959L 0.522H (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 0.107L 0.774H 0.107L (L H)<sup>5</sup> 2L (H L)<sup>5</sup> 0.284L 0.415H 0.284L (L H)<sup>5</sup> 2L (H L)<sup>4</sup> 1.387H 0.214L 1.387H (L H)<sup>4</sup> 2L (H L)<sup>3</sup> 0.574H 1.859L 0.574H *substrate*.

## 7. Acknowledgment

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