Analytical expression for radiation forces on a dielectric cylinder illuminated by a cylindrical Gaussian beam

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Abstract: Radiation forces exerted upon a dielectric, circular-shaped cylinder of infinite length illuminated by a non-paraxial cylindrical Gaussian beam are considered. Vectorial projections of the radiation pressure force on a dielectric, arbitrary- and circular-shaped cylinder are expressed analytically. In particular, the radiation force is expressed through coefficients of the decomposition of the non-paraxial Gaussian beam into the cylindrical functions. Using numerical examples, a possibility to optically trap a circular-shaped cylinder in a non-paraxial cylindrical Gaussian beam is demonstrated.

OCIS codes: (260.1969) Diffraction theory; (260.2160) Energy transfer

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1. Introduction

Diffraction of the electromagnetic wave by a homogeneous sphere can be analyzed by use of the Mie theory. The generalization of the Lorenz-Mie theory onto the diffraction of a Gaussian beam and an arbitrary-shaped beam was discussed in Refs. [1-7] and [8], respectively. The rigorous electromagnetic calculation of the radiation pressure force exerted on a spherical microparticle illuminated by a Gaussian beam with the fifth-order non-paraxiality was considered in Refs. [9,10]. Note that in Refs. [1-7] the Gaussian beam was described using different approaches: Davis-Barton theory [9], standard beams [2], and localized approximation [5]. In Refs. [5,6] all three approaches were numerically compared. Note that the waist radius of the Gaussian beam in a Davis-Barton formulation [9] was much greater than the wavelength of light used. A tightly focused Gaussian beam can be generated by a spherical lens of high numerical aperture and aberration. The calculation of the radiation force pressure on a spherical microparticle located in the lens's focus was dealt with in Refs. [11,12]. Note, however, that in Refs. [11,12] the calculation was performed for the Rayleigh particles, that is, by use of the theory of the second-order scattering. The rigorous calculation of radiation forces exerted upon an arbitrary-radius spherical particle located in the focus of a converging beam with spherical aberration was reported in Refs. [13,14]. However, only the impact of the radiation pressure force acting along the optical axis was analyzed. Forces acting on a spherical particle located in the focus of a converging spherical wave were computer-simulated and rigorously calculated analytically in Refs. [15,16]. In Refs. [17,18], three methods for calculating the radiation pressure force were compared theoretically and numerically: geometric optics approximation, Rayleigh approximation, and rigorous method. Analytical expressions for the radiation pressure forces on a spherical particle with Kerr non-linearity located in the focus of the Gaussian beam were derived in Ref. [19]. Transfer of the angular momentum from a plane, circularly polarized electromagnetic wave to a spherical particle was reported in Ref. [20,7]. TE-polarization-specific analytical relations to calculate the diffraction of a non-paraxial 2D Gaussian beam by a circular dielectric cylinder were derived in Refs. [21,22]. In Ref. [23] the diffraction of the 3D Gaussian beam by an infinite cylinder with arbitrary location and orientation was numerically simulated.

In the present paper, we derive analytical relations and perform computer simulation for calculating the radiation pressure force on an infinite dielectric, circular-shaped cylinder located in the neighborhood of a non-paraxial 2D cylindrical Gaussian beam. In the 2D case, it becomes possible to rigorously describe a non-paraxial cylindrical Gaussian beam as a decomposition in terms of plane waves. The results derived below also apply to cylinders of finite length, provided that the length is much greater than the cylinder's diameter.

2. Radiation pressure force on a microobject

According to Ref. [24], the conservation of the total momentum of a system composed of the electromagnetic field and an object *V* limited by the surface *S* is given by

$$\frac{\partial}{\partial t}\int_{V_1} P_i dV + \frac{\partial}{\partial t}P_{0i} = -\oint_{S_1}\sigma_{ik} n_k dS\ , \tag{1}$$

where P_i are the coordinates of the vector of the electromagnetic field momentum density (V_1 and S_1 are a volume and a boundary surface that envelope the object $V \in V_1$), related to the Poynting vector, P_{0i} are the coordinates of the object momentum vector, $\frac{\partial P_{0i}}{\partial t}$ are the coordinates of vector of the radiation pressure force;

$$\sigma_{ik} = \frac{\varepsilon_0\varepsilon_1|\mathbf{E}|^2 + \mu\mu_0|\mathbf{H}|^2}{2}\delta_{ik} - \varepsilon_0\varepsilon_1 E_i E_k - \mu\mu_0 H_i H_k\ ; \tag{2}$$

σ_{ik} is Maxwell's stress tensor ($\sigma_{ik} = \sigma_{ki}$); $\mathbf{E}$, $\mathbf{H}$ are the vectors of the electromagnetic field strength in free space, and ε_1 is the permittivity of medium.

In the 2D case of TE-polarization and for monochromatic light the time-domain averaging over the period $T = \frac{2\pi}{\omega}$ (ω is the cyclic frequency) yields, in place of Eq. (1) ($F_x = 0$)

$$F_y = \frac{1}{2}\oint_{S_1}\left\{\frac{1}{2}\left[\mu\mu_0|H_y|^2 - \varepsilon_0\varepsilon_1|E_x|^2 - \mu\mu_0|H_z|^2\right]dz + \mu\mu_0 \operatorname{Re}\left(H_y H_z^*\right)dy\right\}, \tag{3}$$

$$F_z = \frac{1}{2}\oint_{S_1}\left\{\frac{1}{2}\left[\mu\mu_0|H_z|^2 - \varepsilon_0\varepsilon_1|E_x|^2 - \mu\mu_0|H_y|^2\right]dy + \mu\mu_0 \operatorname{Re}\left(H_z H_y^*\right)dy\right\}, \tag{4}$$

here S_1 is a contour that envelops the object cross-section in the YOZ-plane, μ is the permeability of medium (μ =1), and μ_0 is the permeability of vacuum. In the 2D case for TE-polarization ($H_x = E_y = E_z = 0$), the electric field is parallel to the X-axis: $E_x \neq 0$, *z* is the optical axis, with the 2D object presented by an arbitrary-shaped cylinder of infinite length along the X-axis. The YOZ-plane is where light propagates (see Fig.1).

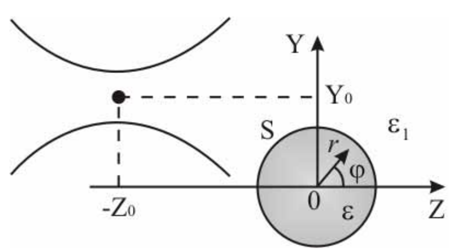

Fig. 1. The Gaussian beam with focus at (-Z_0, Y_0) falls on a circular cylinder with its center at (0;0).

3. Diffraction of a cylindrical non-paraxial Gaussian beam by a circular homogeneous cylinder

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Following Ref. [18], we will consider the diffraction of a 2D non-paraxial cylindrical Gaussian beam by a circular homogeneous cylinder. It is noteworthy that in the 2D case, the diffraction problem is solved independently for TE ($E_x \neq 0$) and TM ($H_x \neq 0$) polarization. In homogeneous space, the fields E_x and H_x satisfy the scalar 2D Helmholtz equation. Any function that satisfies the 2D Helmholtz equation can be represented by an integral decomposition into plane waves. Therefore, for TE-polarization, when (E_x^i, H_y^i, H_z^i) are non-zero, the electric field strength for the non-paraxial Gaussian beam can be given by

$$E_x^i(\rho,\varphi) = \frac{E_0\omega_0\sqrt{\pi}}{\lambda}\int_{-\infty}^{\infty}\exp\left[-\frac{k^2\omega_0^2q^2}{4} + ik\left(Z_0 p - Y_0 q\right) + ikr\cos\left(\varphi-\gamma\right)\right]dq\,, \tag{5}$$

where (ρ,φ) are the coordinates in the plane (Y,Z), $\gamma = \arcsin q$, $p^2+q^2=1$, $p=\cos\gamma$, and $q=\sin\gamma$, ω_0 is the waist's radius, E_0 = const, $k=\frac{2\pi}{\lambda}\sqrt{\varepsilon_1}=\frac{\omega}{c}\sqrt{\varepsilon_1}$ is the wavenumber in the medium, c is the light speed in free space, and λ is the wavelength. The series expansion of the electric field strength (5) in terms of cylindrical harmonics is

$$E_x\left(\rho,\varphi\right) = E_0\sum_{n=-\infty}^{\infty} i^n C_n J_n(kr)e^{in\varphi}\,, \tag{6}$$

$$C_n = \frac{\omega_0\sqrt{\pi}}{\lambda}\int_{-\infty}^{\infty}\exp\left[-\frac{k^2\omega_0^2q^2}{4} + ik\sqrt{1-q^2}\,Z_0 - ikqY_0 - in\arcsin q\right]dq\,, \tag{7}$$

where $J_n(x)$ is the Bessel function of the n-th order. From Maxwell's equation we go on to calculate the magnetic field strength, H_r and H_φ. Then, the magnetic field projection of the incident wave takes the form:

$$H_\varphi^i(r,\varphi) = iH_0\sum_{n=-\infty}^{\infty} i^n C_n J_n'(kr)e^{in\varphi}\,,\ J_n'(kr) = \frac{d}{d(kr)}J_n(kr)\,, \tag{8}$$

$$H_r^i(r,\varphi) = H_0\sum_{n=-\infty}^{\infty} i^n nC_n\frac{J_n(kr)}{kr}e^{in\varphi}\,,\ H_0 = \sqrt{\frac{\varepsilon_1\varepsilon_0}{\mu_0}}E_0\,. \tag{9}$$

Similarly, the expansions of the electromagnetic fields scattered outside the cylinder ($\vec{E}^S$, $\vec{H}^S$) functions are:

$$E_x^S = E_0\sum_{n=-\infty}^{\infty} i^n C_n^S H_n^{(1)}(kr)e^{in\varphi}\,,\ H_\varphi^S = iH_0\sum_{n=-\infty}^{\infty} i^n C_n^S H_n'^{(1)}(kr)e^{in\varphi}\,, \tag{10}$$

$$H_r^S = H_0\sum_{n=-\infty}^{\infty} ni^n C_n^S\frac{H_n^{(1)}(kr)}{kr}e^{in\varphi}\,,$$

where $C_n^S = a_n C_n$,

$$a_n = -\frac{k_1 J_n'(k_1R)J_n(kR) - kJ_n(k_1R)J_n'(kR)}{k_1 J_n'(k_1R)H_n^{(1)}(kR) - kJ_n(k_1R)H_n'^{(1)}(kR)}\,, \tag{11}$$

where also R is the radius of the cylinder's circular cross-section, $k=\frac{2\pi}{\lambda}\sqrt{\varepsilon}$, ε is the permittivity of the cylinder. Substituting Eqs. (5)-(11) into Eqs. (3)-(4), integrating over a circumference of radius $R'>R$ and tending the radius R' to infinity, we can obtain an analytical expression for projections of the radiation pressure force on the circular homogeneous dielectric cylinder ($\mu=1$):

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$$F_y = -\frac{i\varepsilon_0\varepsilon_1|E_0|^2}{k}\sum_{n=-\infty}^{\infty} C_n (C^*_{n+1}a^*_{n+1} + C^*_{n+1}a_n +$$

$$+ 2C^*_{n+1}a_n a^*_{n+1} - C^*_{n-1}a^*_{n-1} - C^*_{n-1}a_n - 2C^*_{n-1}a_n a^*_{n-1}), \quad (12)$$

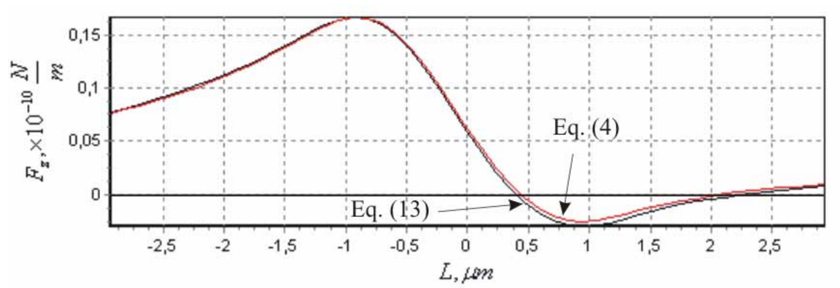
$$F_z = \frac{\varepsilon_0\varepsilon_1|E_0|^2}{k}\sum_{n=-\infty}^{\infty} C_n (C^*_{n+1}a^*_{n+1} + C^*_{n+1}a_n +$$

$$+ 2C^*_{n+1}a_n a^*_{n+1} + C^*_{n-1}a^*_{n-1} + C^*_{n-1}a_n + 2C^*_{n-1}a_n a^*_{n-1}). \quad (13)$$

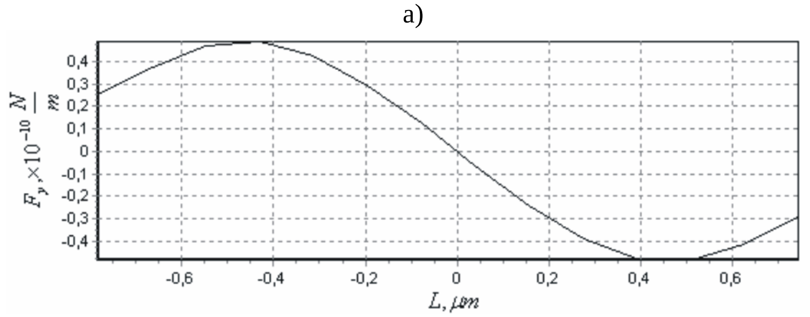
Note that for TM-polarization the relationships for the force projections are identical to Eqs. (12) and (13) for TE-polarization. We only need to replace the coefficients in Eq. (11) with the similar Mie coefficients for TM-polarization.

4. Numerical simulation

To compare computation results for the radiation forces on a microcylinder derived from Eqs. (9), (10) and from Eqs. (19), (20), we computed the radiation force on a circular microcylinder near the Gaussian beam focus at the following parameters: wavelength is $\lambda = 1\,\mu m$, $\varepsilon = 1.2$, the particle diameter is $2\,\mu m$, the Gaussian beam waist's diameter is $1\,\mu m$, and the power is 100 mW/m. It should be noted that in our case the confinement factor $S=1/(k\omega_0)=1/\pi$ (Ref. [5]) is not a small quantity, and, thus, the Gaussian beam can not be described in the Davis-Barton approximation [9]. Figure 2 shows the forces on a microparticle against the displacement L, acting along (F_z) and transversely to (F_y) the light propagation axis. The displacement L is the spacing between the beam's waist and the cylinder's center. At L>0, the cylinder's center is located in the diverging beam and at L<0 the cylinder is in the converging beam (note that in Refs. [5,6] the opposite was the case). The discrepancy between the corresponding plots for the F_z is less than 5%. In calculation, all the series are truncated to the first 15 terms. Figure 2 demonstrates the optical trapping of a dielectric circular cylinder in a single Gaussian beam.



a)



b)

Fig. 2. The Z-axis (a) and Y-axis (b) projections of the pressure force on a circular cylinder $\varepsilon = 1.2$ by the Gaussian beam (medium permittivity is $\varepsilon_1 = 1$).

From Fig. 2(a) it is seen that the maximal absolute value of the reverse radiation pressure [5,6] is attained when L is close to the cylinder's radius R=1 μm. This conclusion agrees with the results reported by other researchers (see Fig. 8 in [5], Fig. 4 in [6] and Table 2 in [14]). Also, from Fig. 2 it follows that for the beam power of 0.1W/m the order of the force is 10^{-10} N/m, which is equivalent to the 3D situation when for the beam power of 0.1W and sphere radius of R=1 μm the order of the force is 10^{-10} N. This is in compliance with Fig. 2 in Ref. [19].

5. Conclusions

Summing up, as a result of this work we have: (A) deduced relationships for the radiation pressure force (in particular, for a non-paraxial cylindrical Gaussian beam with TE-polarization) exerted on an infinite dielectric, circular-shaped cylinder for: an arbitrary radius of integration (Eqs. (3)-(4)) and infinite radius of integration (Eqs. (12)-(13)) and (B) numerically demonstrated that the dielectric circular cylinder can be optically trapped in a single cylindrical Gaussian beam, providing restrictions are imposed on the cylinder's permittivity (Fig. 2).

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